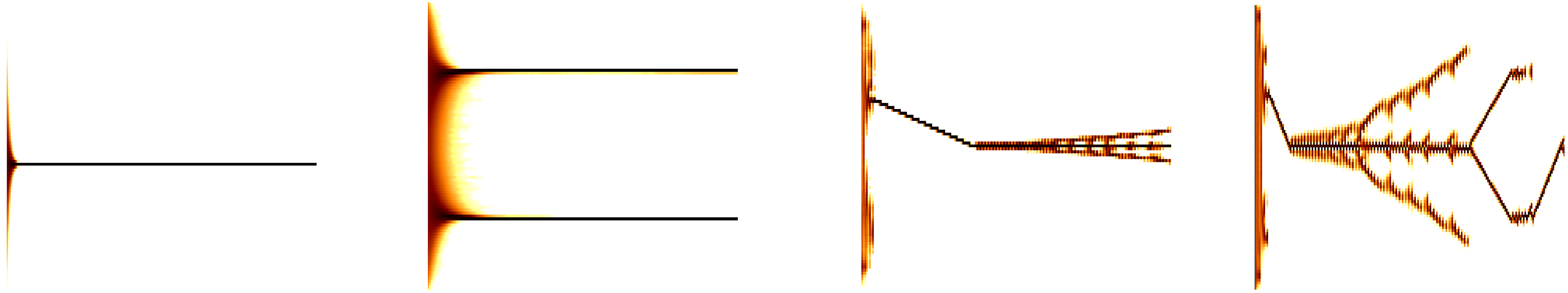
AAAI-25 / IAAI-25 / EAAI-25



Replicating Electoral Success

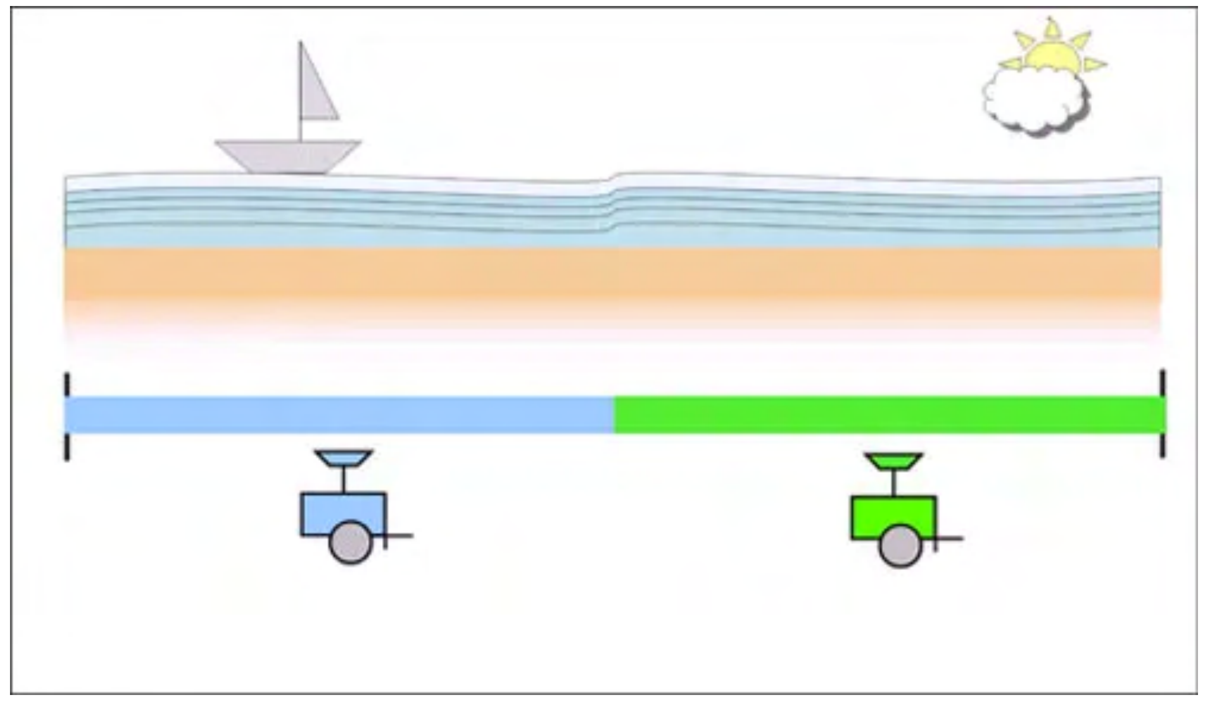
Kiran Tomlinson
Microsoft Research

Tanvi Namjoshi
Princeton University

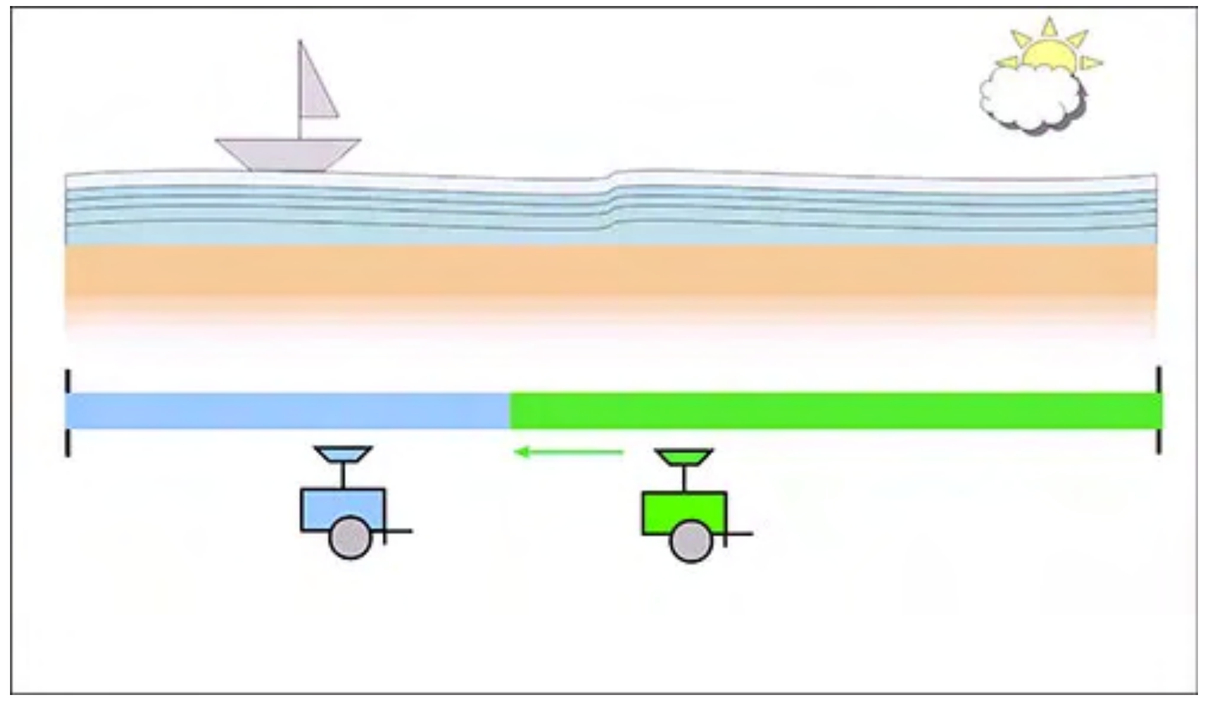
