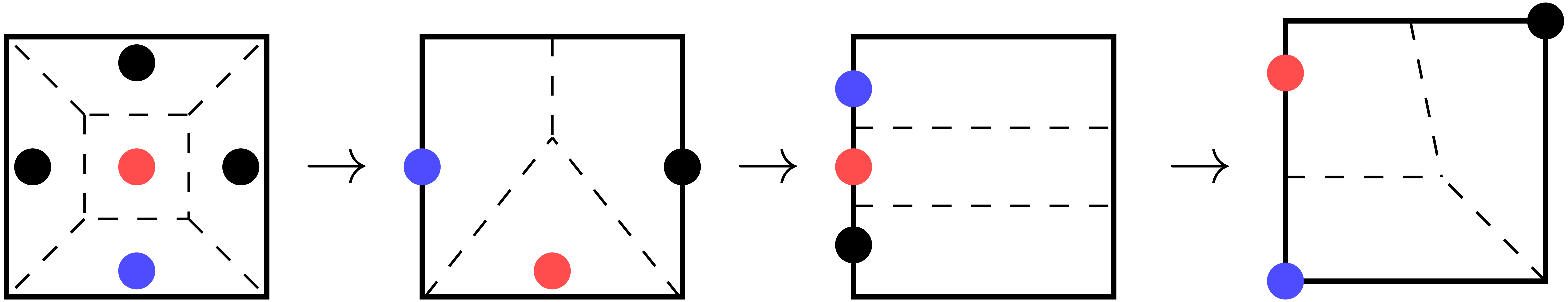
AAAI 2026



Exclusion Zones of Instant Runoff Voting

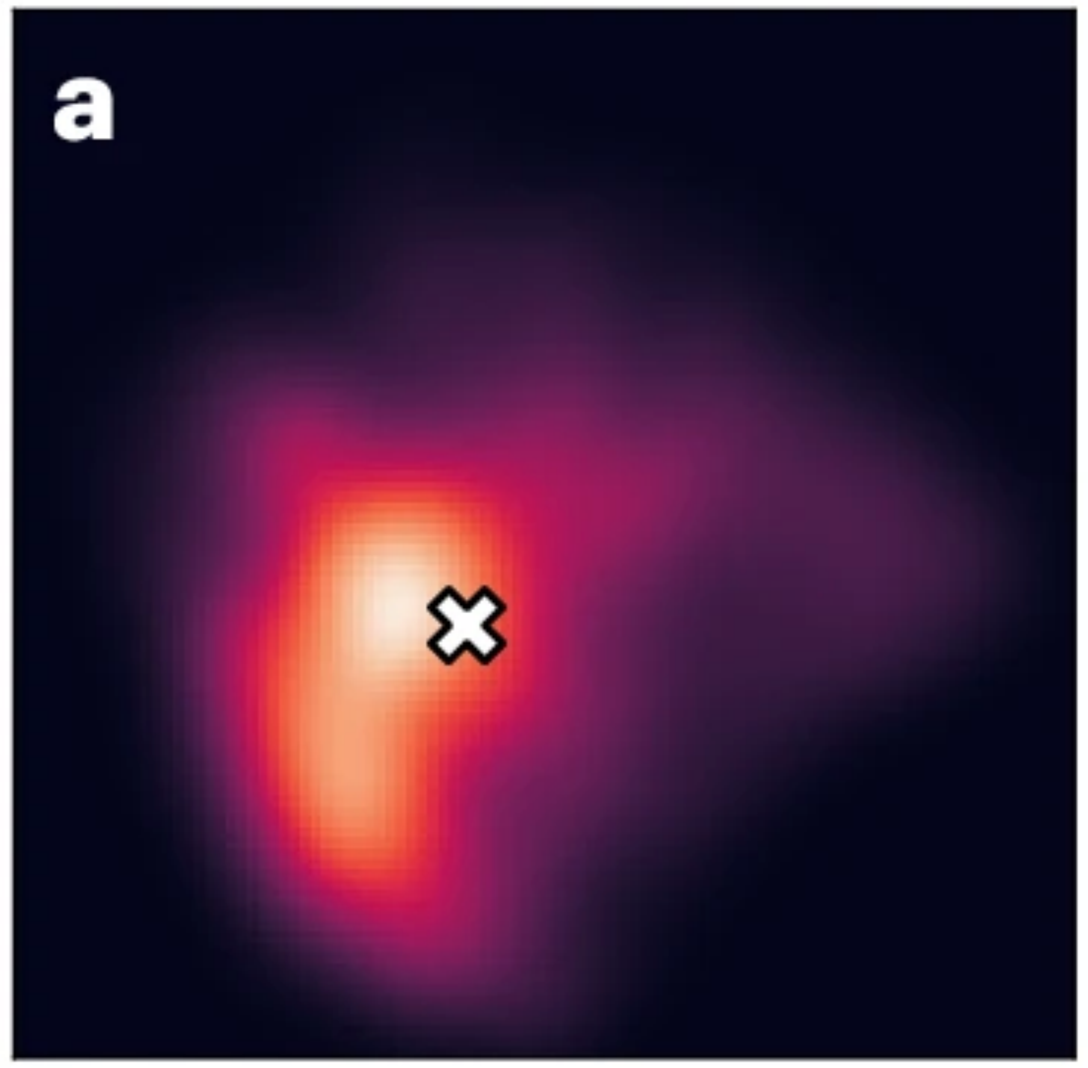
Kiran Tomlinson
Microsoft Research

Johan Ugander
Yale University

Jon Kleinberg
Cornell University

Given a distribution of voters in a metric space, what regions in the space does a voting algorithm favor?

Pew Research Center, 2004



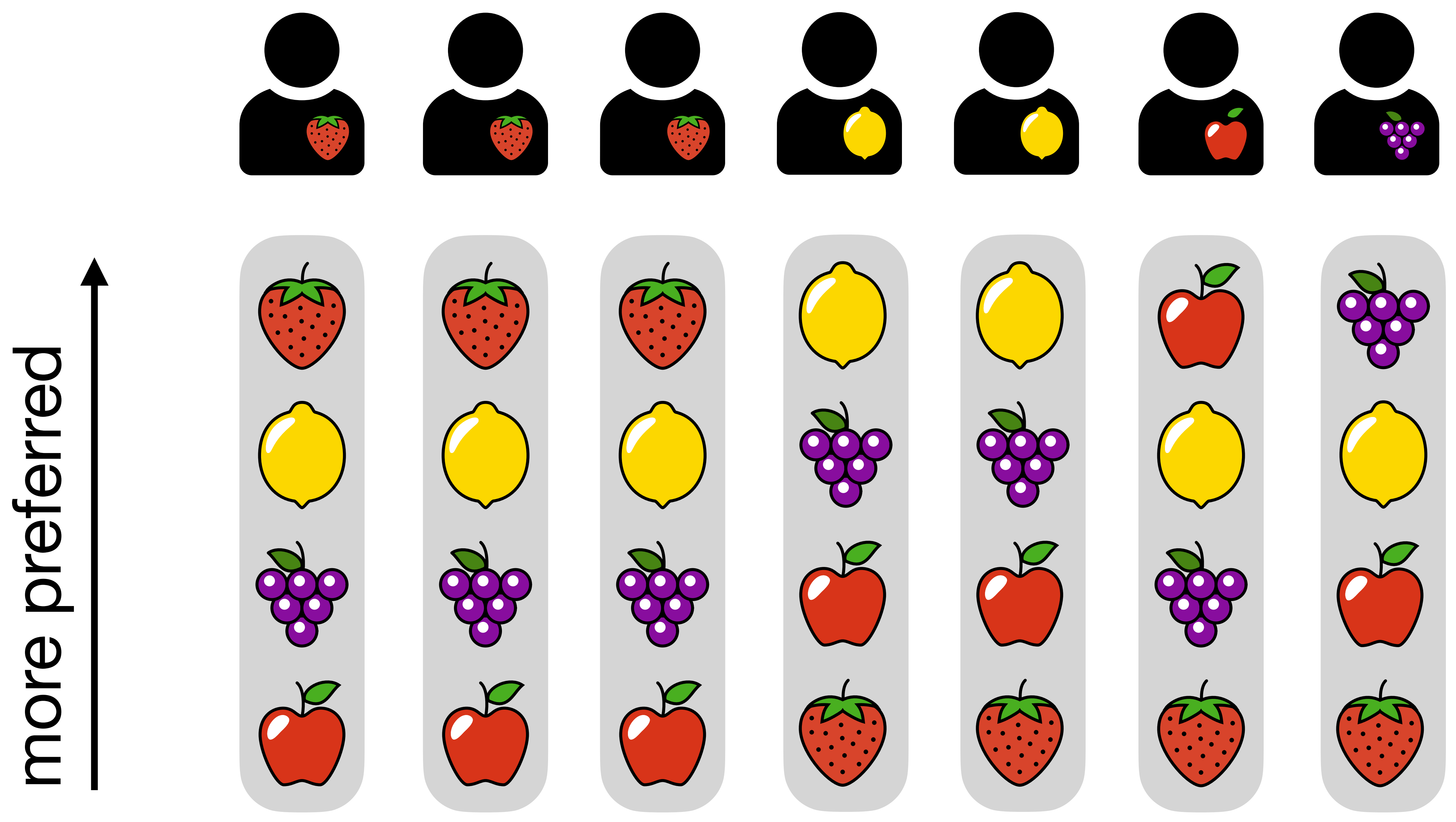
Ojer et al, *Nature Human Behavior*, 2025

E.g., will a given voting algorithm tend to elect moderates?

Instant runoff voting (IRV)

repeatedly eliminate the candidate with fewest first-place votes

Elimination order



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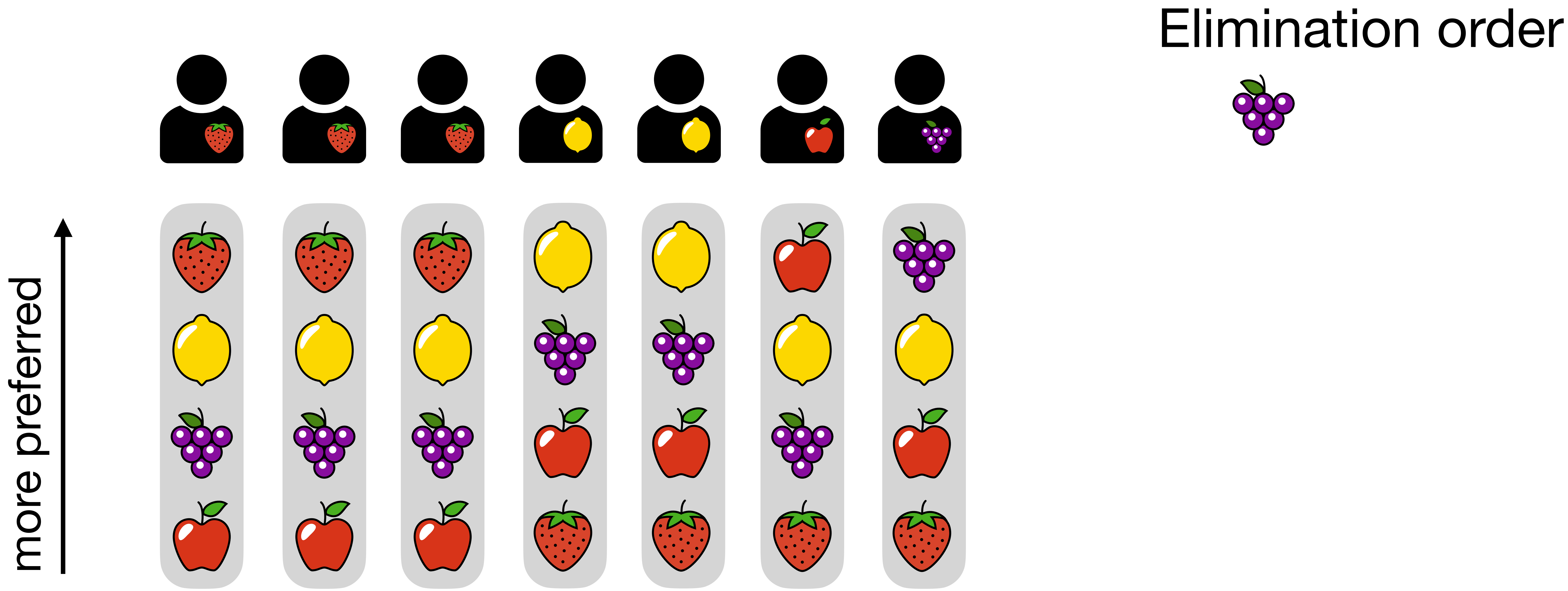

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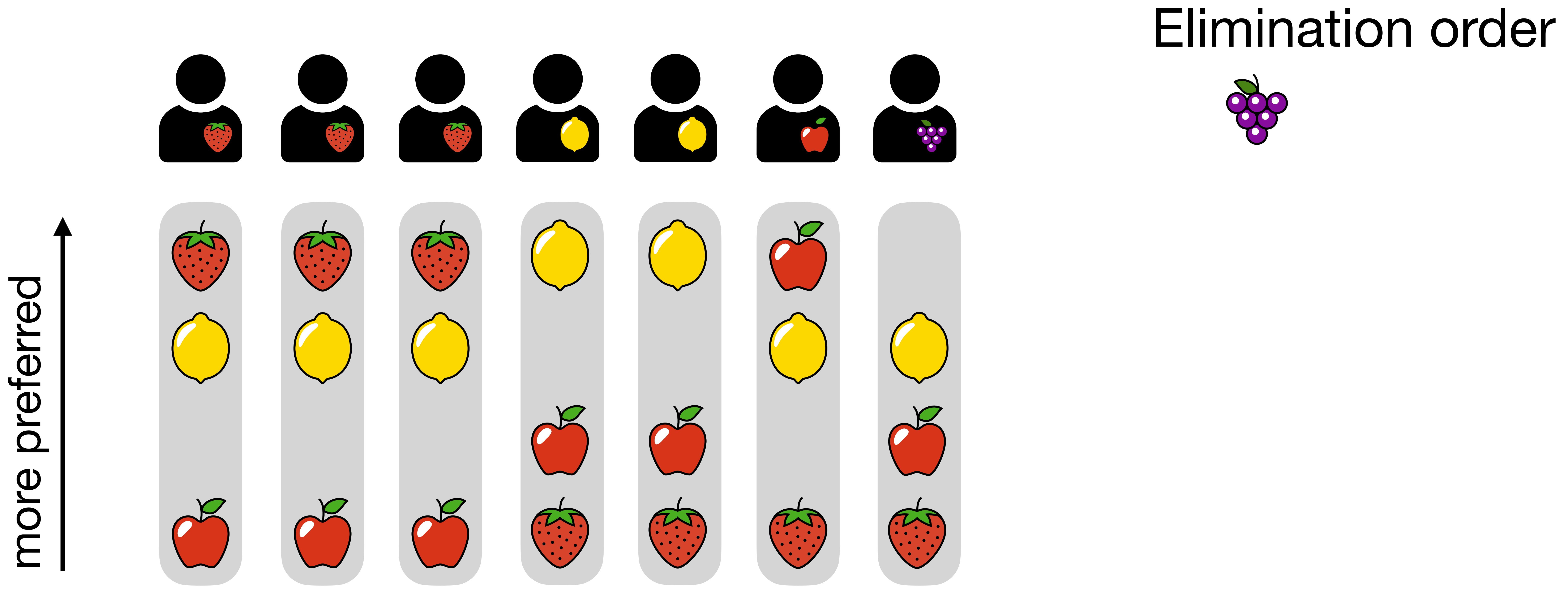

