Exclusion Zones of Instant Runoff Voting

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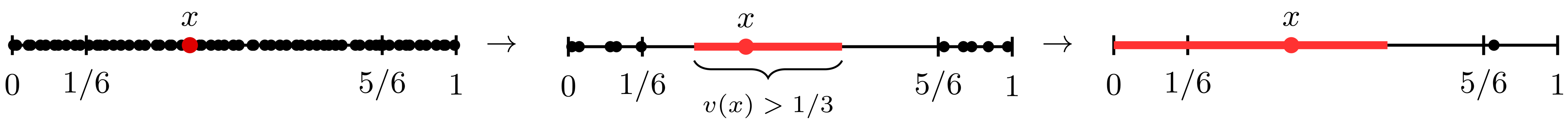
Background

What positions does a voting system favor?

d-Euclidean preferences
voters and candidates in $[0, 1]^d$,
voters rank candidates by distance

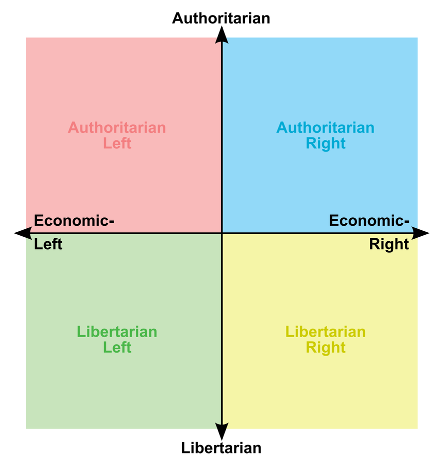
IRV
voters rank all candidates; repeatedly eliminate candidate with fewest first-place votes, last left wins

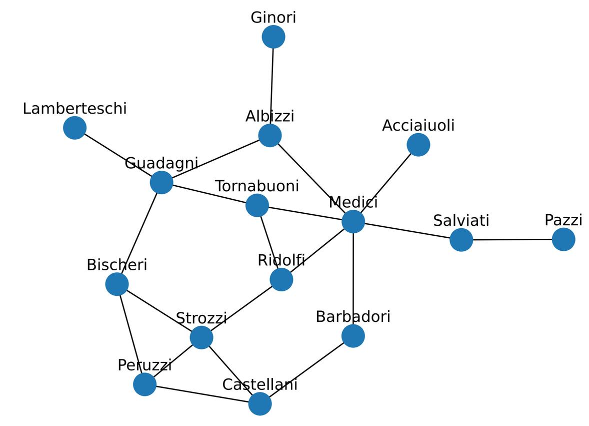
Theorem [TUK, AAAI '24] With uniform 1-Euclidean voters, IRV can only elect candidates in [1/6, 5/6] (unless no such candidate is present). This is the smallest such *exclusion zone.*



Does IRV have exclusion zones in higher dimensions?

In other metric spaces?





Exclusion Zone Properties

Key Definition Given a distribution V of voters over a metric space M, an *exclusion zone* of a voting rule r is a set $S \subseteq M$ that is guaranteed to contain the winner of any election over r, M, V containing at least one candidate from S.

1. For any two exclusion zones S, T, either $S \subset T$ or $T \subset S$.
2. The intersection of all exclusion zones gives the unique *minimal exclusion zone*.
3. Any weak Condorcet position is in the minimal exclusion zone.
4. The only exclusion zone containing a weak anti-Condorcet position the trivial one.
5. If S is an exclusion zone containing x and wins some election with x, then $y \in S$.