Johan Ugander
Stanford University

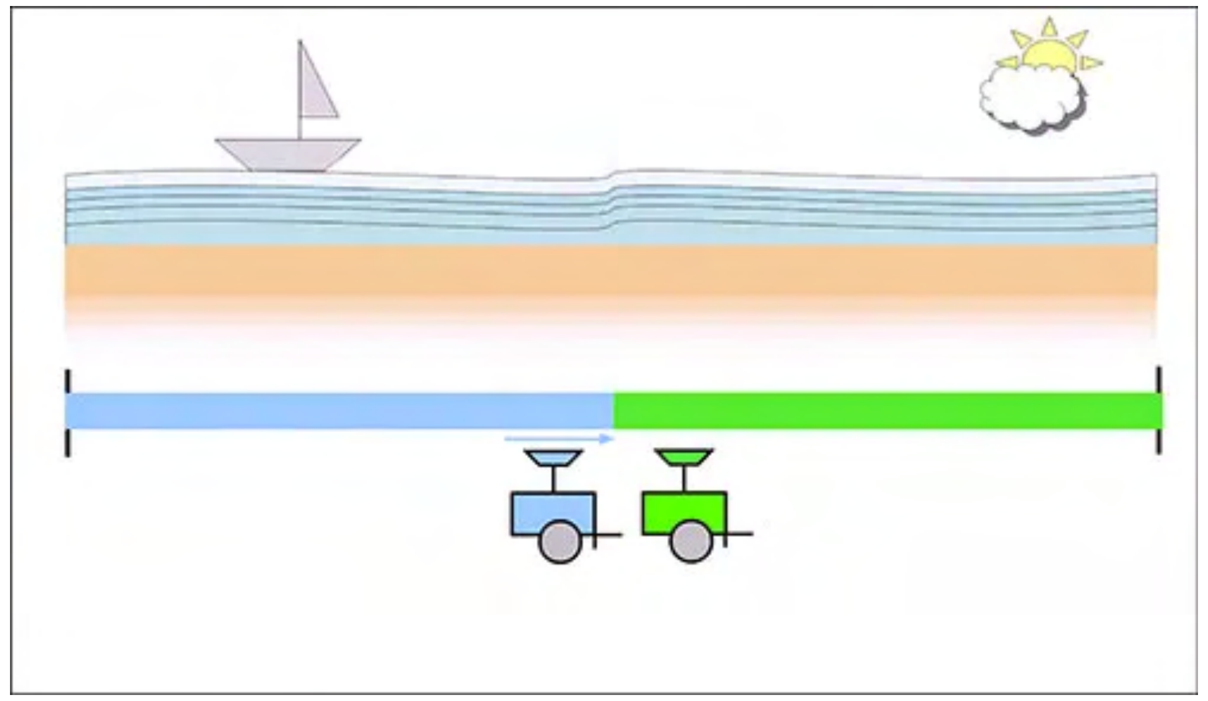
Jon Kleinberg
Cornell University

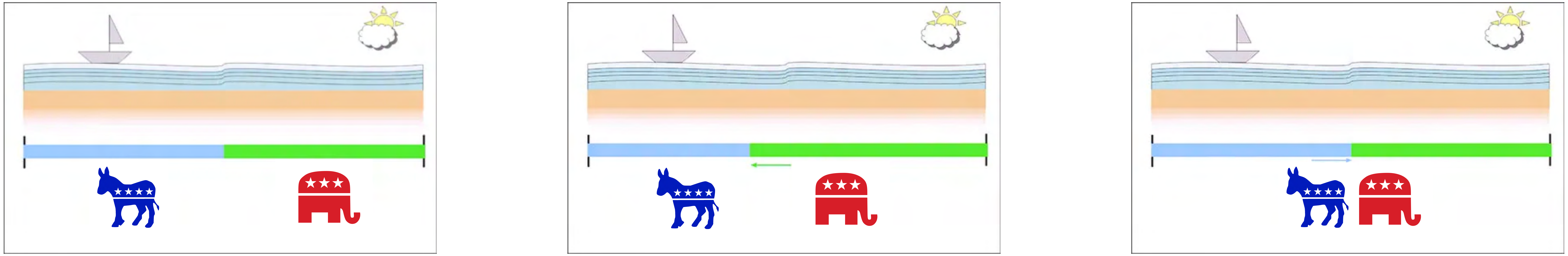
What positions do candidates choose if they want to win elections?