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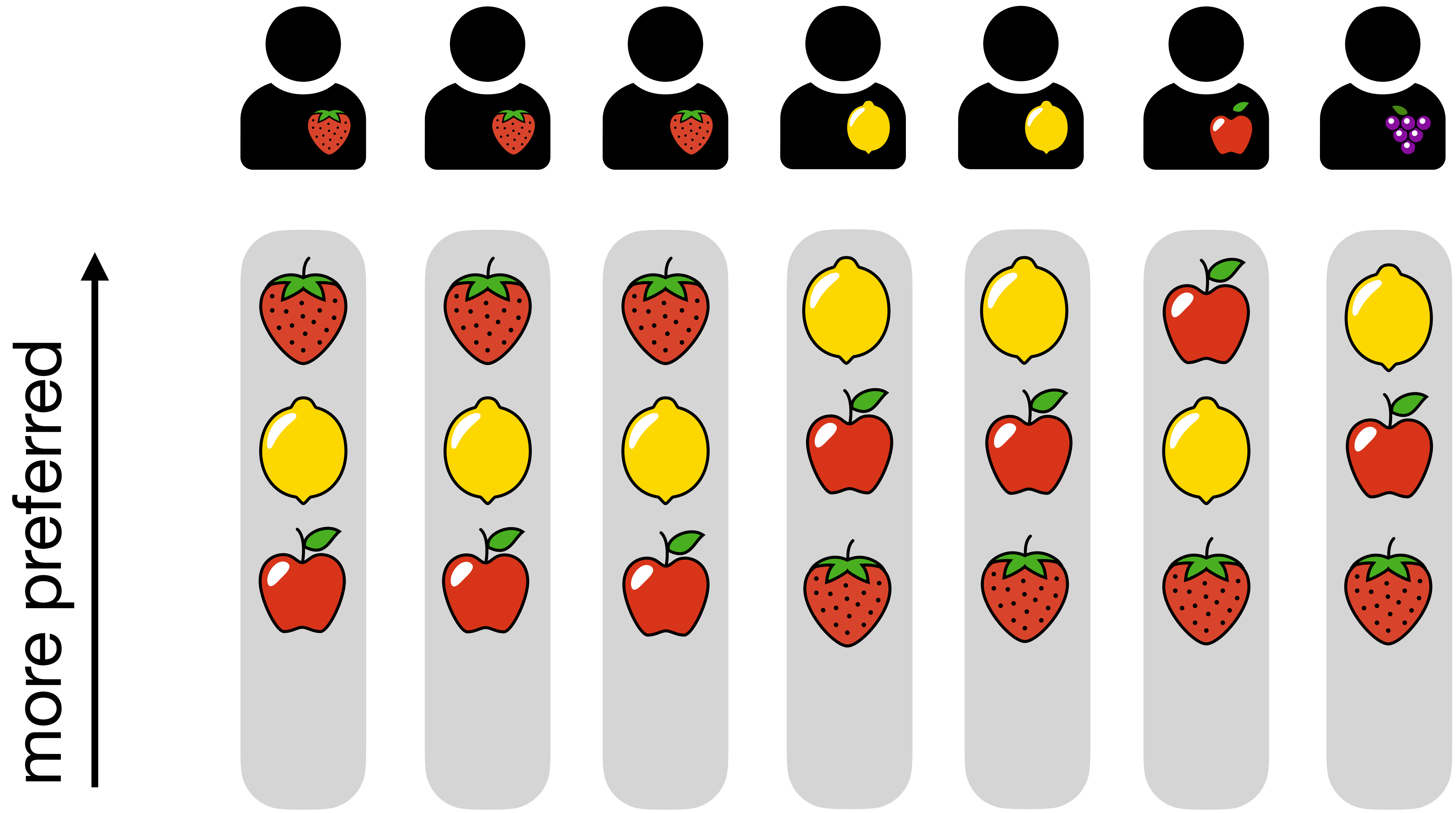

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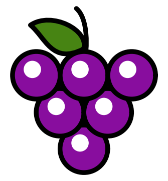

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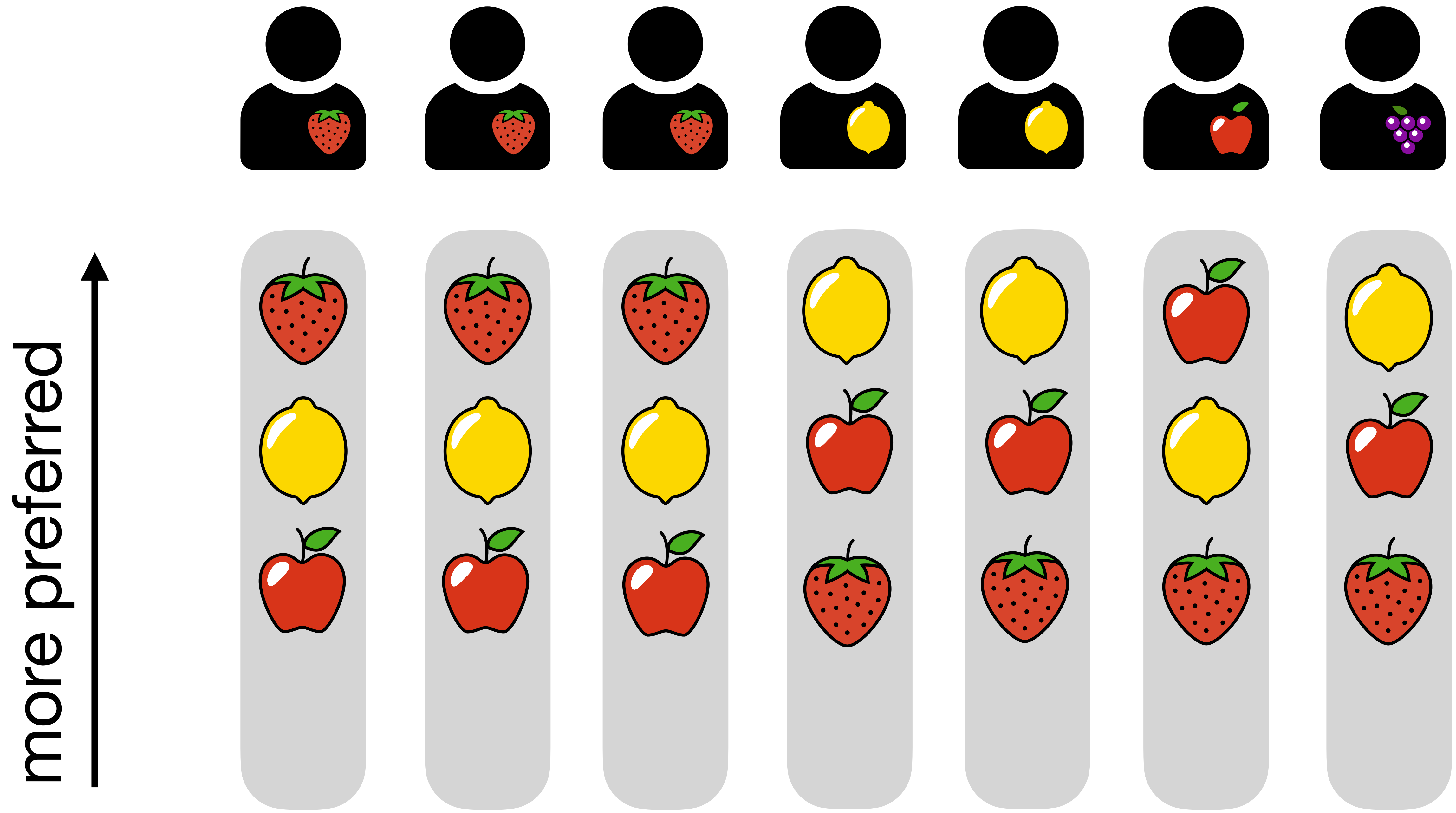

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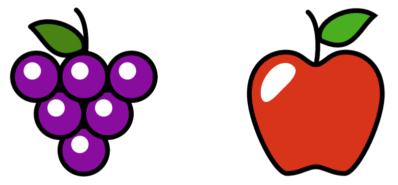

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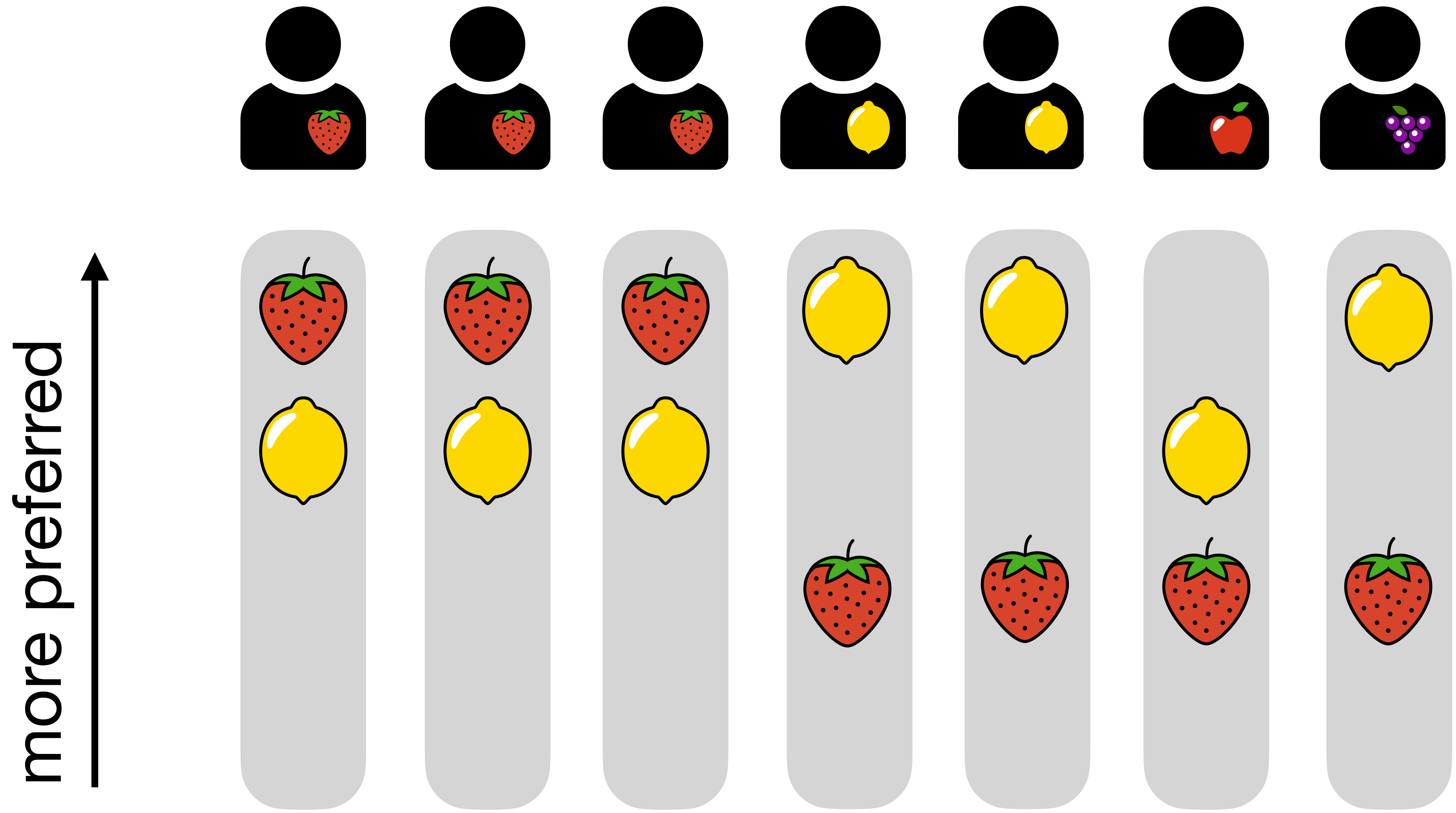

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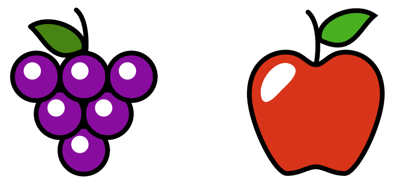

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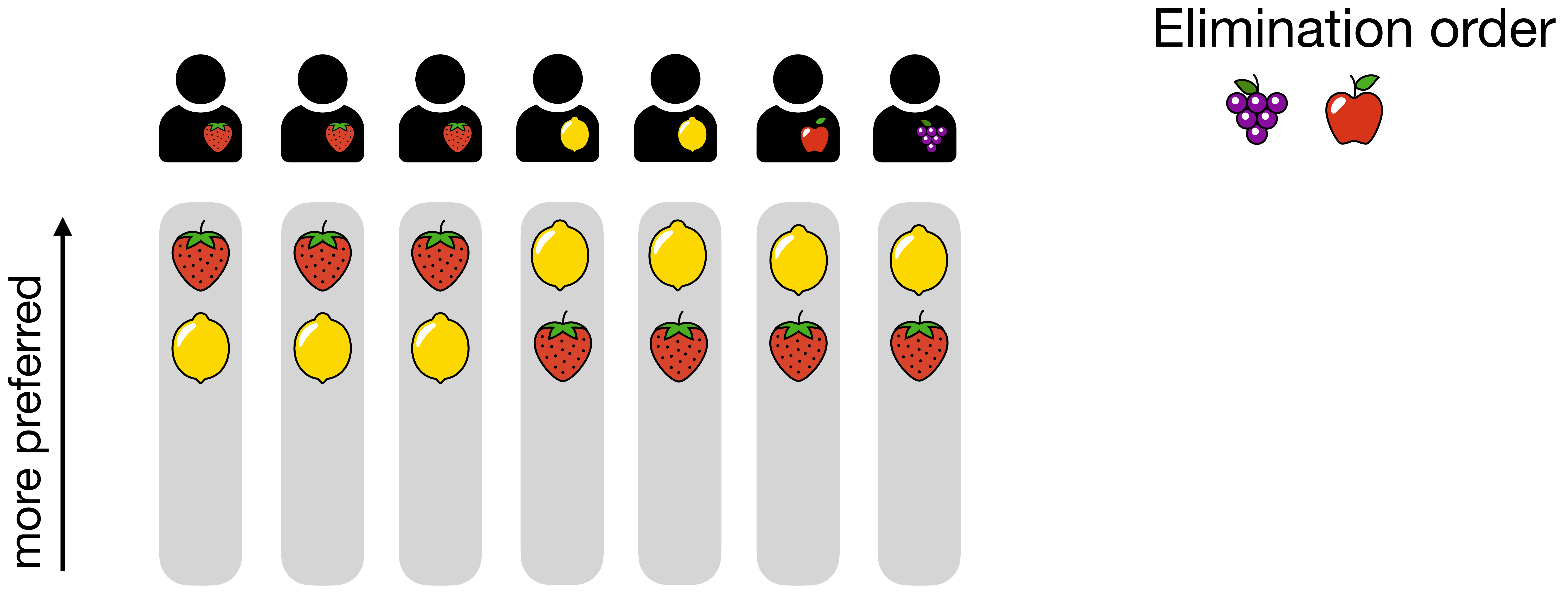

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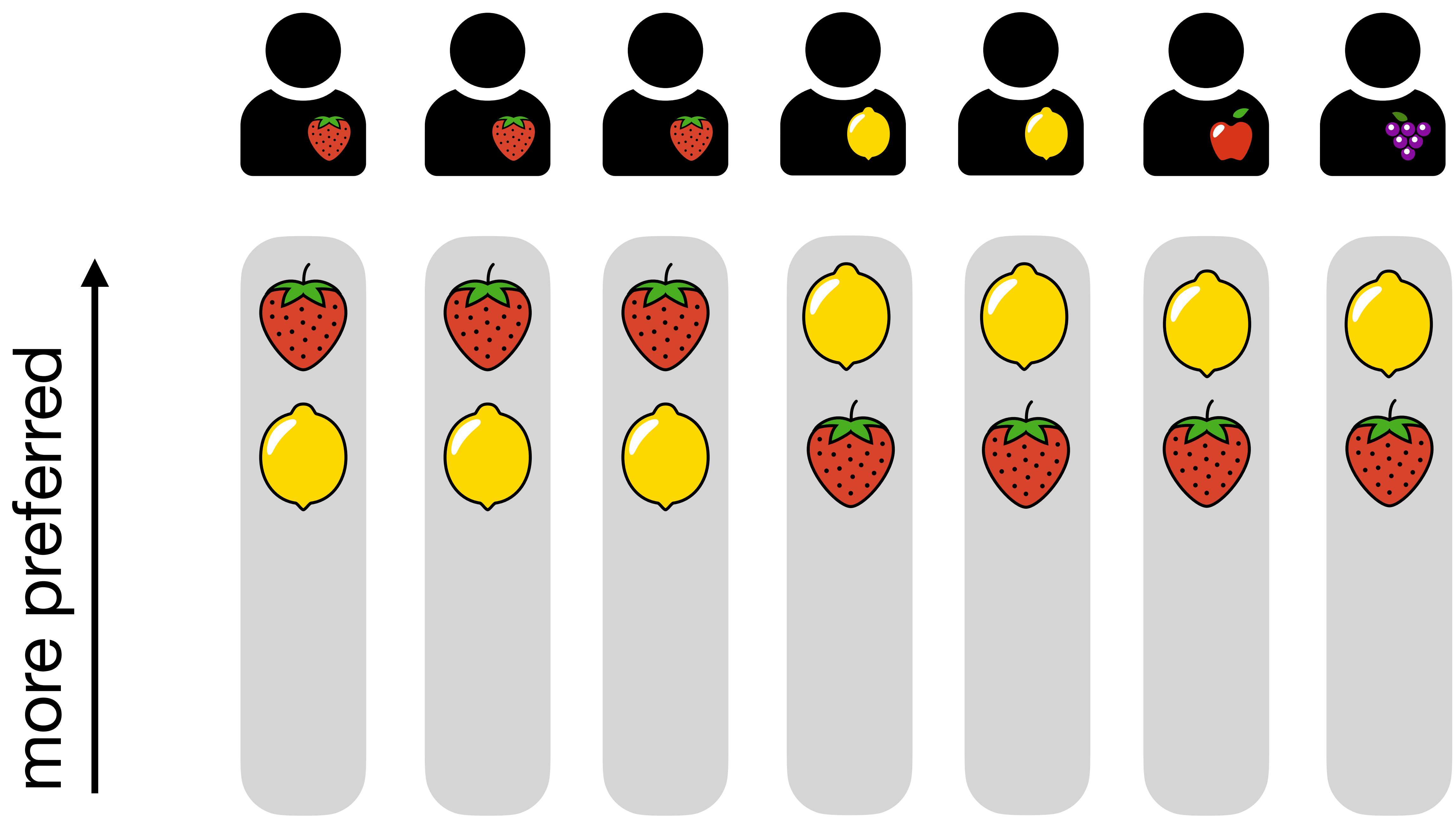