trivial exclusion zone (M)

minimal exclusion zone

Condorcet position (= core)
majority-preferred to all other positions

anti-Condorcet position
all other positions majority-preferred

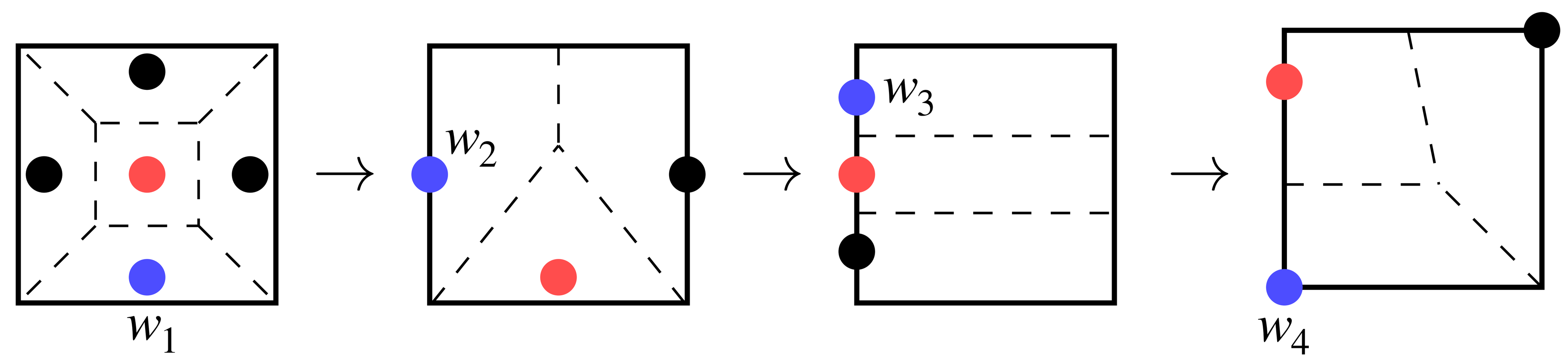
Condorcet Chain Lemma Given r, M, V, if there exists some sequence of elections $C_1, \ldots, C_n$ won by $w_1, \ldots w_n$ such that:
1. C_1 contains a weak Condorcet position that isn't w_1
2. For $i = 1, \ldots, n-1$, $w_i \in C_{i+1}$, but $w_{i+1} \neq w_i$
3. w_n is a weak anti-Condorcet position

then r, M, V has no nontrivial exclusion zones.

Proposition For any Condorcet method and symmetric 1-Euclidean voters, the collection of all exclusion zones is:

$S = [c, 1-c]$ for any $c \in [0, 1/2]$
$S = (c, 1-c)$ for any $c \in [0, 1/2]$
$S = \{1/2\}$

Higher Dimensions



Proposition The square with uniform L_2 voters has no nontrivial IRV exclusion zone.