https://roughlydaily.com/tag/hotelling/







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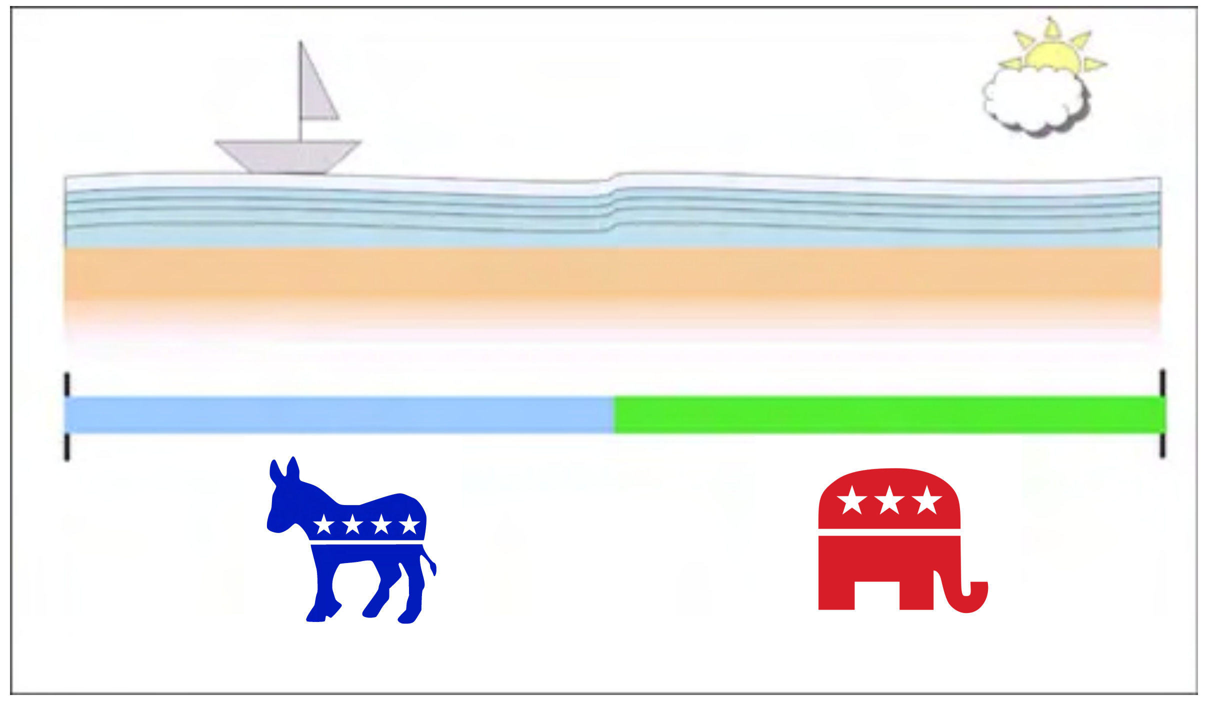