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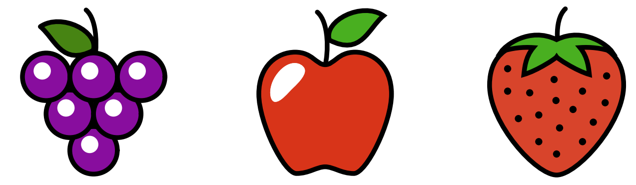

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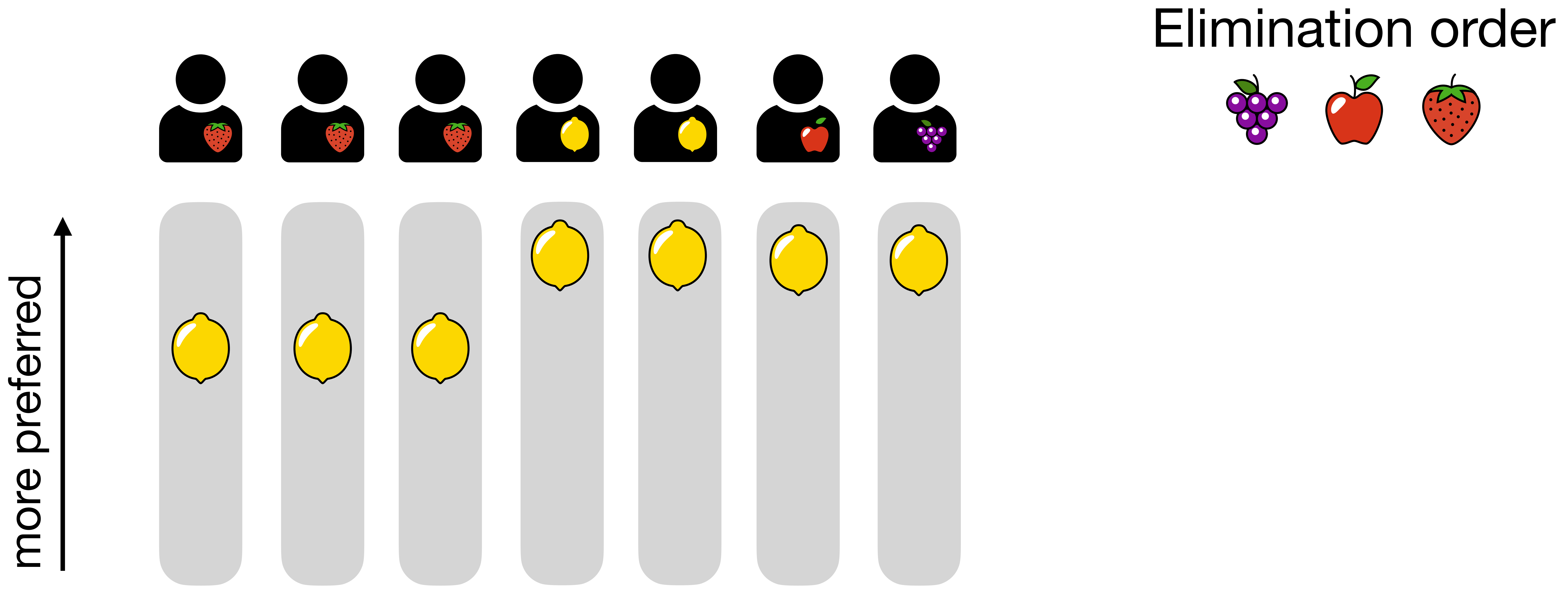

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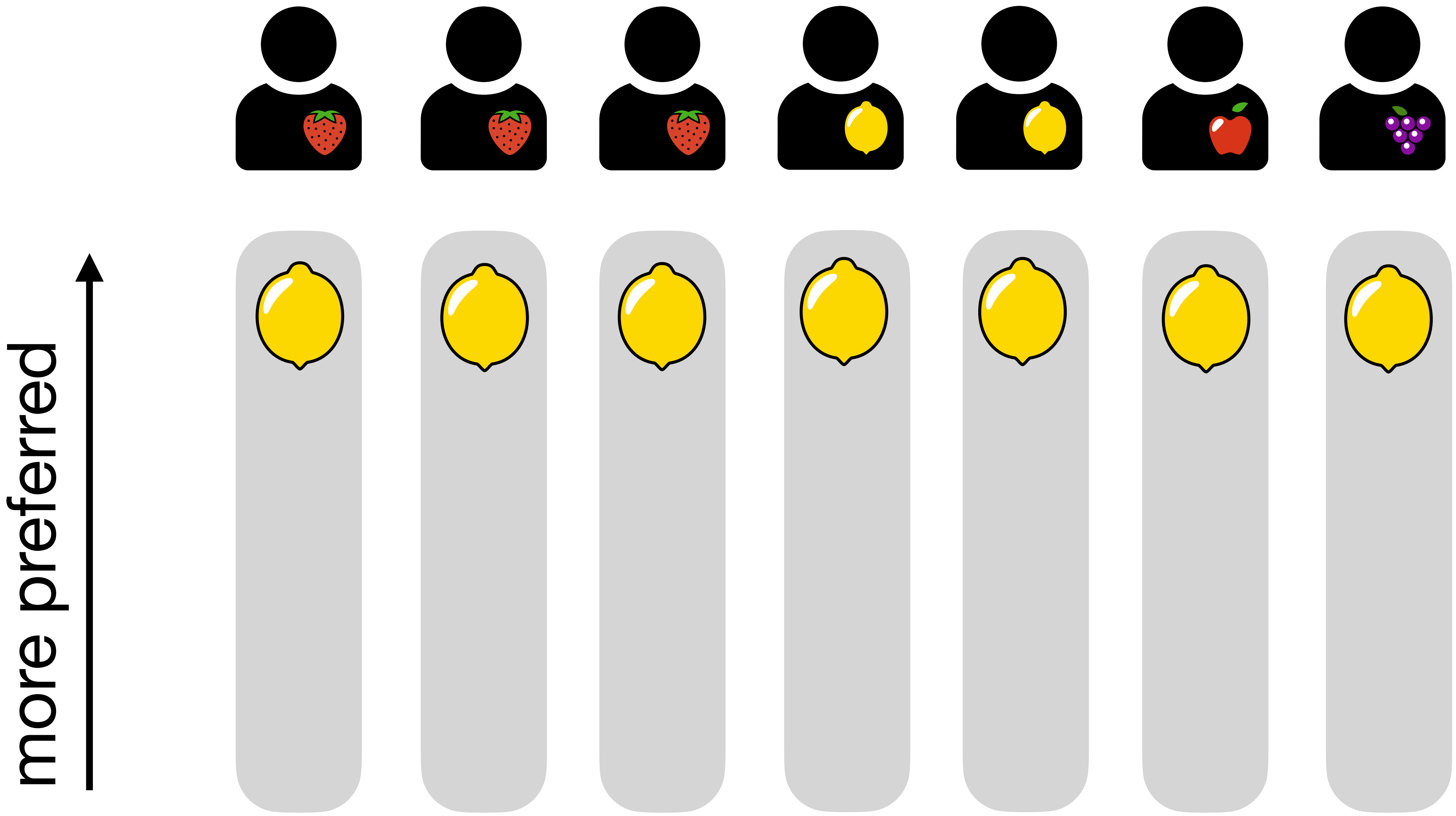

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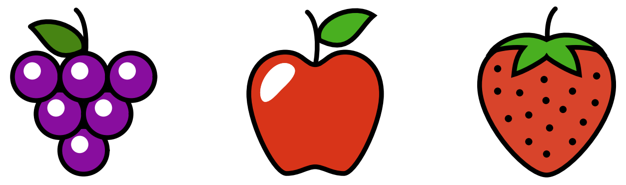

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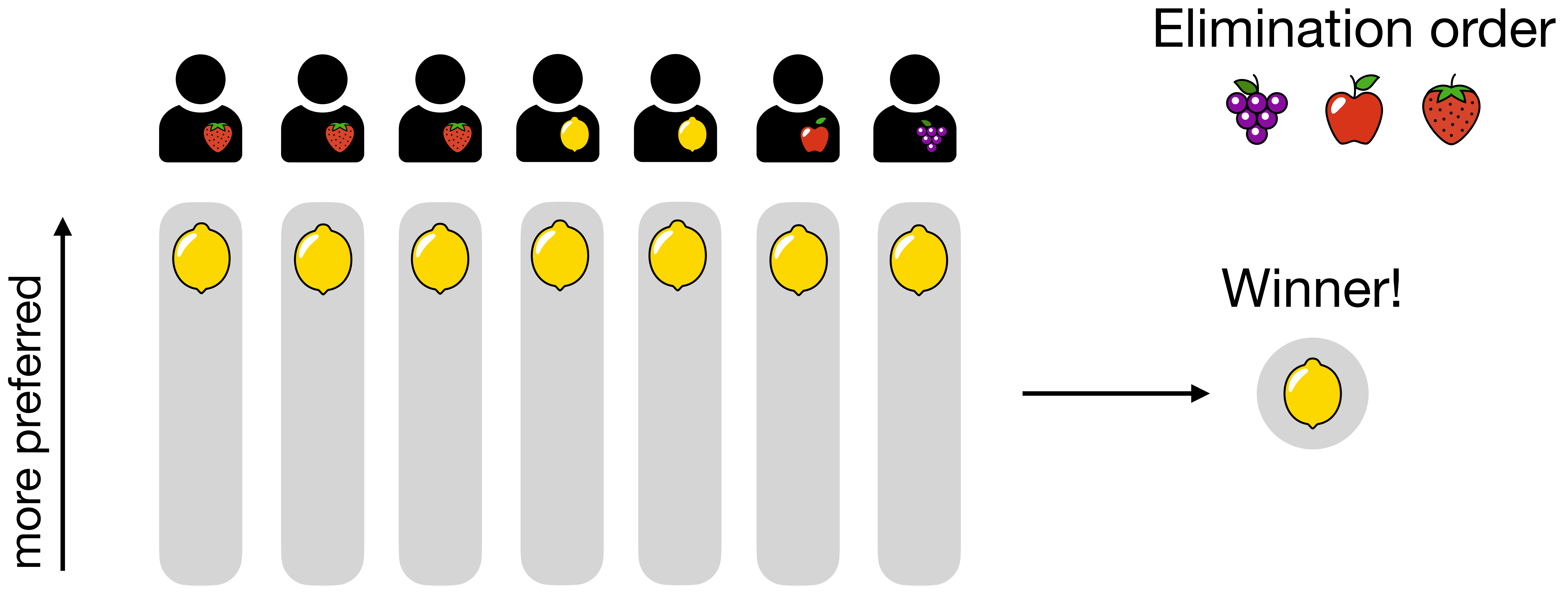

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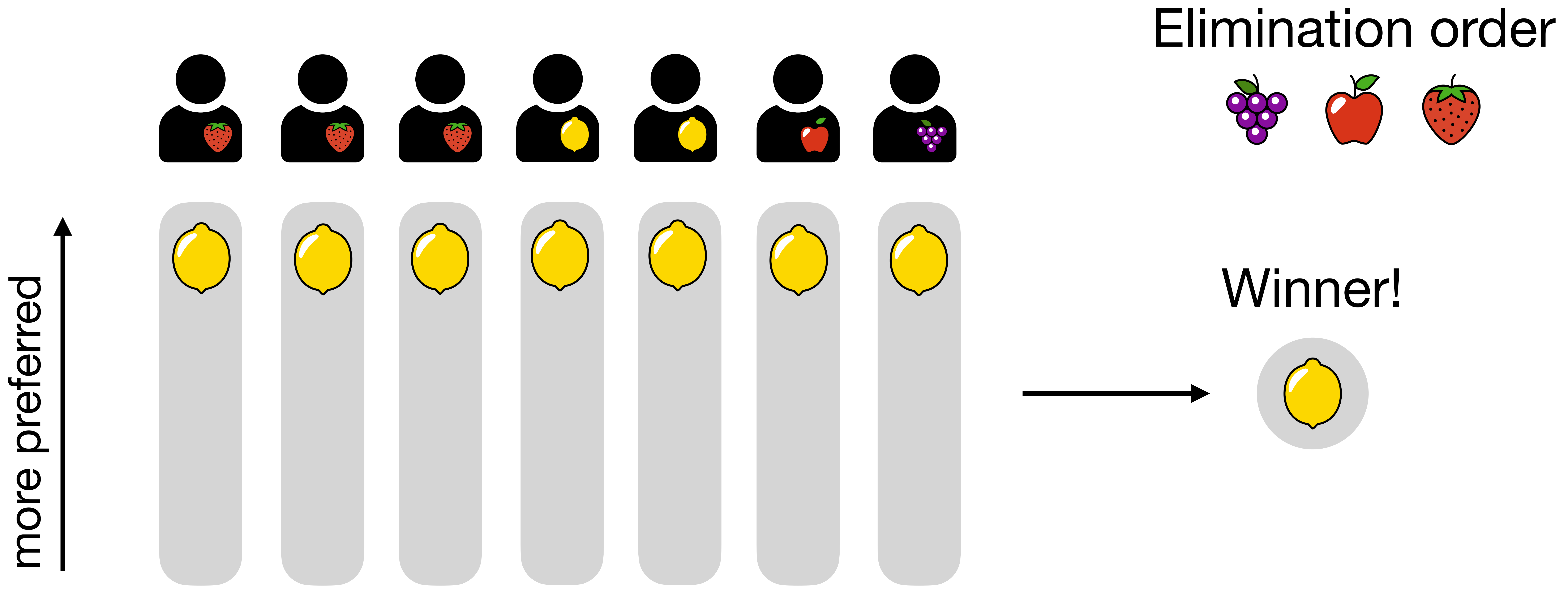
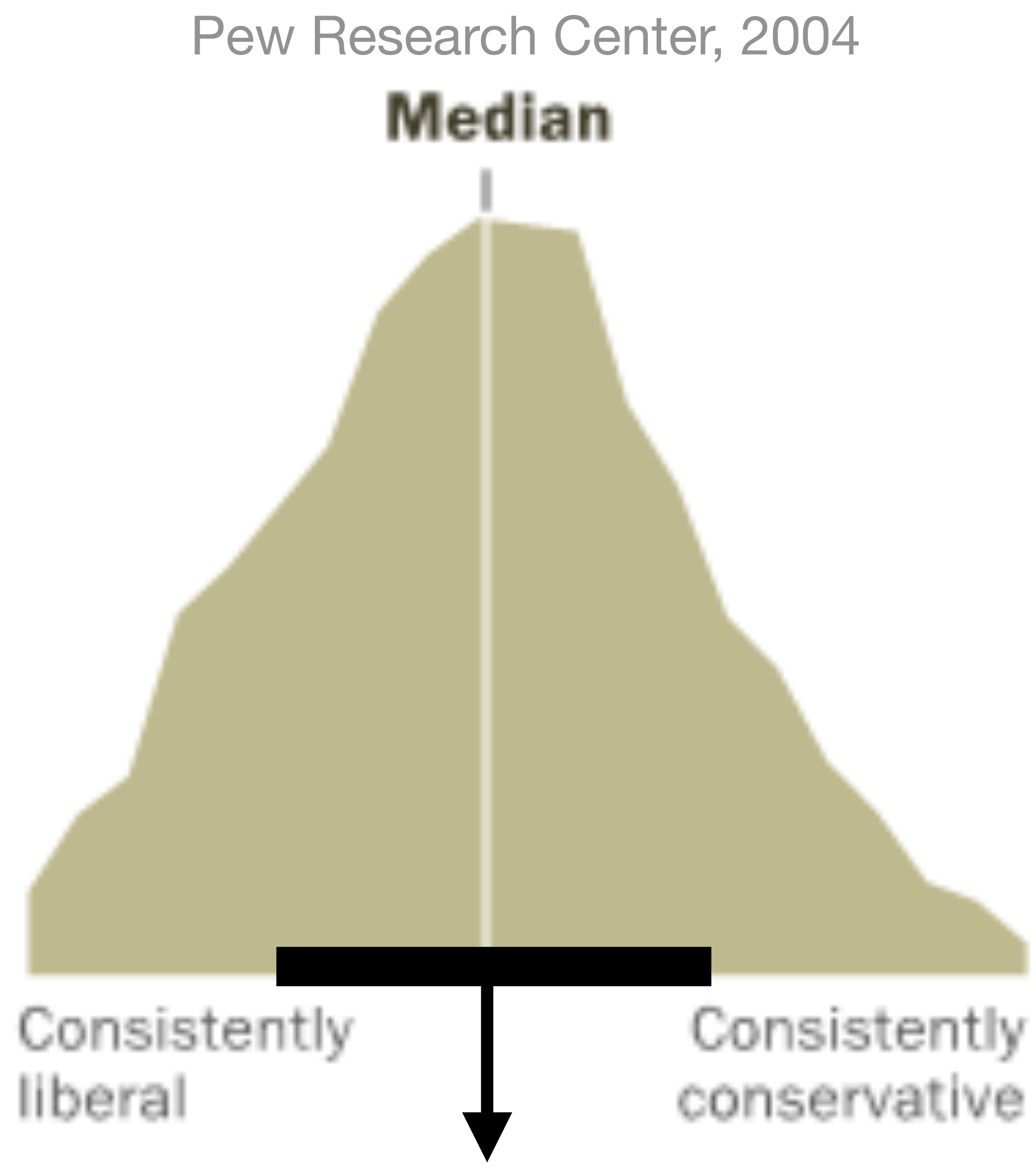