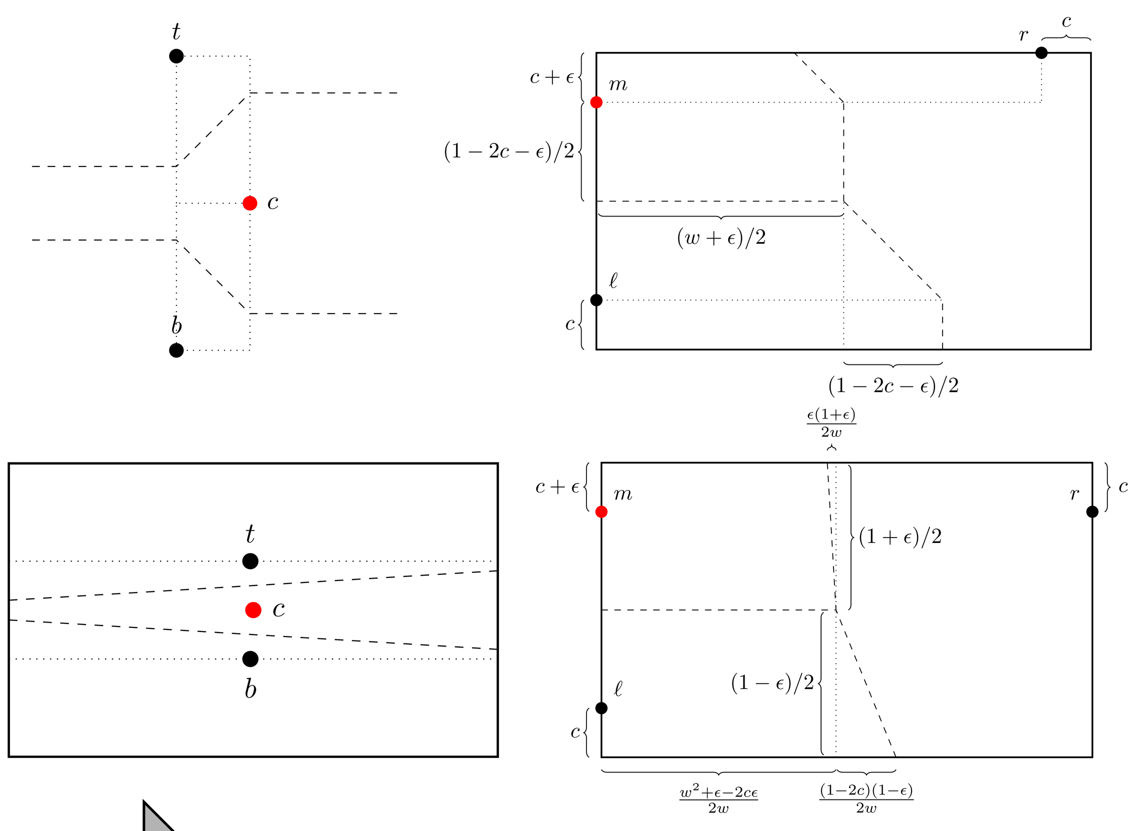
The Condorcet Chain Lemma converts a nonexistence proof into a construction exercise!

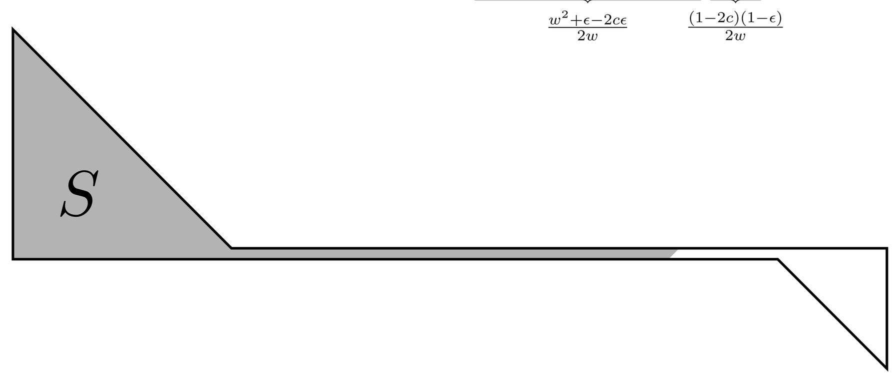
With some more geometry (see right), this idea generalizes a lot. 1d was special!

Theorem All hyperrectangles with $d \geq 2$ and uniform L_1 or L_2 voters have no nontrivial IRV exclusion zones.

So do IRV exclusion zones only exist in one dimension? No!

Theorem With uniform L_1 voters over the shape to the right, the shaded region is an IRV exclusion zone.