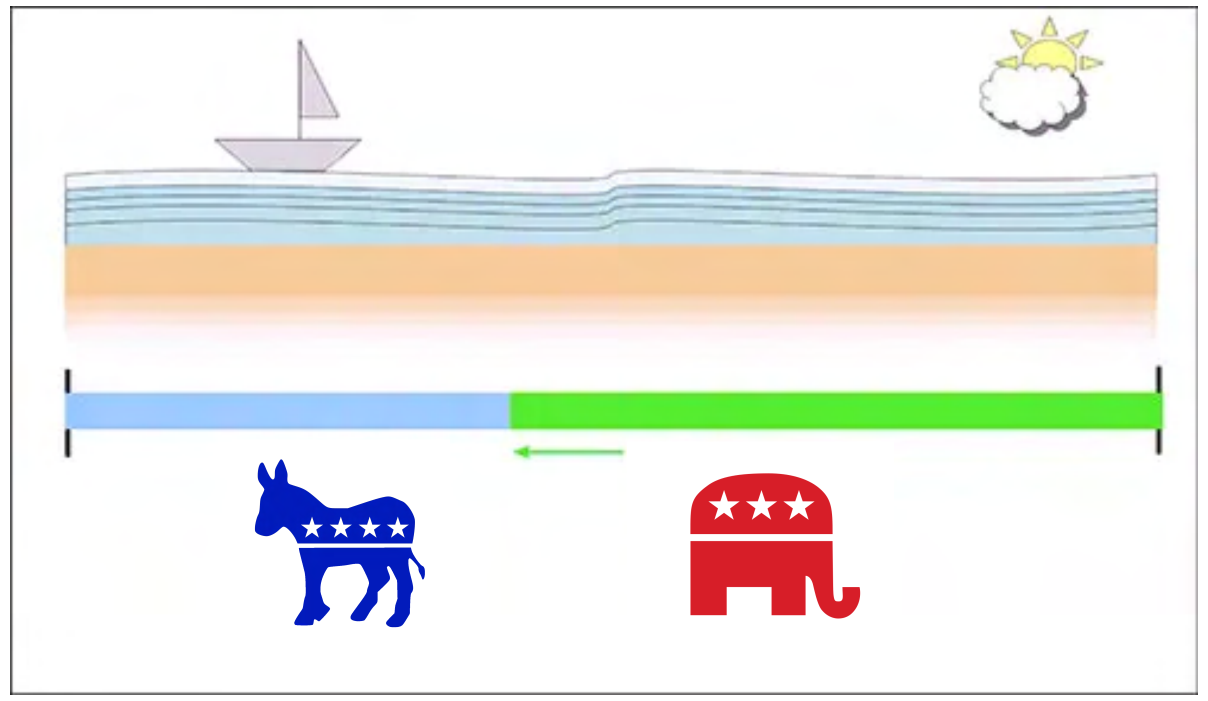
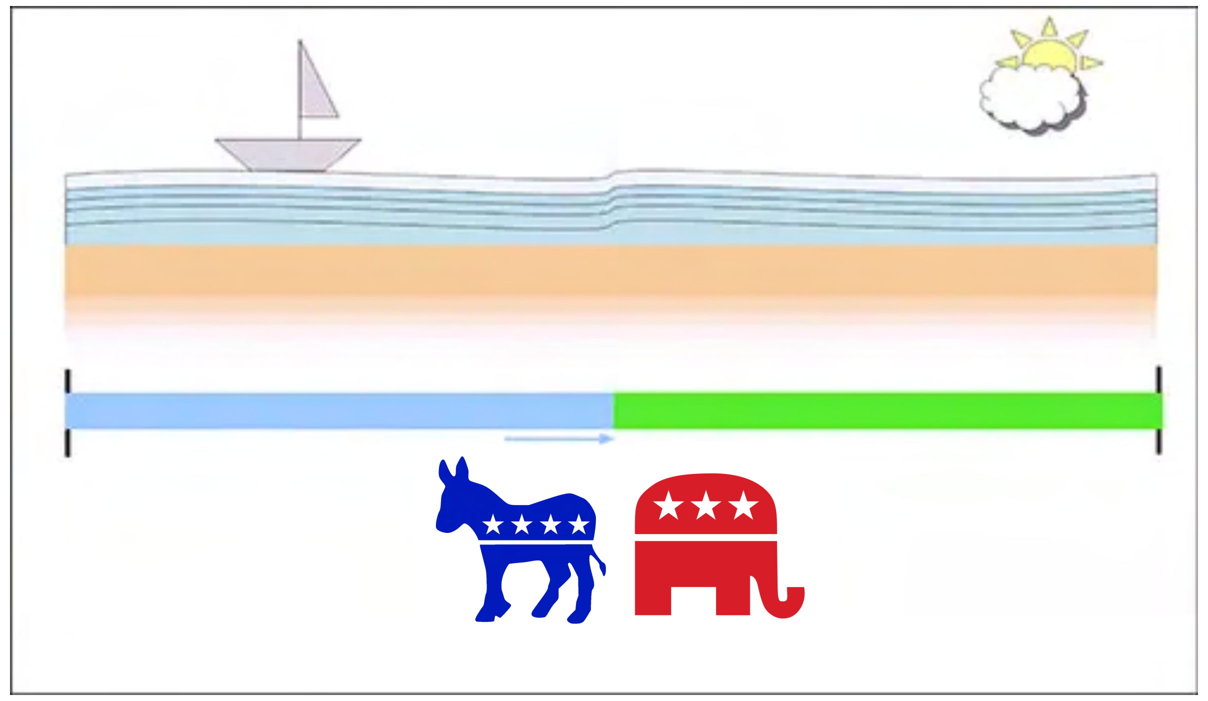
Hotelling-Downs model of candidate positioning

(Hotelling 1929, Downs 1957)