a.k.a. RCV, STV, AV, Hare method, preferential voting

Two years ago….



IRV always favors candidates from here

The Thirty-Eighth AAAI Conference on Artificial Intelligence (AAAI-24)

The Moderating Effect of Instant Runoff Voting

Kiran Tomlinson[1], Johan Ugander[2], Jon Kleinberg[1]

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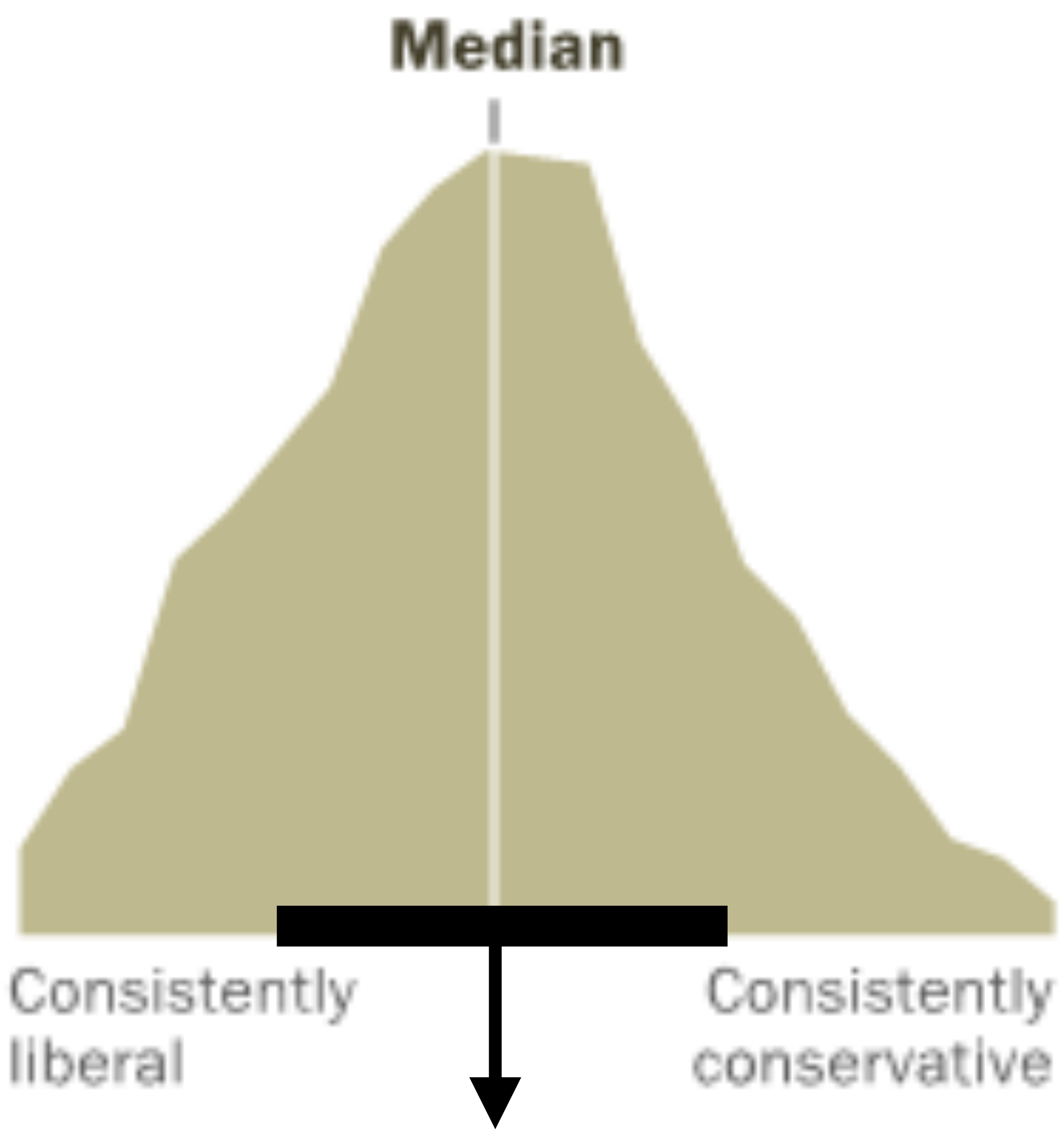

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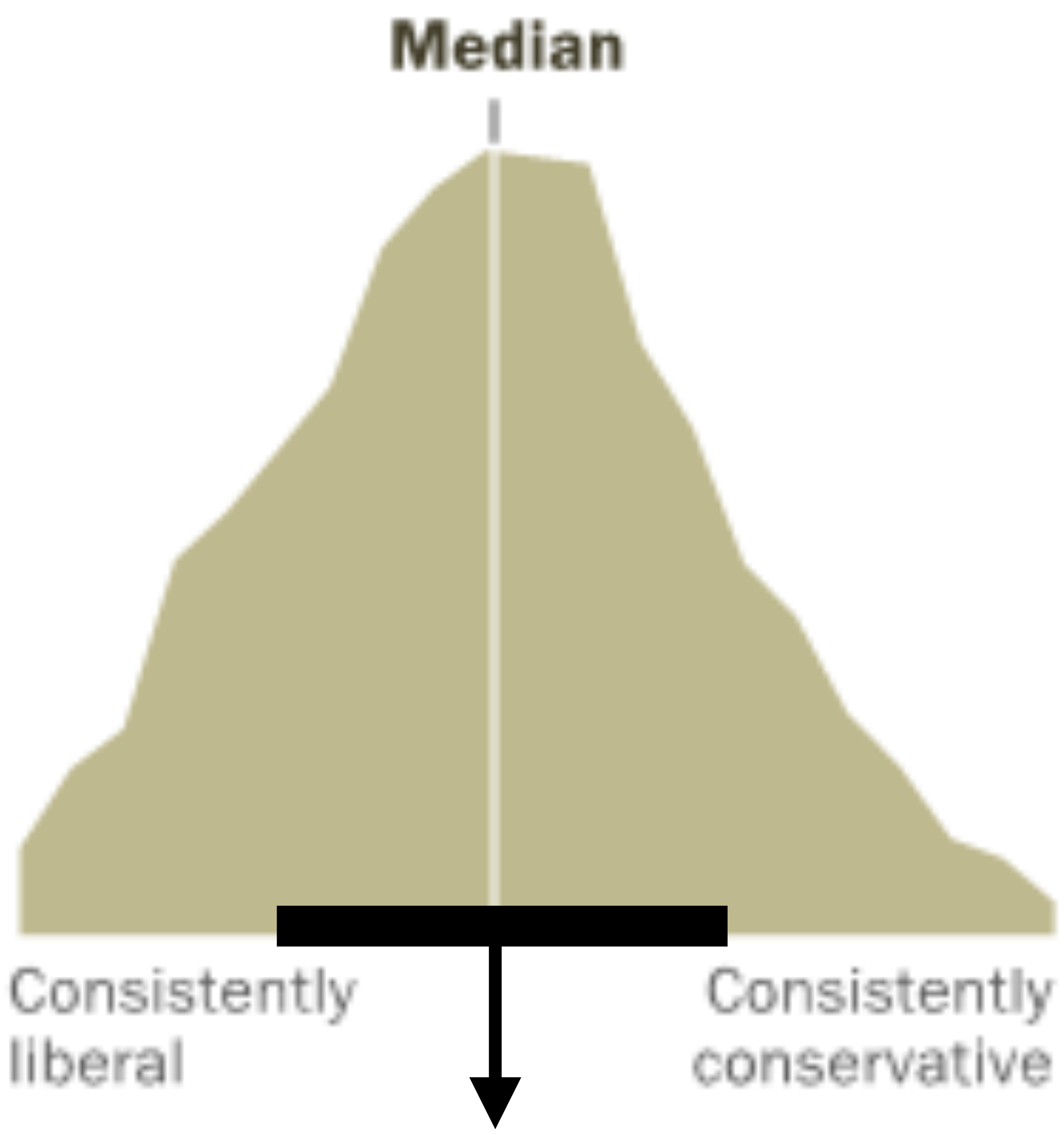

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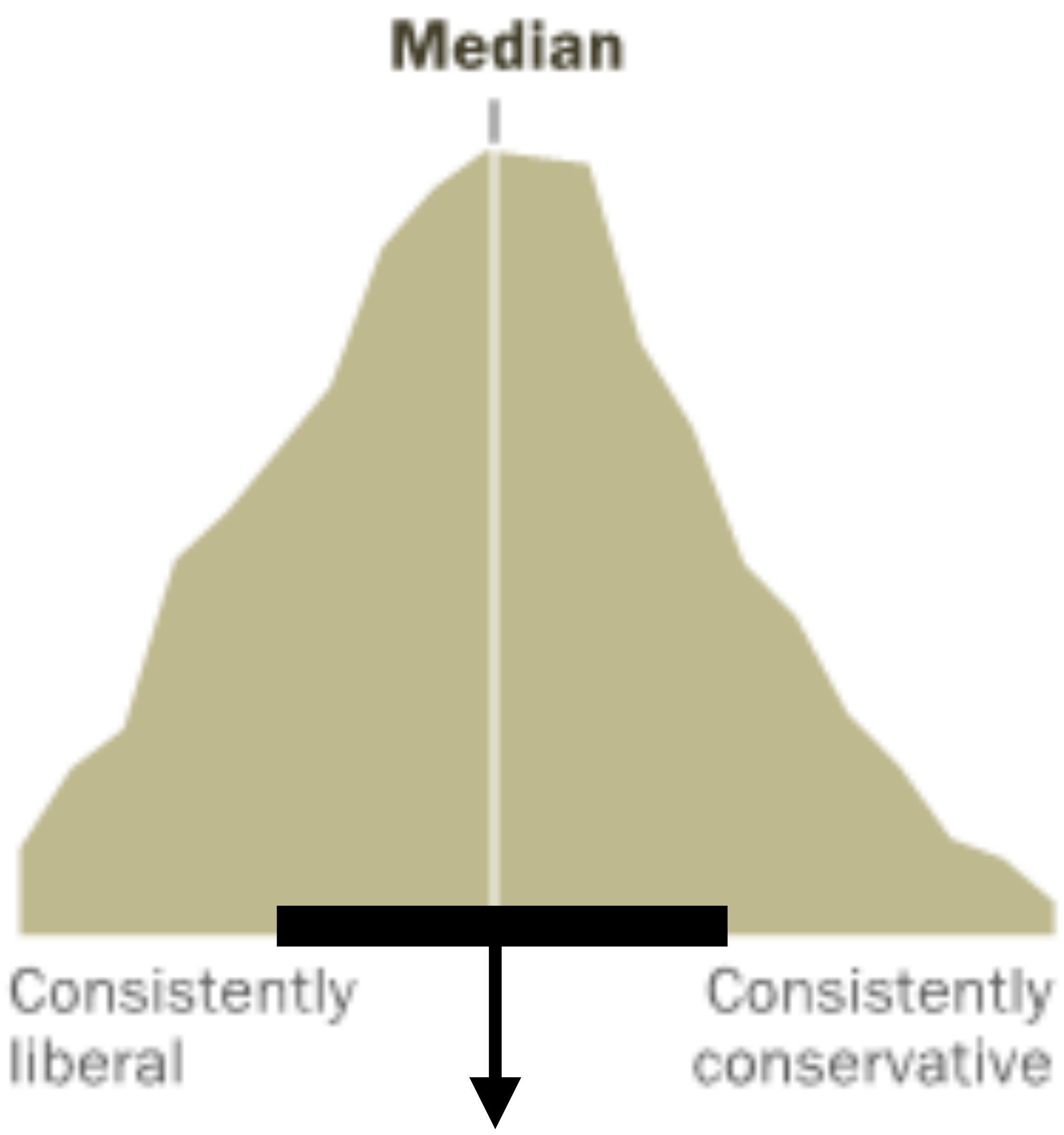

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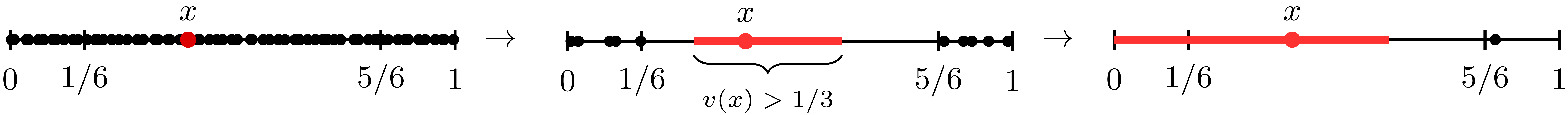

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Definition
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trivial exclusion zone (all positions)

minimal exclusion zone

The space of all Condorcet method exclusion zones in 1d

Median Voter Theorem (Black, 1948)
With 1-Euclidean preferences, the candidate closest to the median voter is the Condorcet winner.