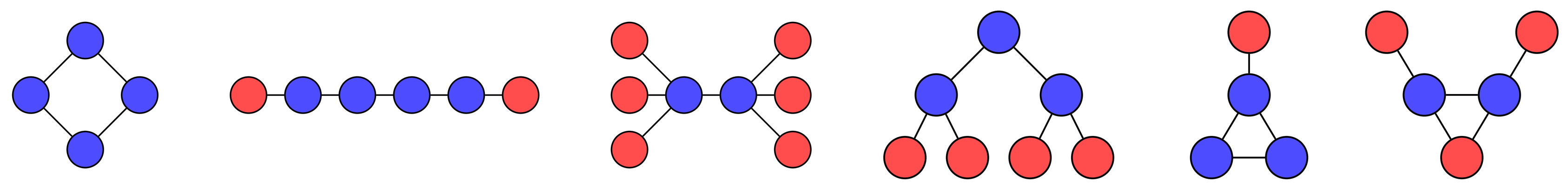




The Graph Metric

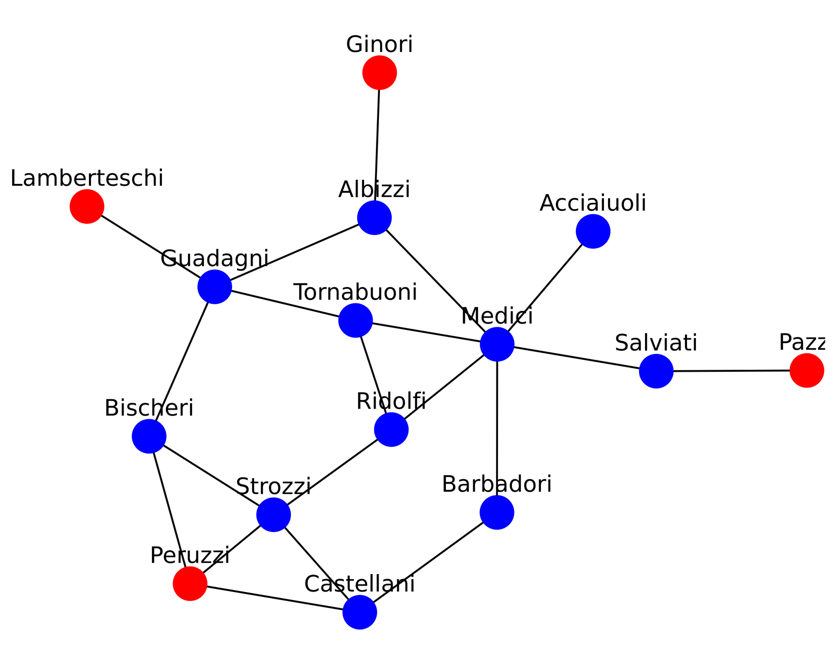
Voting on graphs
Nodes = voters
Candidates = subset of nodes
Voters prefer closer candidates by path distance
Voters have vote share 1 split among equidistant nodes

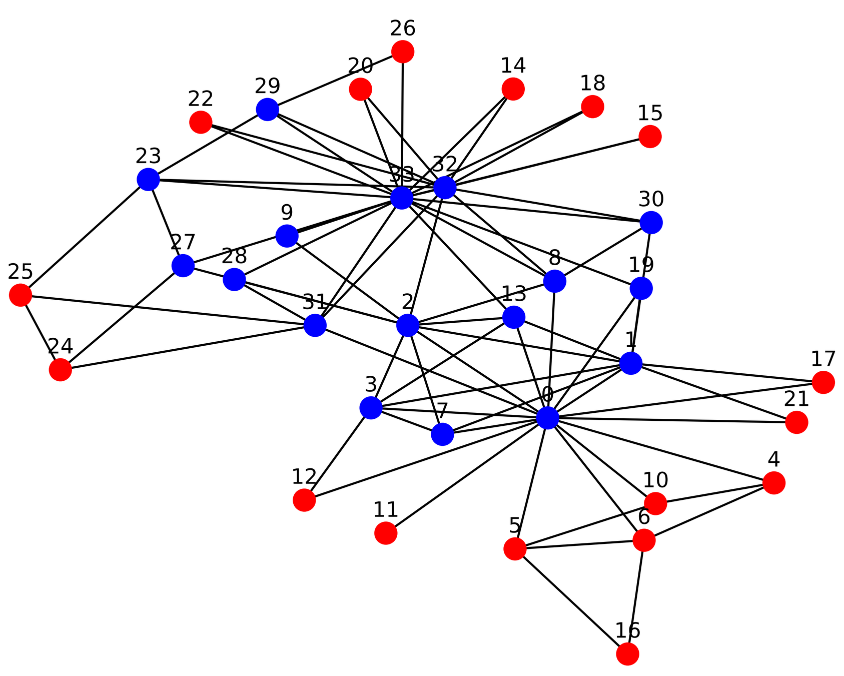
Minimal IRV exclusion zones of small graphs:



Theorem Perfect binary trees with odd height have no nontrivial IRV exclusion zones. For perfect binary trees with even height $h > 0$, the set of internal nodes is the minimal IRV exclusion zone.