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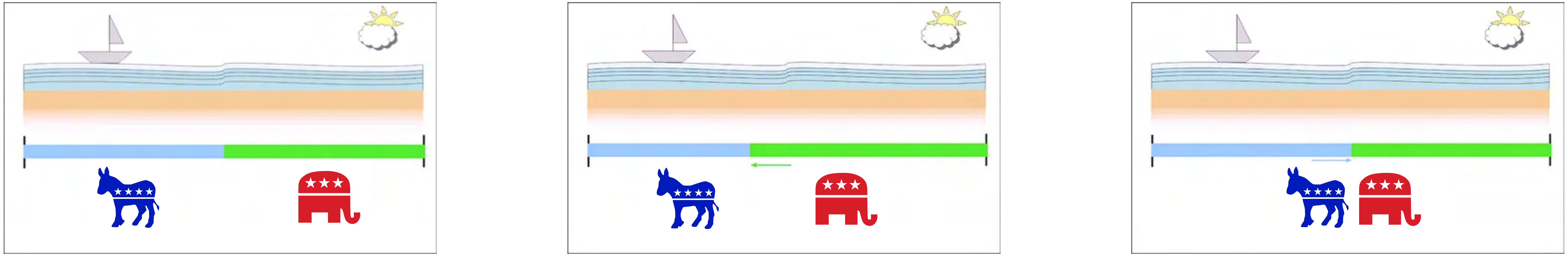

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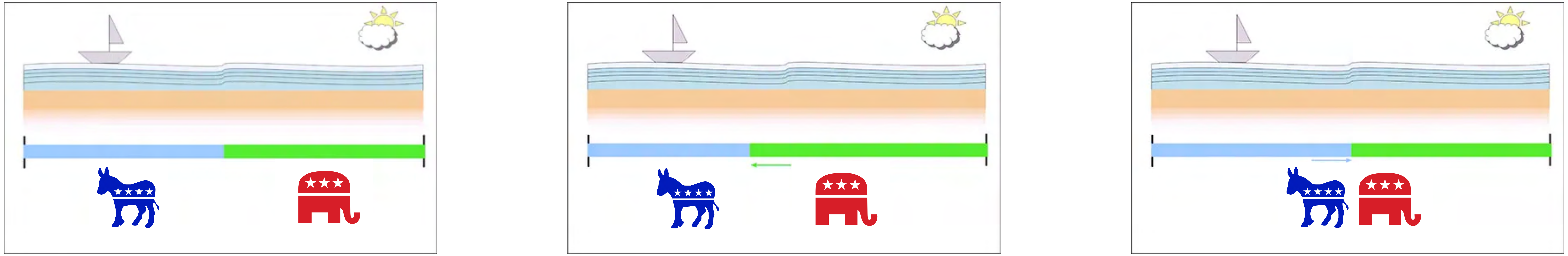

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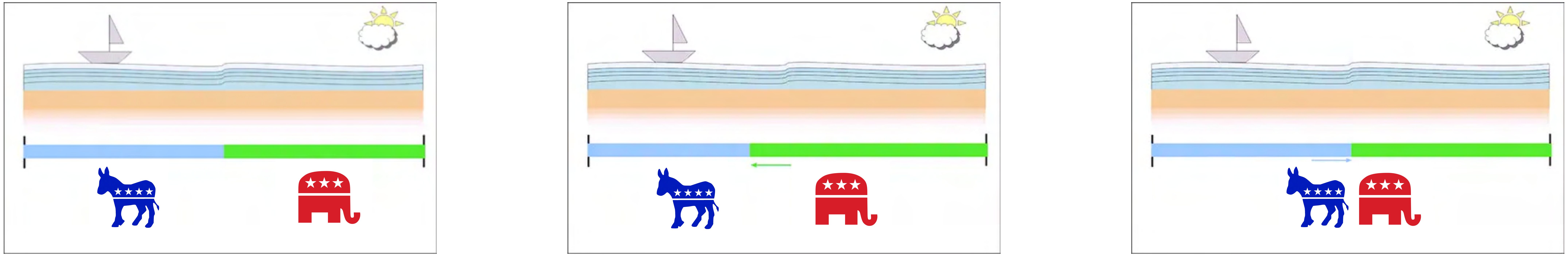