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Condorcet methods with symmetric 1-Euclidean voters have the exclusion zones:
$S = [c, 1 - c]$ for any $c \in [0, 1/2)$,
$S = (c, 1 - c)$ for any $c \in [0, 1/2)$,
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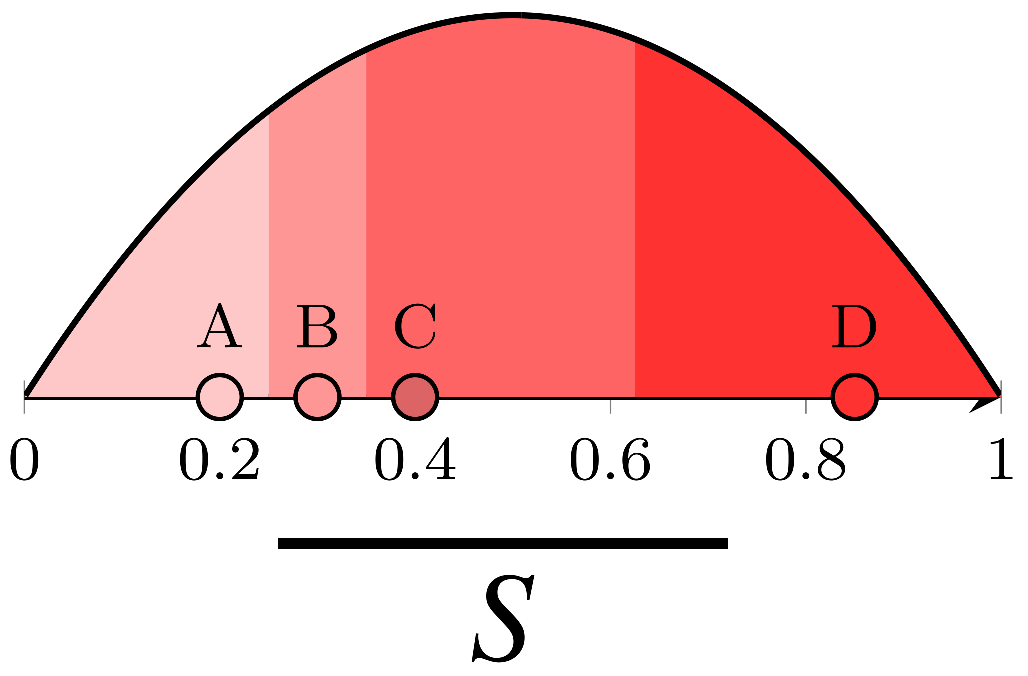
e.g.,



Does the 1-Euclidean moderating effect for IRV extend to *d*-Euclidean space?

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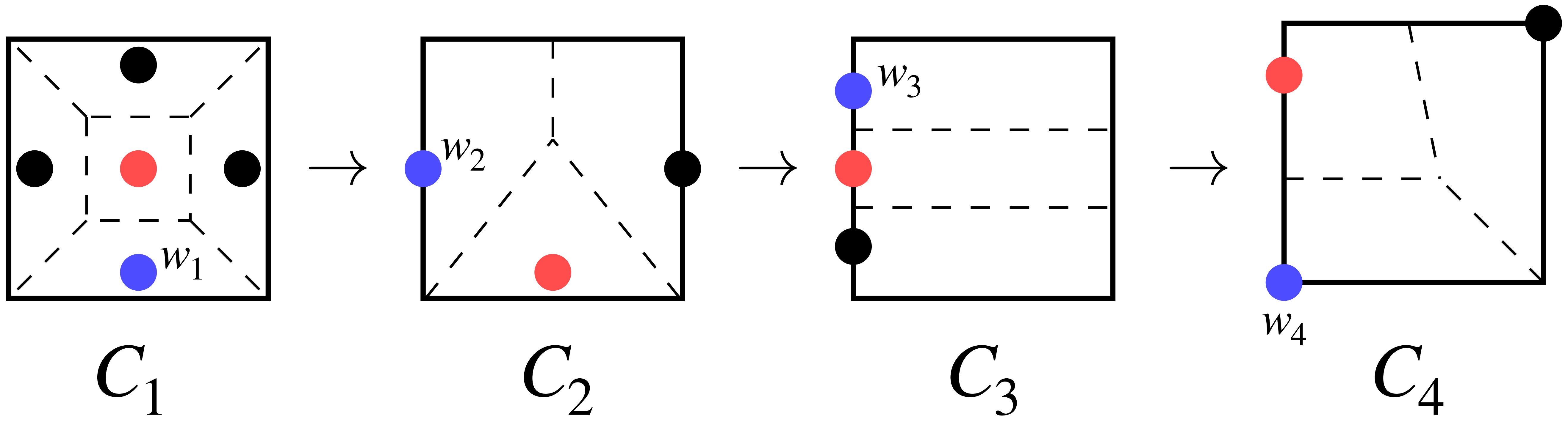

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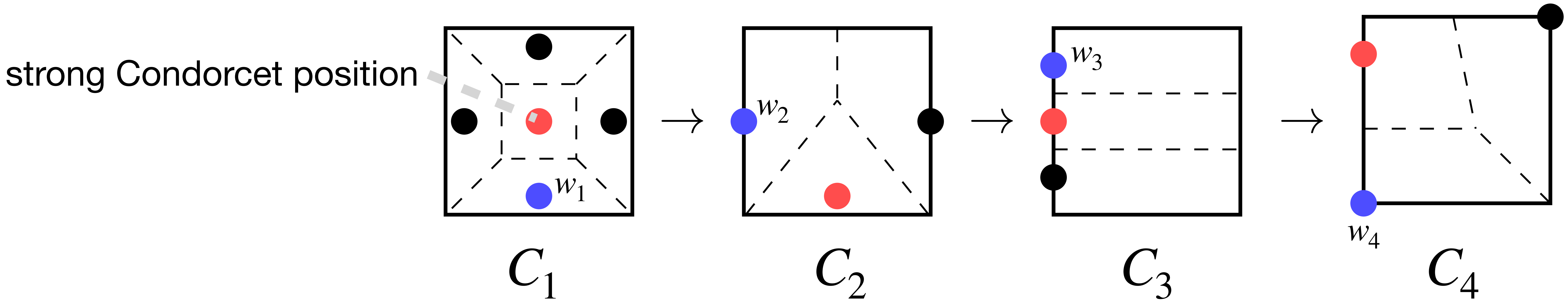

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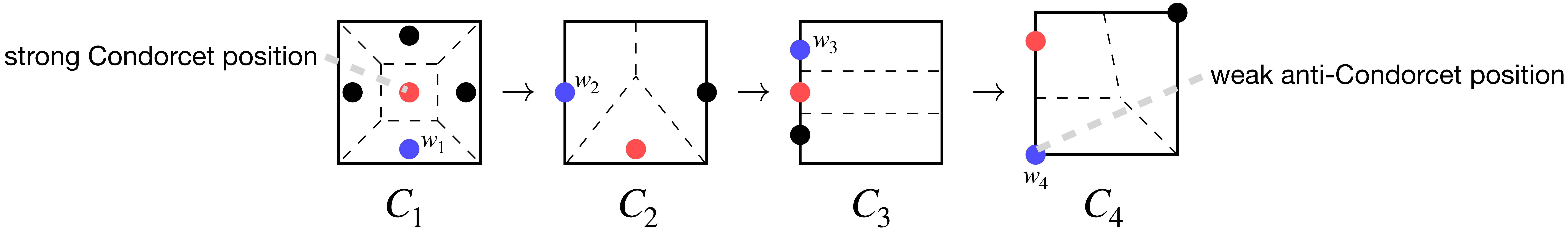

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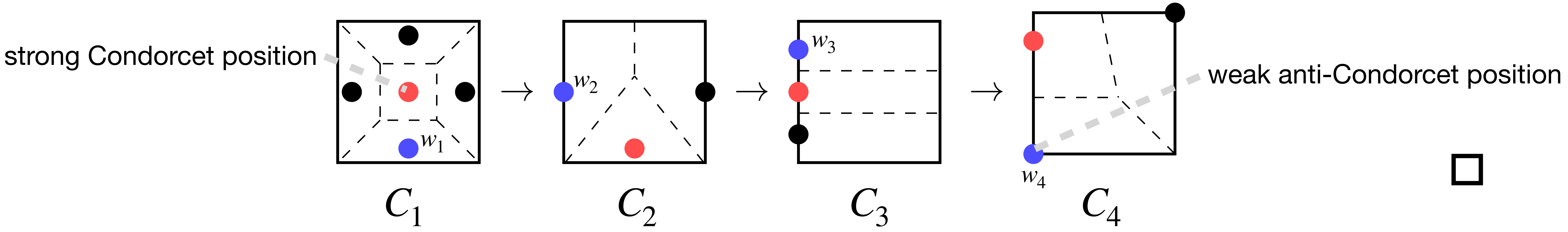

□

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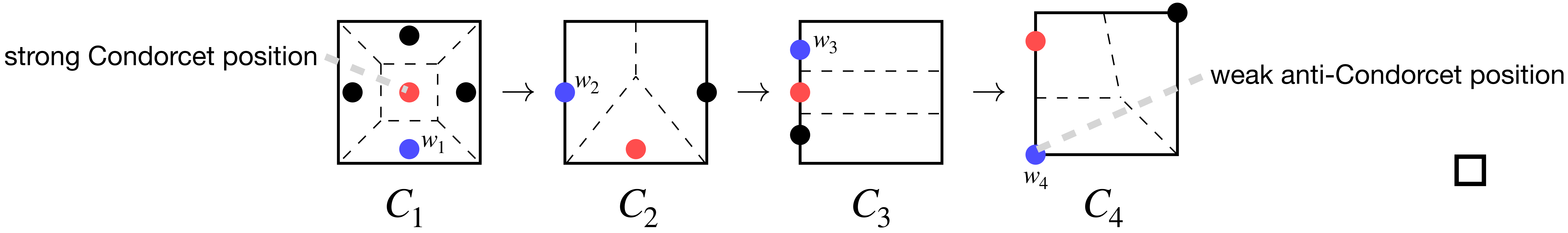

$\square$

Theorem

Every d-dimensional hyperrectangle ($d \geq 2$) with uniform L_1 or L_2 voters has no nontrivial IRV exclusion zone.

… So does IRV only have exclusion zones in one dimension?

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The shaded region is an IRV exclusion zone with uniform L_1 voters over this shape:

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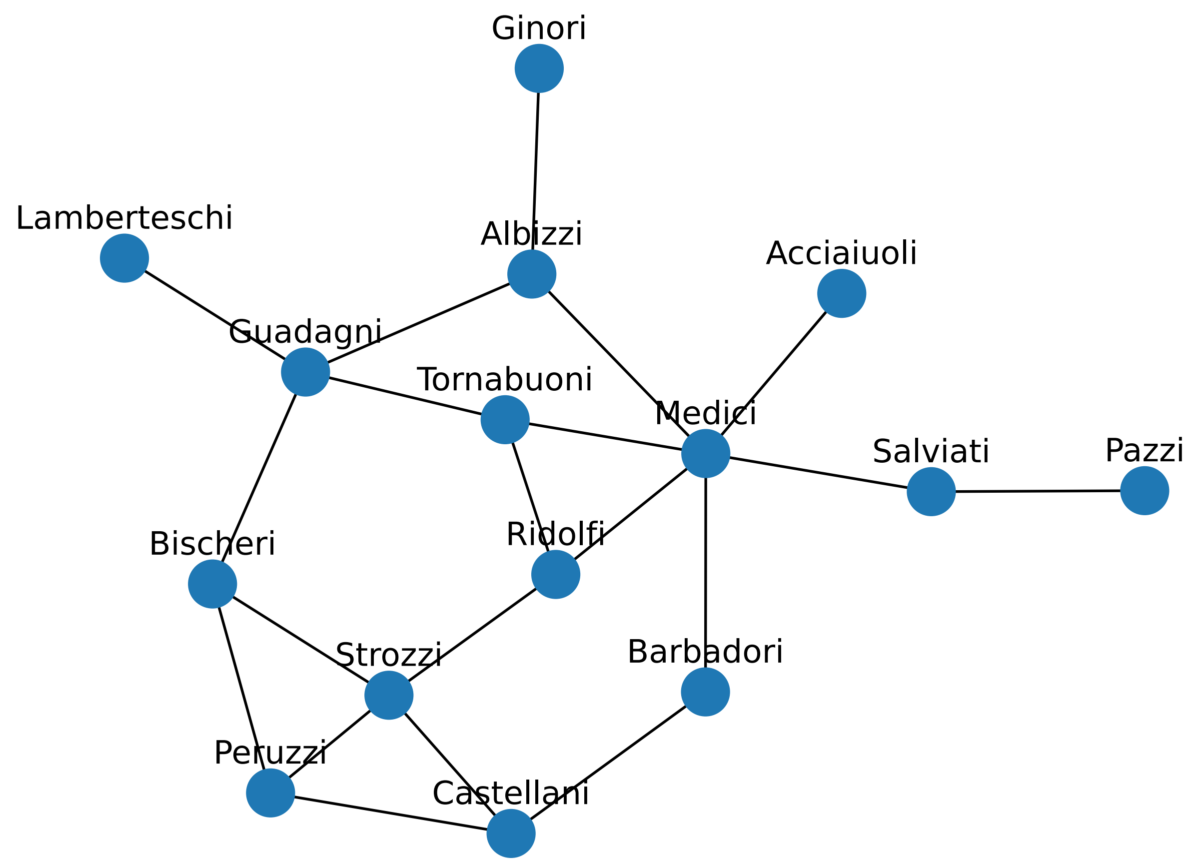
The shaded region is an IRV exclusion zone with uniform L_1 voters over this shape:

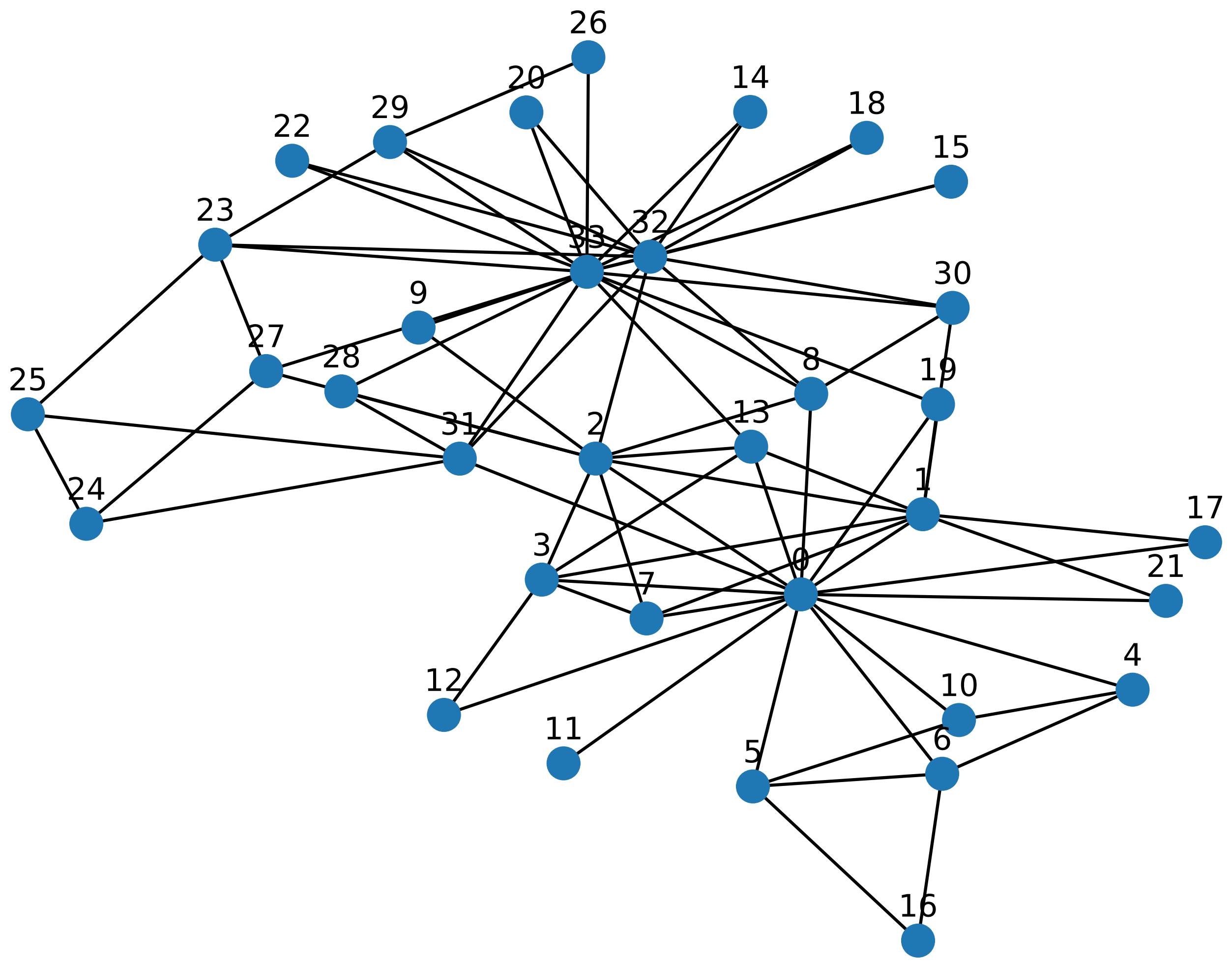
hyperrectangles have too many symmetries