Minimal IRV exclusion zones of real social networks:



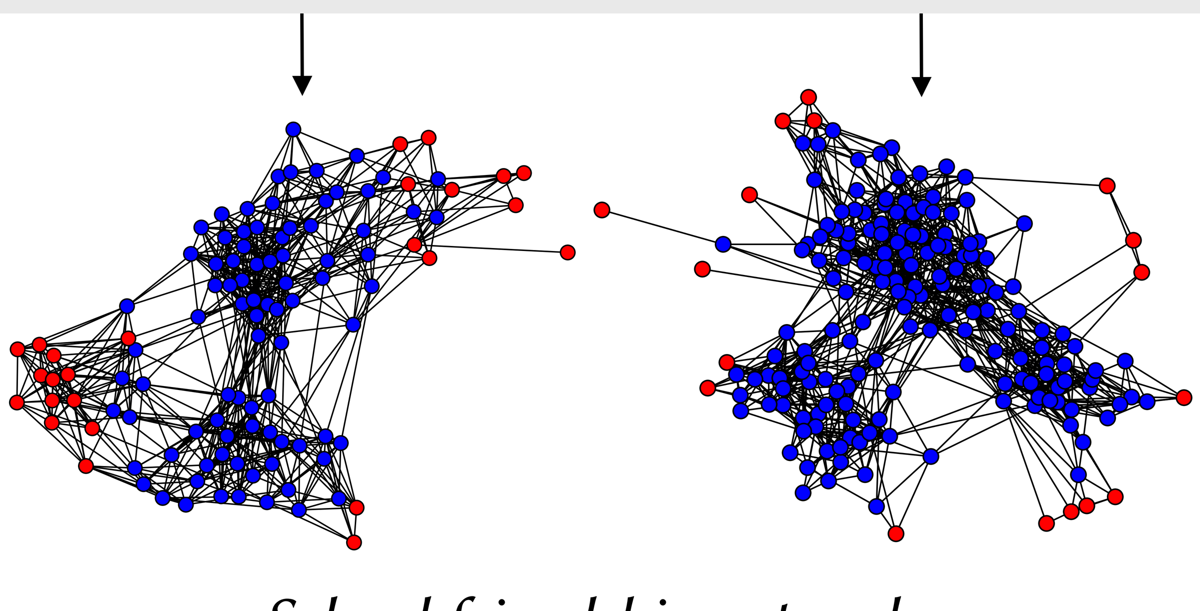


IRV-Exclusion Given a graph G and a set of nodes S, is S an IRV exclusion zone of G?

Theorem IRV-Exclusion is co-NP-complete.

Definition A set of nodes S is a $(1-\epsilon)$-approximate exclusion zone for a voting rule if, drawing a uniformly random election $C \subseteq V$ with $C \cap S \neq \emptyset$, the winner is in S w.p. at least $1 - \epsilon$.

Theorem Let $\epsilon, \delta \in (0, 1)$ and G be an n-node, m-edge graph. There is a randomized algorithm with runtime $O((n^3 + n^2 m)\log(1/\delta)/\epsilon^2)$ returning a set S such that:
1. S is a subset of the minimal IRV exclusion zone of G
2. S is a $(1-\epsilon)$-approximate IRV exclusion zone w.p. at least $1 - \delta$.



School friendship networks (Paluck et al, 2020)

Paper:

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