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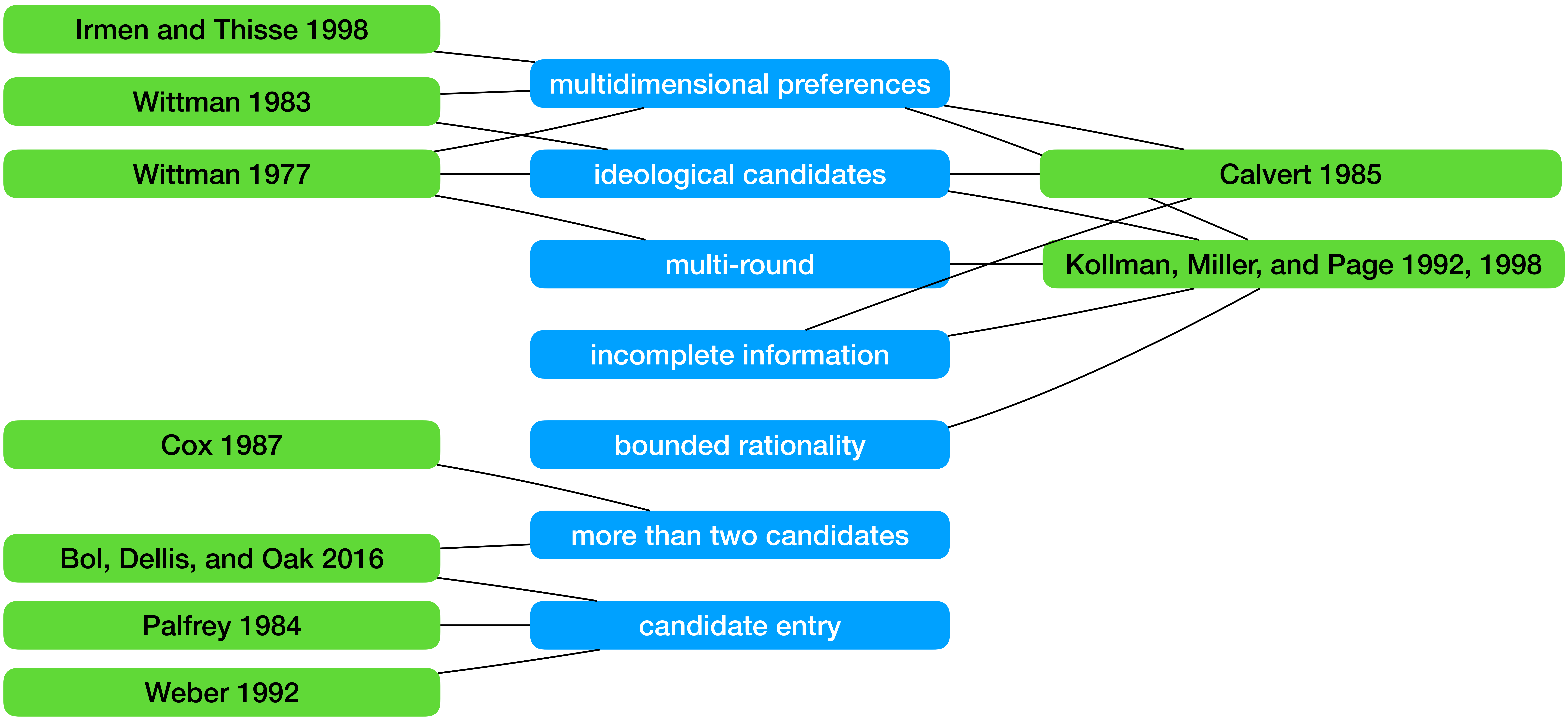
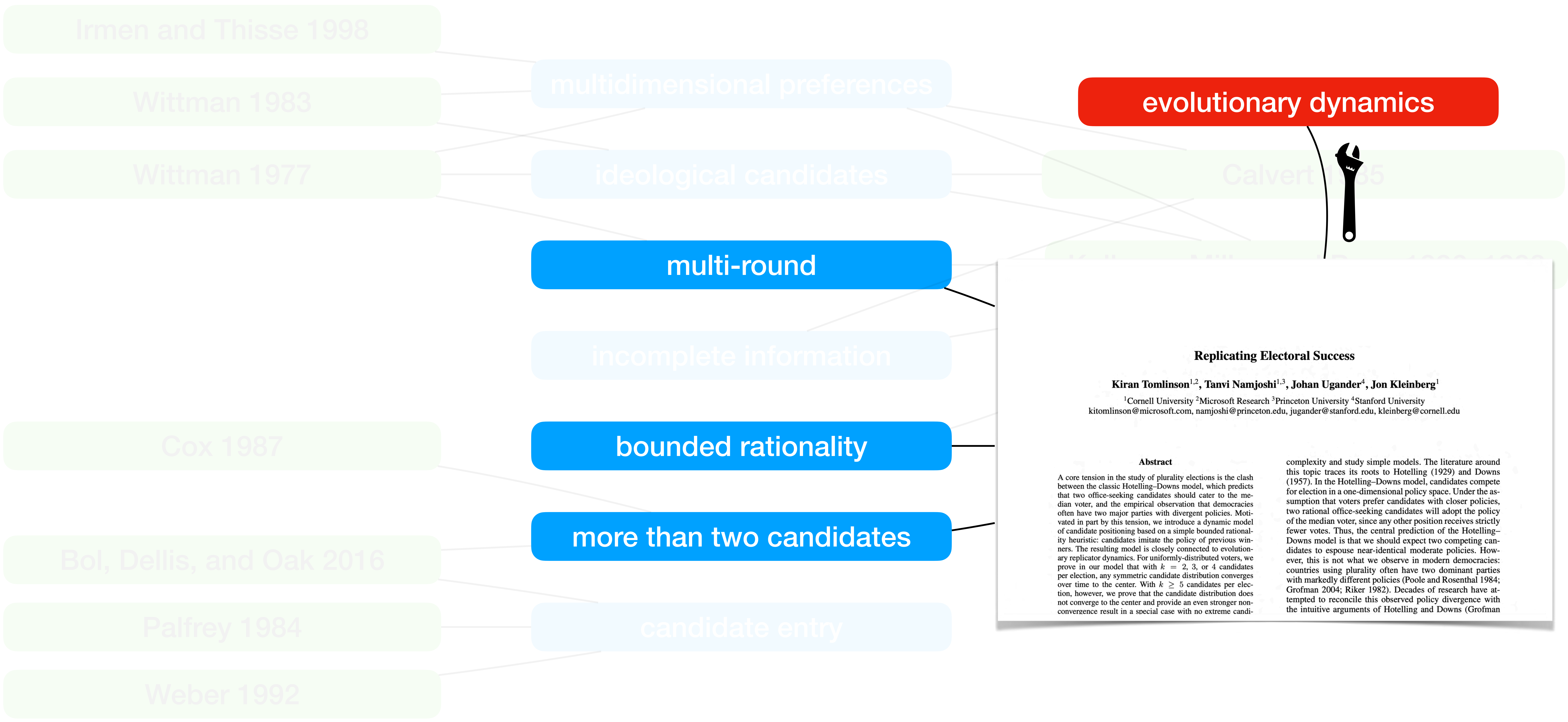
Assumes full information and perfect rationality