Voting with the graph metric

- Nodes = voters
- Some subset of voters run for office
- Voters prefer closer candidates
- Resolve ties with Split-IRV (vote share 1 evenly split among equidistant candidates)



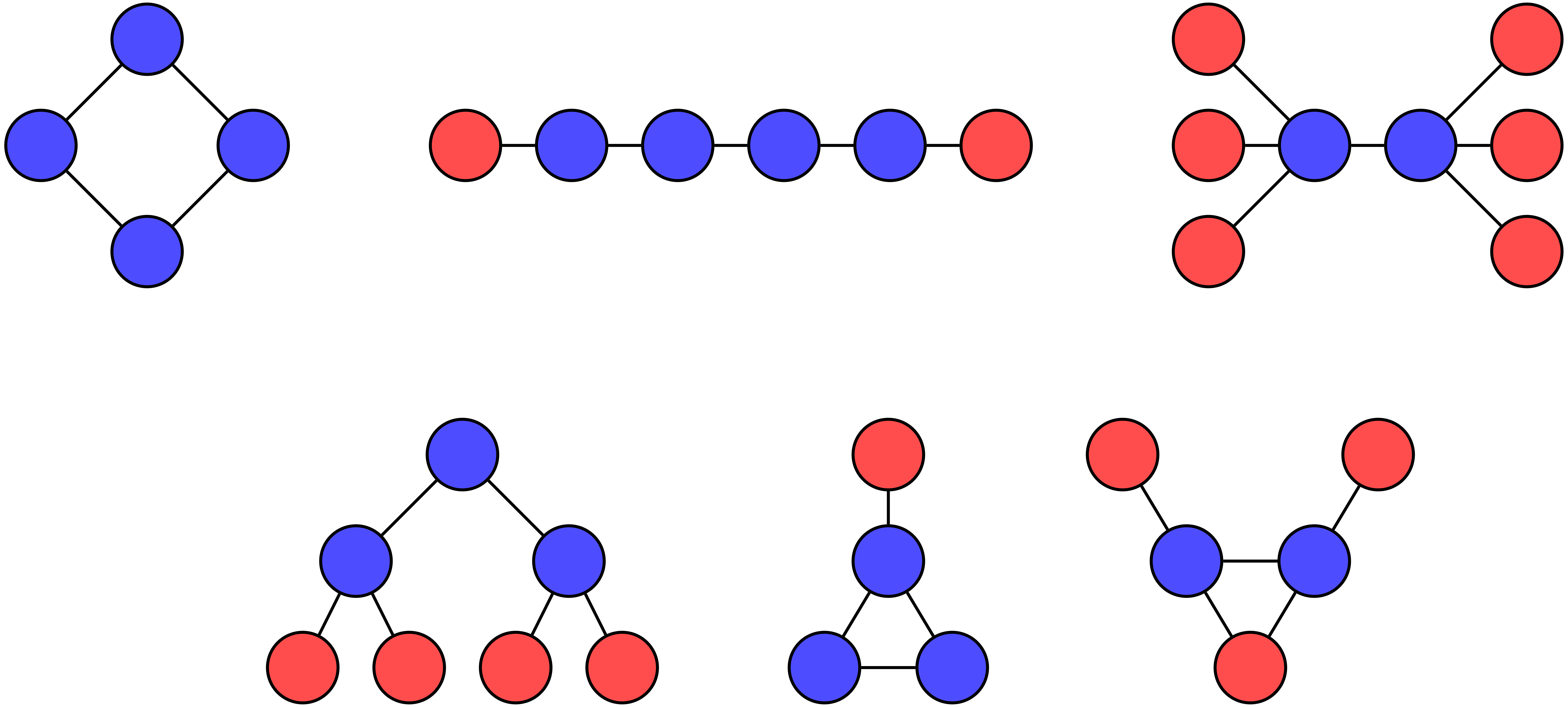
15th century Florentine marriages



Zachary's karate club

IRV exclusion zones in graphs

minimal exclusion zone



Finding IRV exclusion zones in graphs

IRV-Exclusion
Given a graph *G* and a set of nodes *S*, is *S* an IRV exclusion zone of *G*?

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IRV-Exclusion is co-NP-complete.

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Given a graph *G* and a set of nodes *S*, is *S* an IRV exclusion zone of *G*?

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IRV-Exclusion is co-NP-complete.

Theorem
Let *G* be a graph with *n* nodes and *m* edges. For any $\epsilon, \delta \in (0,1)$, there is a randomized algorithm returning a set *S* in time $O((n^3 + n^2m)\log(1/\delta)/e^2)$ s.t.

1. *S* is a subset of the minimal IRV exclusion zone of *G* and
2. *S* is a $(1-\epsilon)$-approximate IRV exlusion zone of *G* w.p. at least $1-\delta$.

IRV exclusion zones in graphs

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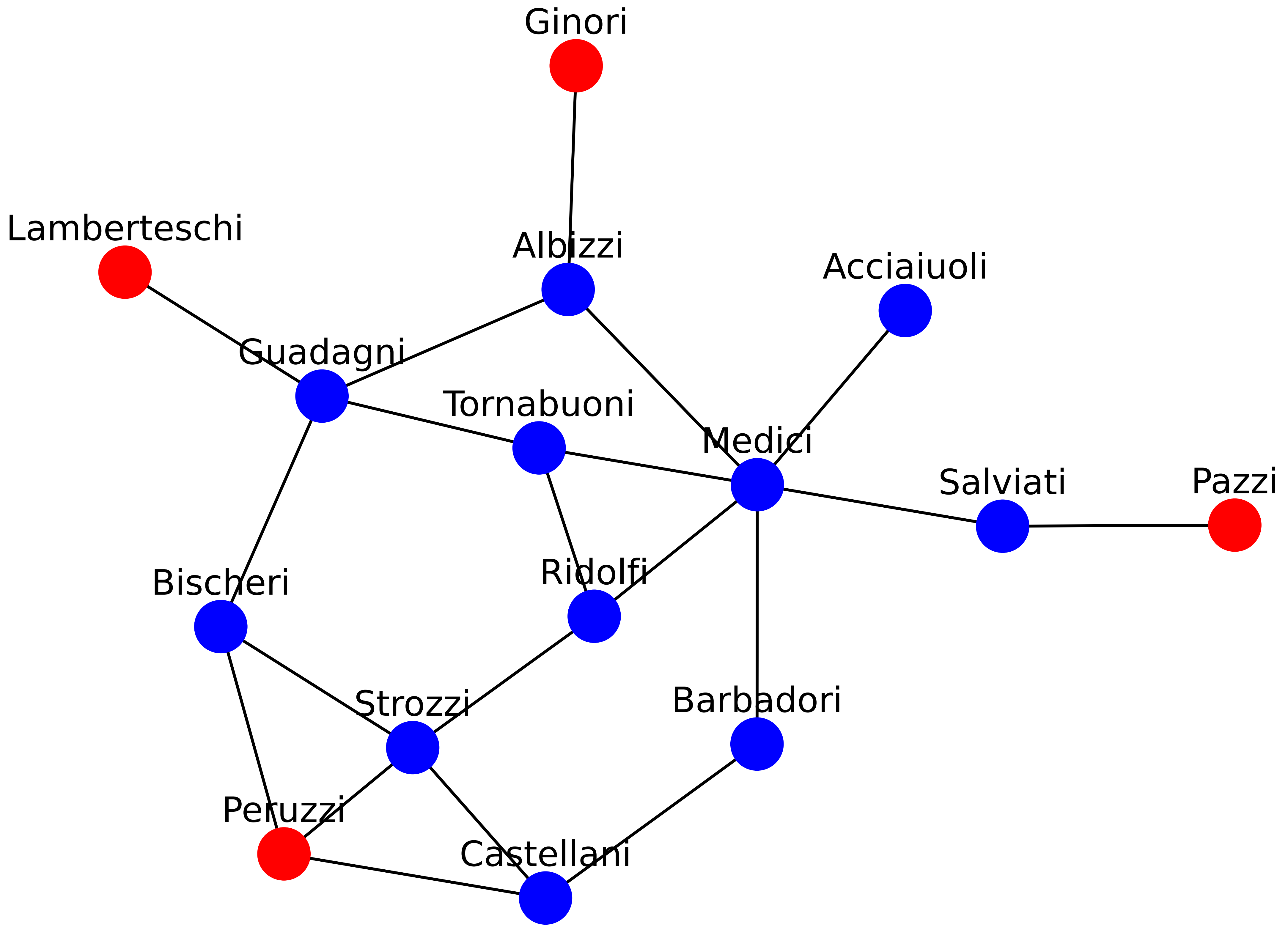

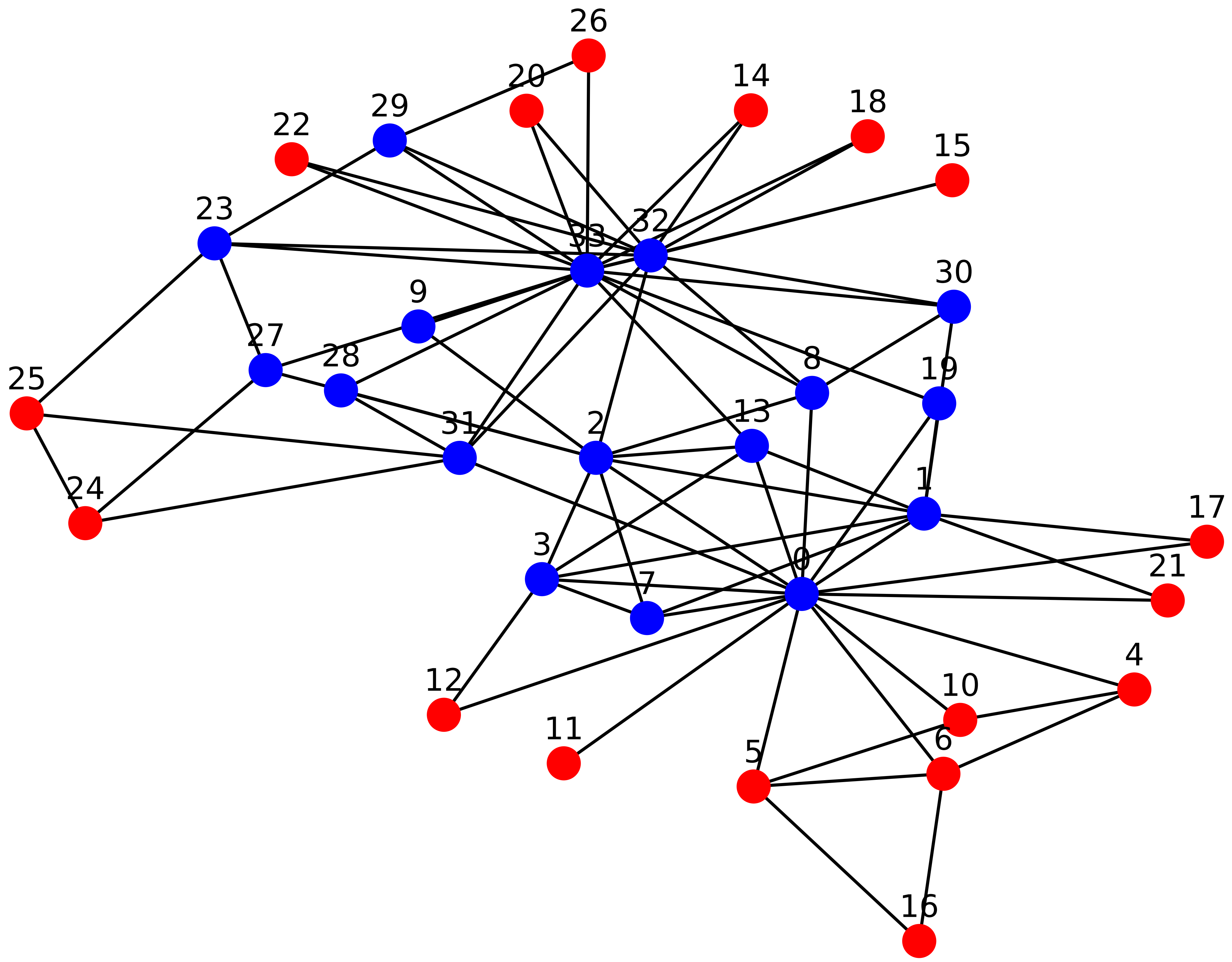

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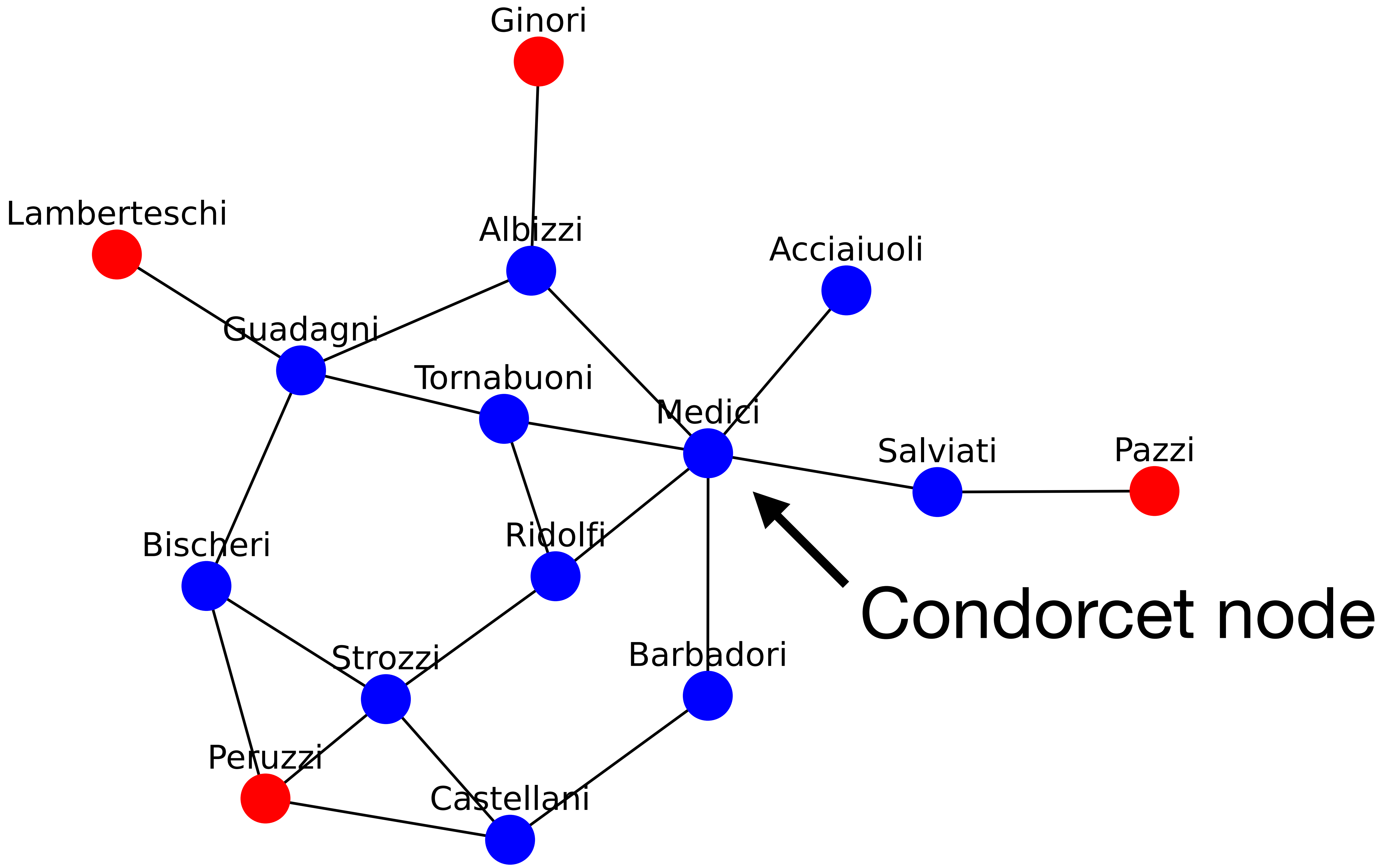