Decades of research on candidate positioning



Our work



Replicating Electoral Success

Kiran Tomlinson[1,2], **Tanvi Namjoshi**[1,3], **Johan Ugander**[4], **Jon Kleinberg**[1]

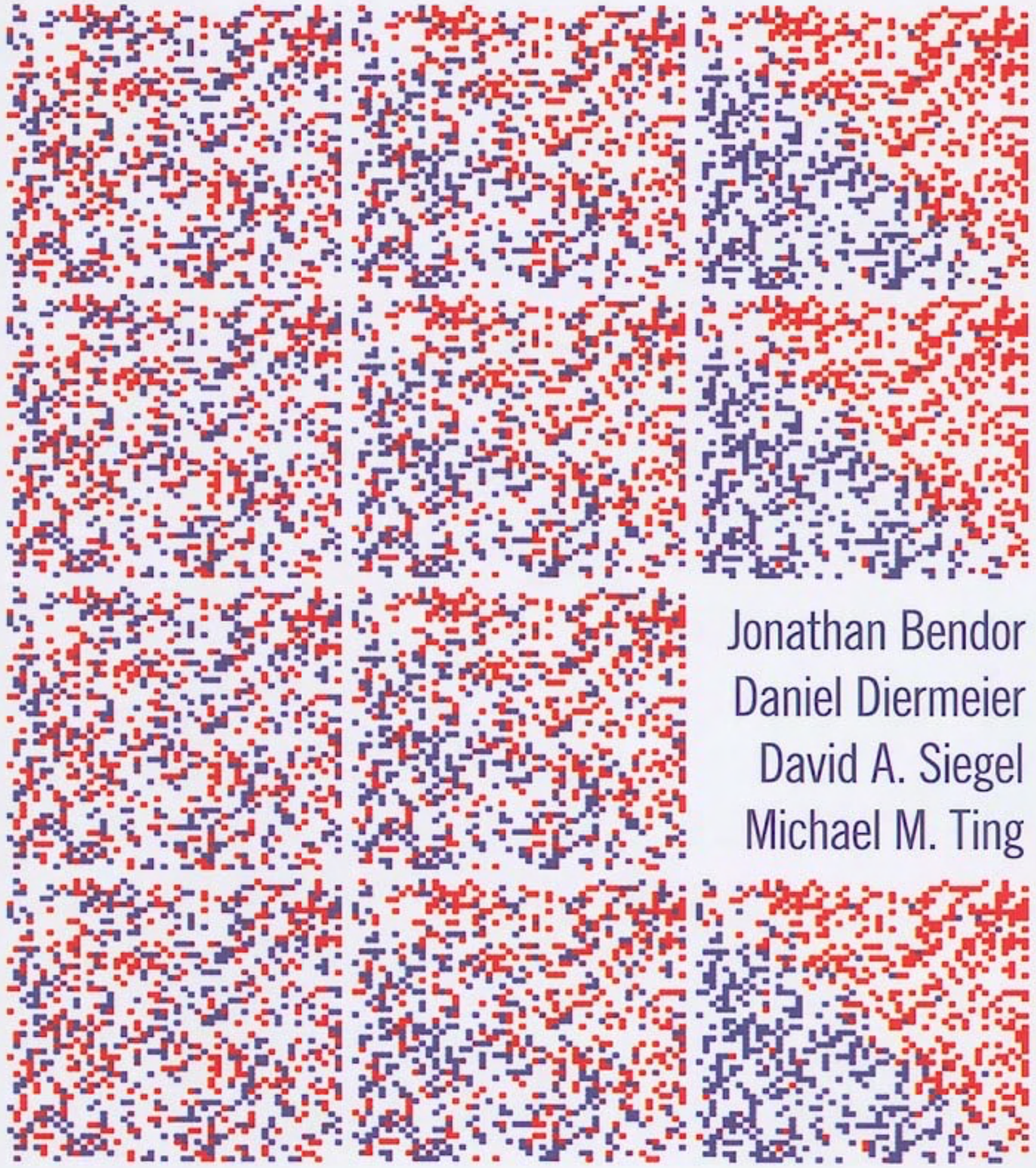
[1]Cornell University [2]Microsoft Research [3]Princeton University [4]Stanford University
kitomlinson@microsoft.com, namjoshi@princeton.edu, jugander@stanford.edu, kleinberg@cornell.edu