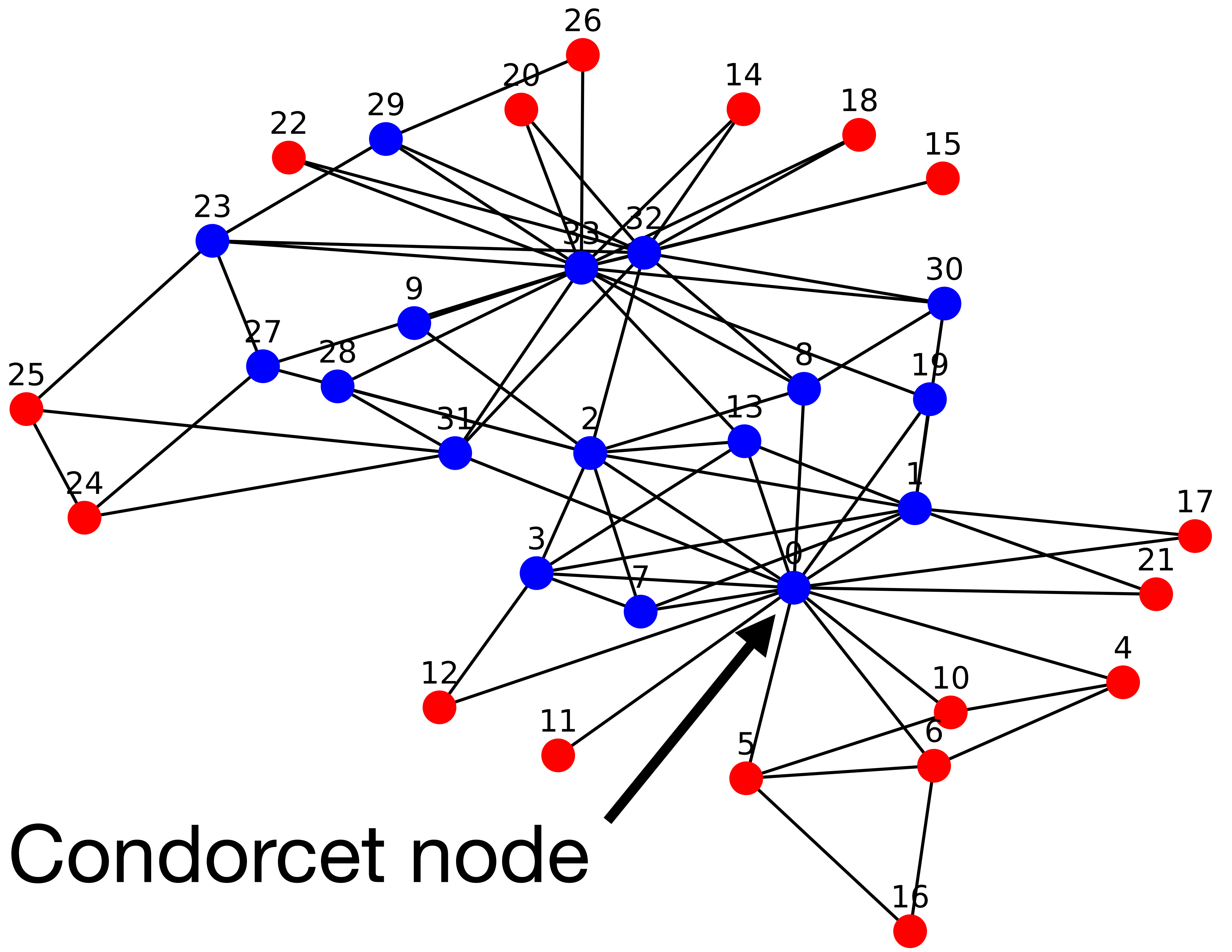

Other results

- Properties of exclusion zones
- Minimal IRV exclusion zones of paths, stars, bistars, and perfect binary trees
- Experiments: approximation algorithm on real-world networks

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Open questions

- Do other voting rules have nontrivial exclusion zones with 1-Euclidean preferences? d-Euclidean? Graphs?
- In higher dimensions, are there “natural” voter distributions with nontrivial IRV exclusion zones?

Thank you!

Supported in part by:


Kiran Tomlinson
Microsoft Research
kitomlinson@microsoft.com


Johan Ugander
Yale University


Jon Kleinberg
Cornell University

Exclusion Zones of Instant Runoff Voting, arxiv.org/abs/2502.16719

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A **Condorcet method** elects the Condorcet winner when one exists.

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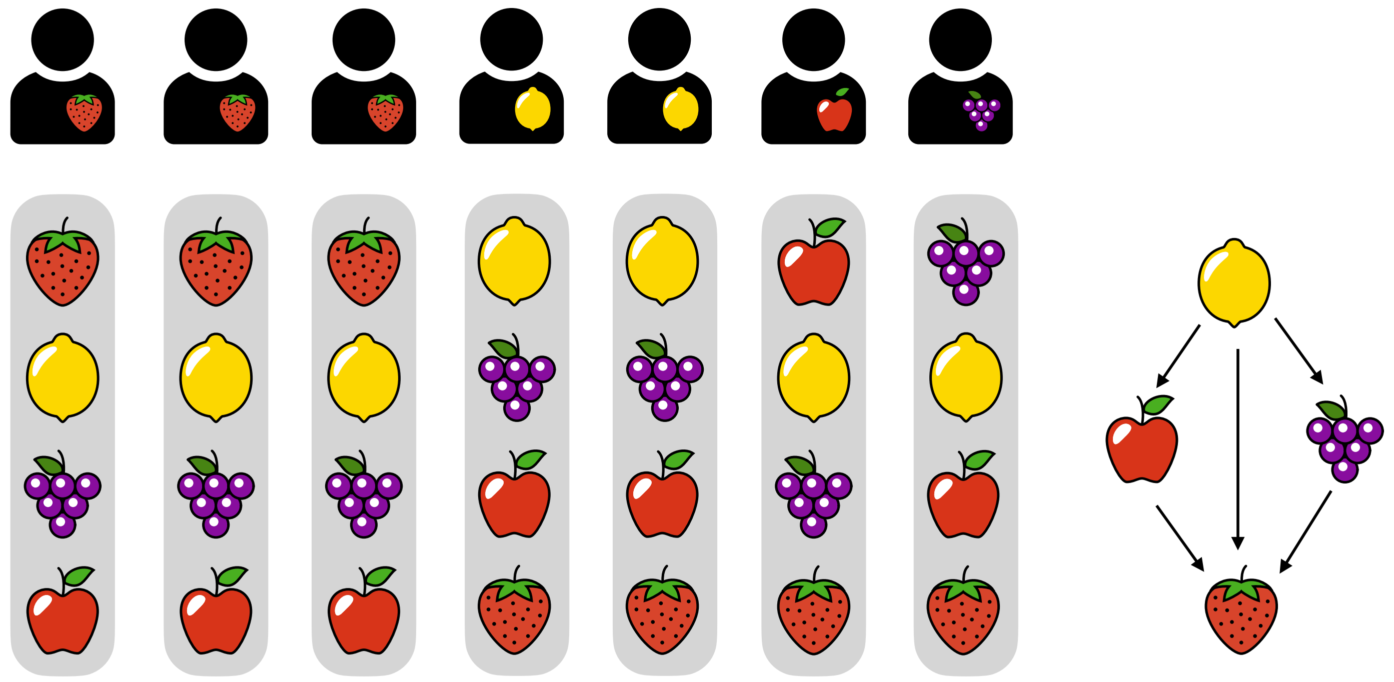
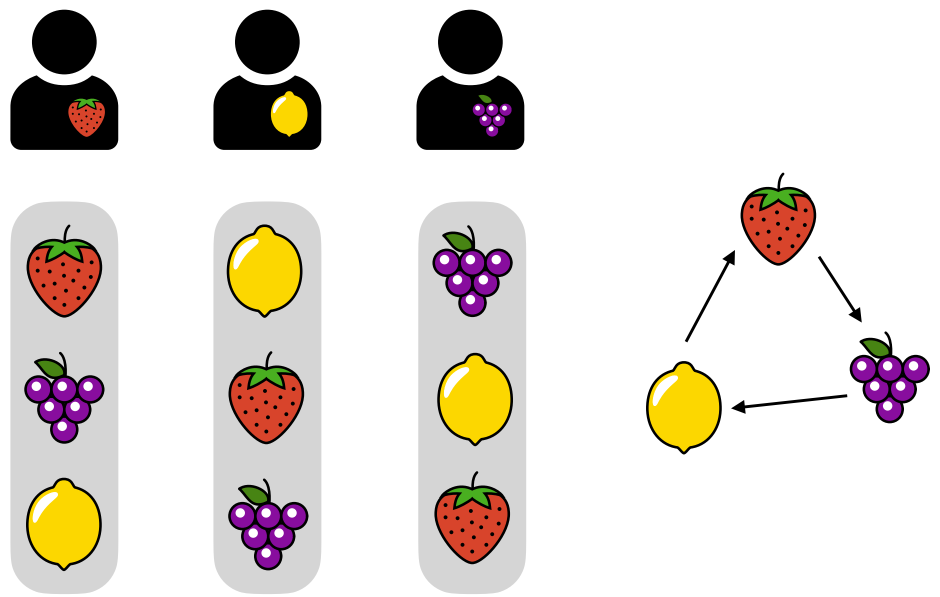
(1743 - 1794)





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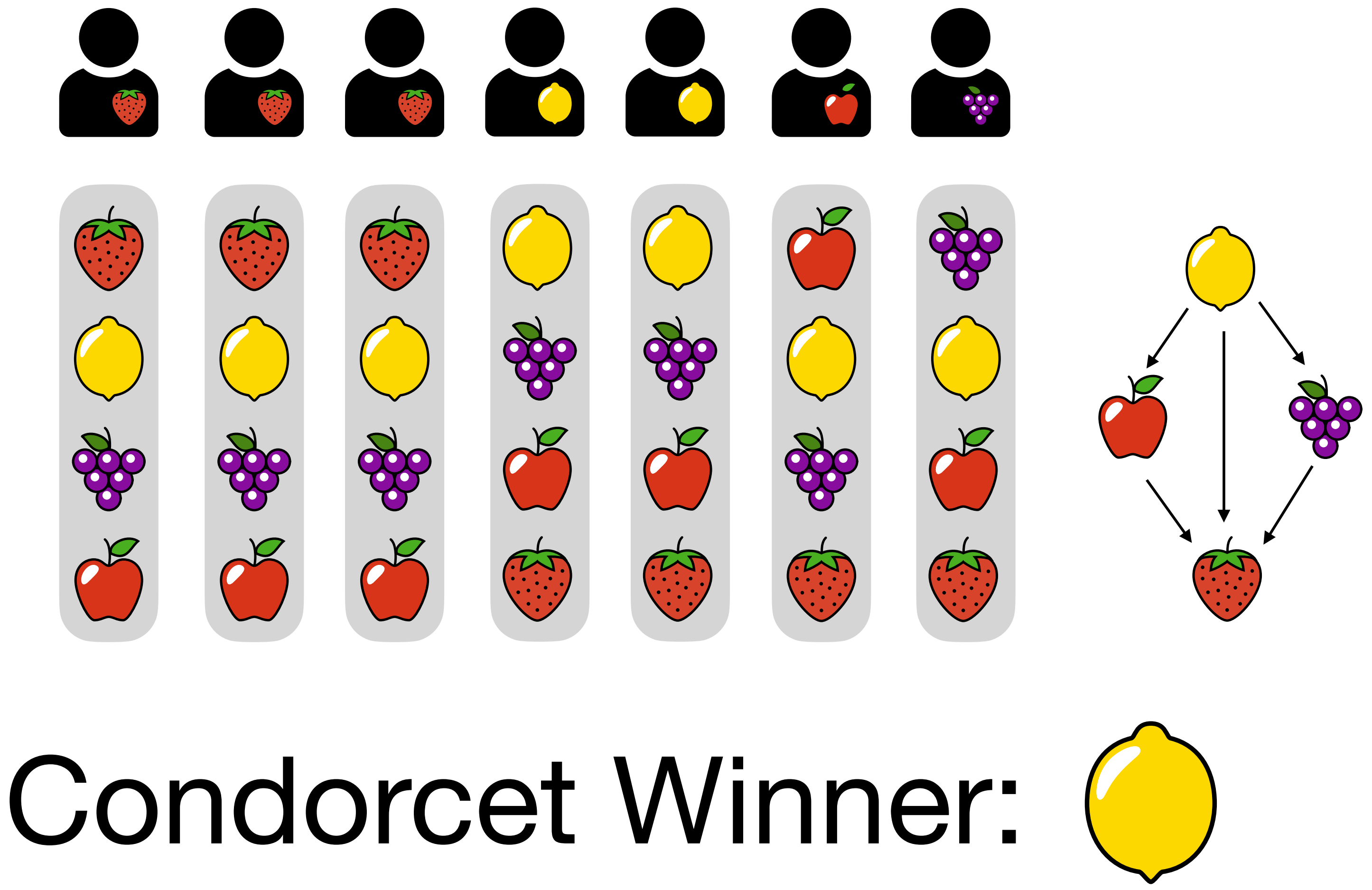
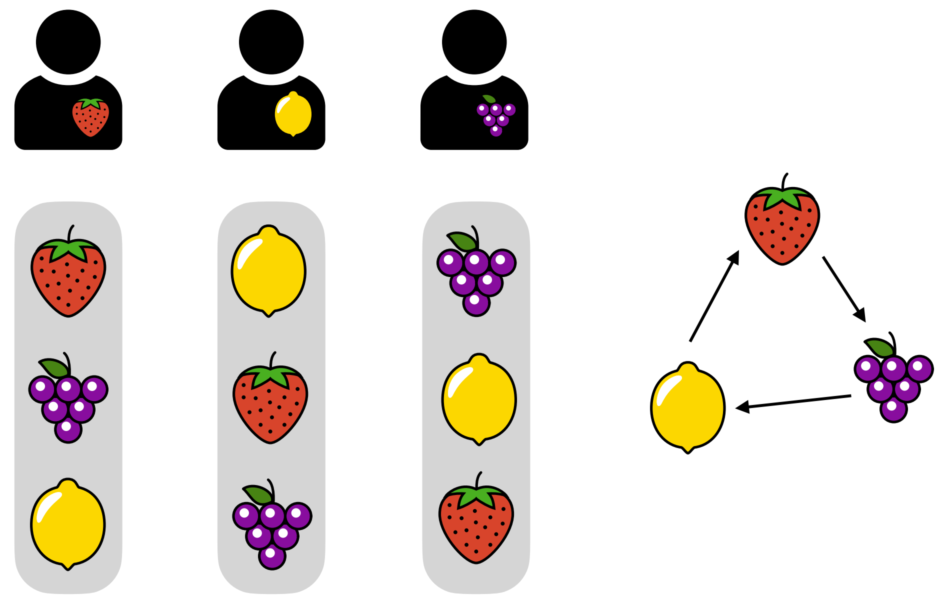


Condorcet Winner:



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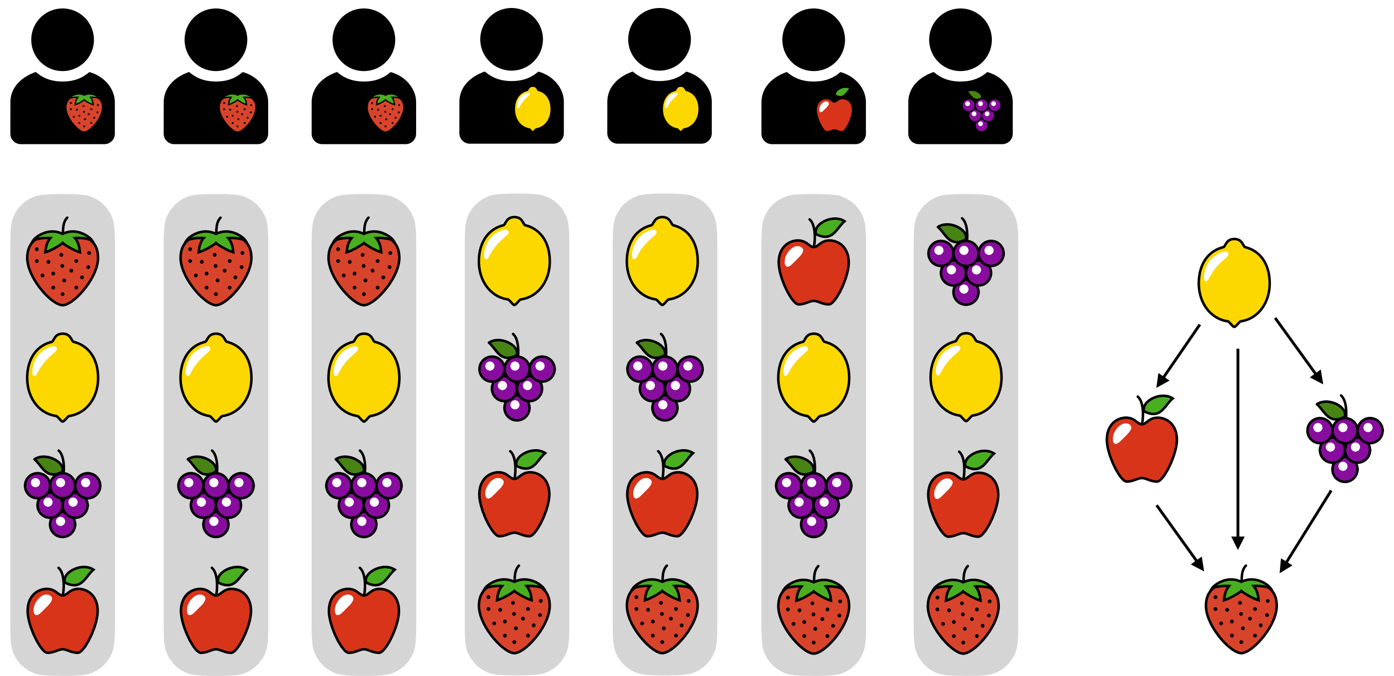
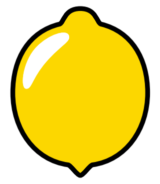
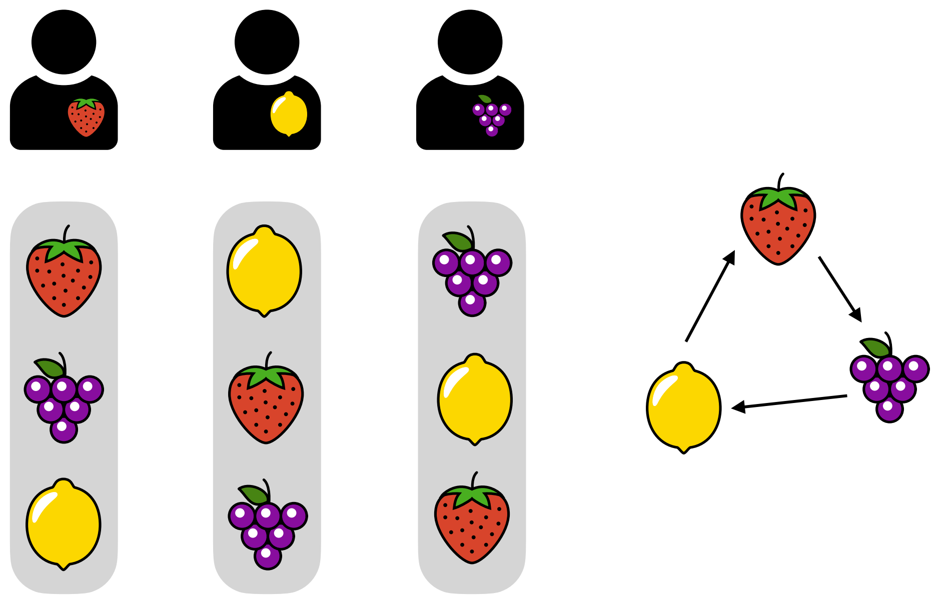


Condorcet Winner: 



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Condorcet Chain Lemma
Given an election setting, if there exist elections $C_1, \ldots, C_n$ with candidates $w_1 \in C_1, \ldots, w_1 \in C_n$ such that:

1. C_1 includes a weak Condorcet position, but a different candidate w_1 wins
2. each C_{i+1} includes w_i, but some other candidate w_{i+1} wins
3. w_n is a weak anti-Condorcet position,

then the election setting has no nontrivial exclusion zones.

Condorcet and anti-Condorcet positions

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Condorcet and anti-Condorcet positions

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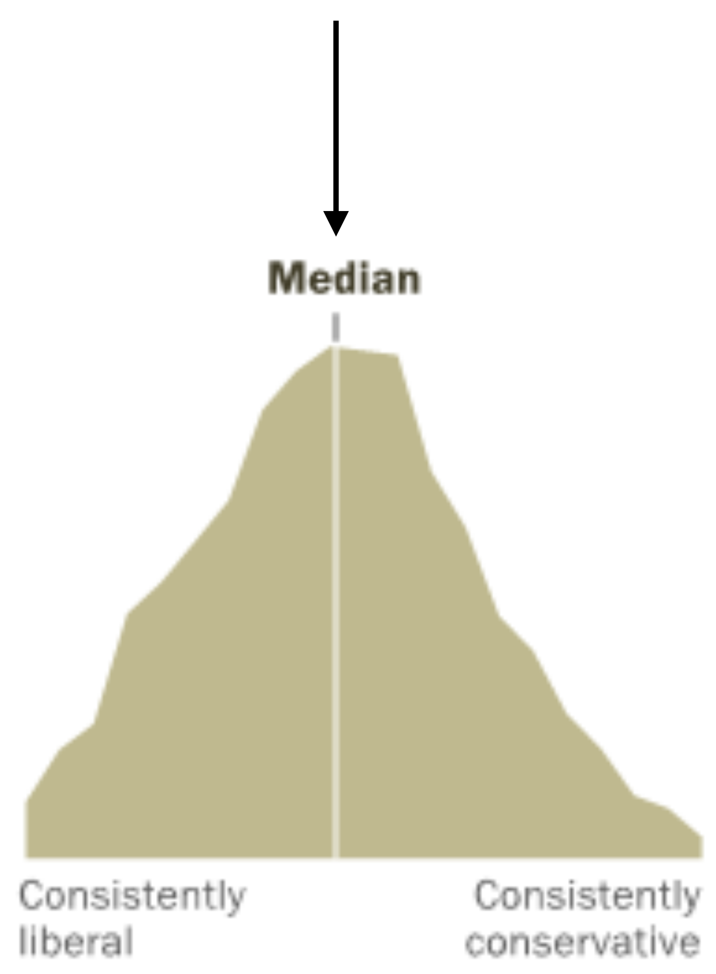
strong Condorcet position



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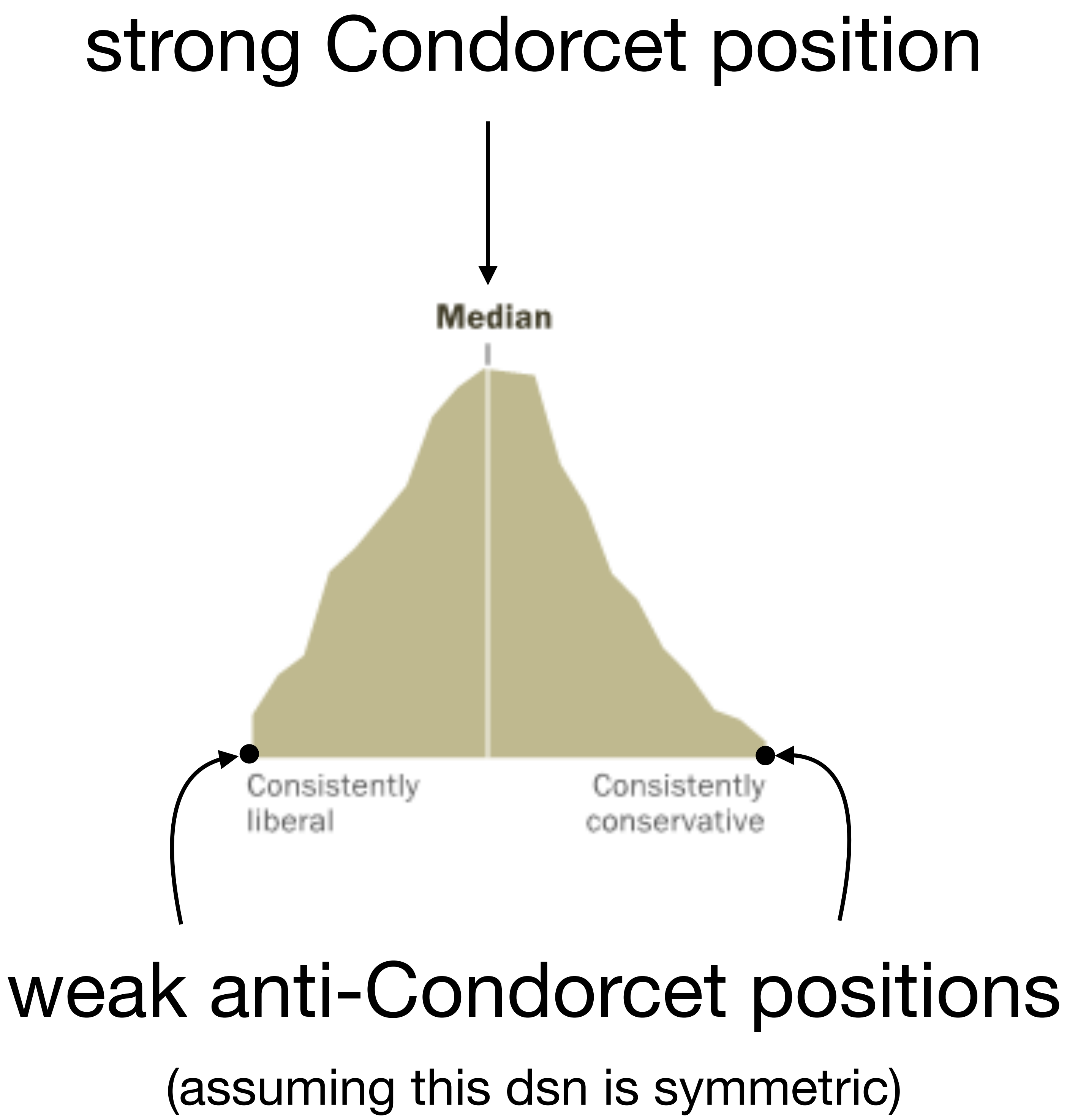

Condorcet and anti-Condorcet positions

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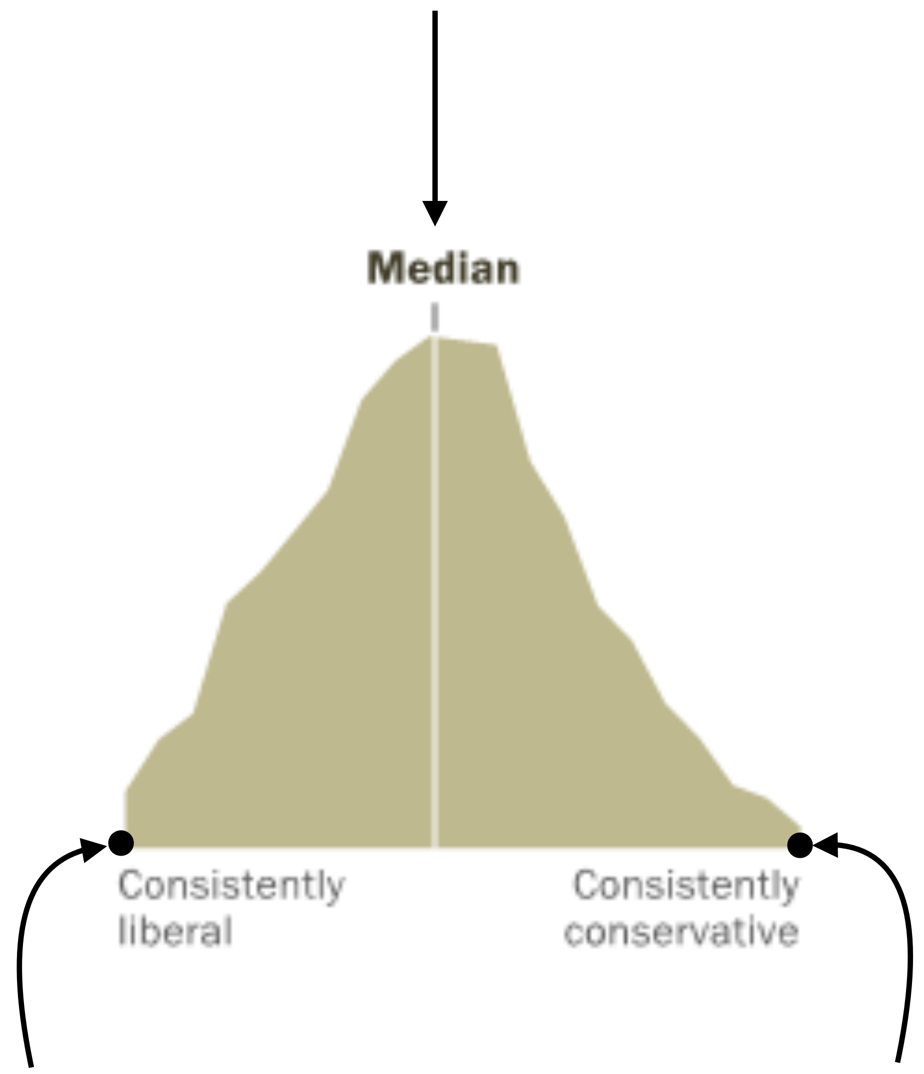

weak anti-Condorcet positions

(assuming this dsn is symmetric)

Proposition

For any reasonable* voting rule:

1. Any weak Condorcet position is in the minimal exclusion zone.
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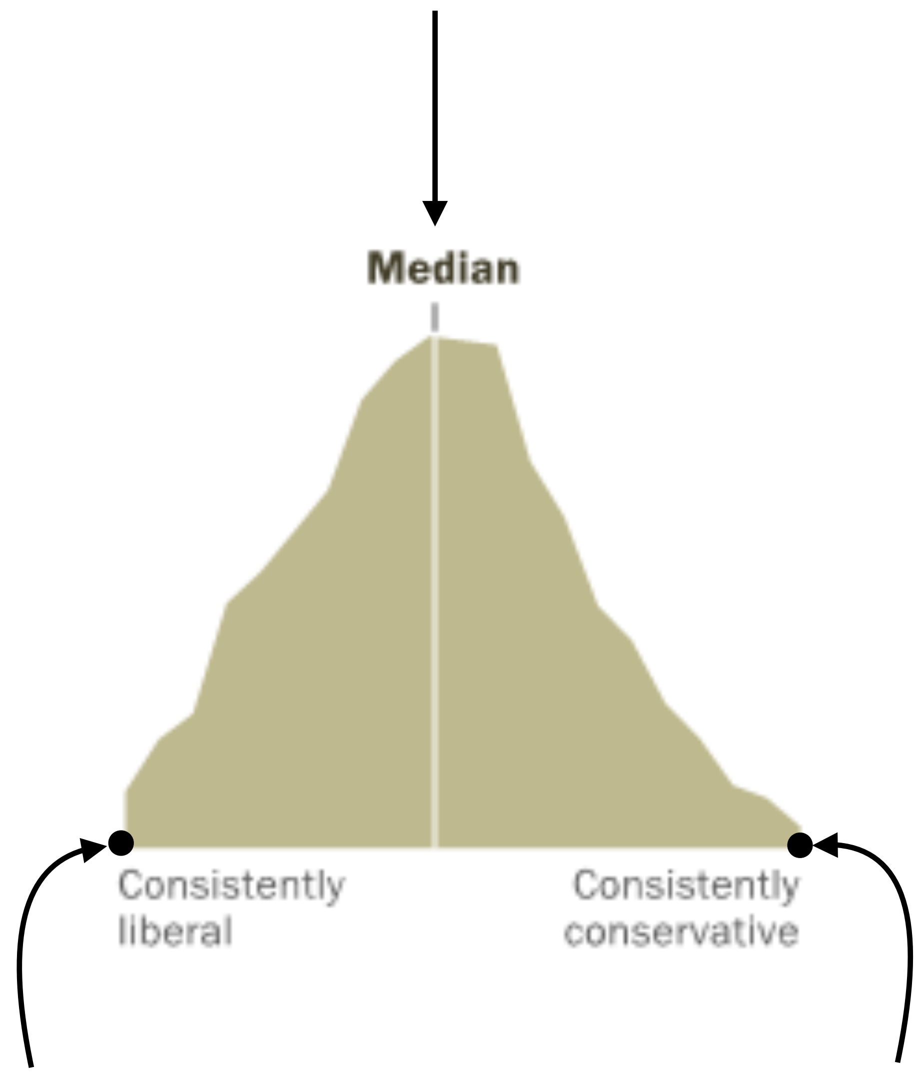

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