Abstract

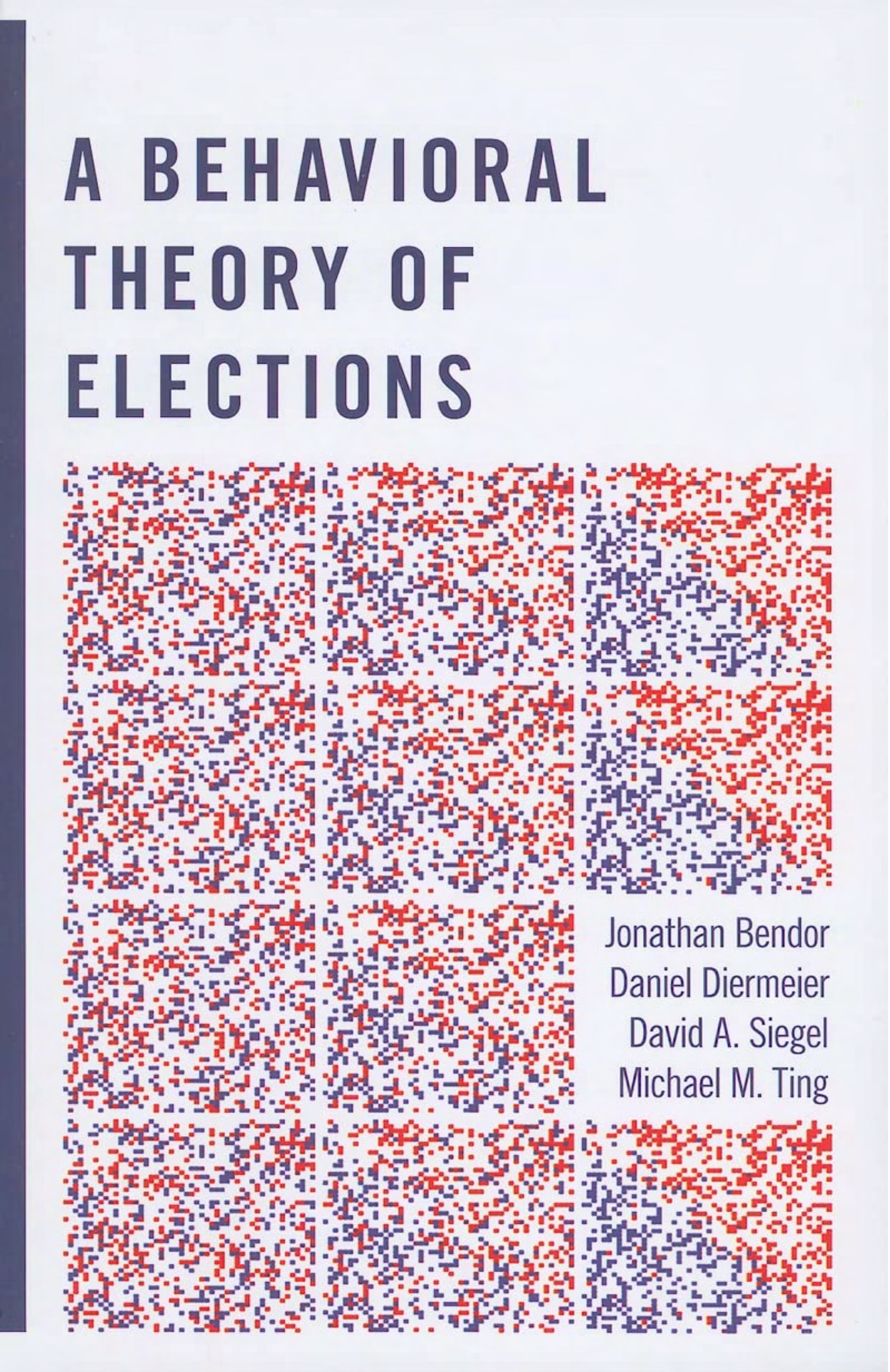
A core tension in the study of plurality elections is the clash between the classic Hotelling–Downs model, which predicts that two office-seeking candidates should cater to the median voter, and the empirical observation that democracies often have two major parties with divergent policies. Motivated in part by this tension, we introduce a dynamic model of candidate positioning based on a simple bounded rationality heuristic: candidates imitate the policy of previous winners. The resulting model is closely connected to evolutionary replicator dynamics. For uniformly-distributed voters, we prove in our model that with $k = 2$, 3, or 4 candidates per election, any symmetric candidate distribution converges over time to the center. With $k \geq 5$ candidates per election, however, we prove that the candidate distribution does not converge to the center and provide an even stronger non-convergence result in a special case with no extreme candi-

complexity and study simple models. The literature around this topic traces its roots to Hotelling (1929) and Downs (1957). In the Hotelling–Downs model, candidates compete for election in a one-dimensional policy space. Under the assumption that voters prefer candidates with closer policies, two rational office-seeking candidates will adopt the policy of the median voter, since any other position receives strictly fewer votes. Thus, the central prediction of the Hotelling–Downs model is that we should expect two competing candidates to espouse near-identical moderate policies. However, this is not what we observe in modern democracies: countries using plurality often have two dominant parties with markedly different policies (Poole and Rosenthal 1984; Grofman 2004; Riker 1982). Decades of research have attempted to reconcile this observed policy divergence with the intuitive arguments of Hotelling and Downs (Grofman

A BEHAVIORAL THEORY OF ELECTIONS



Jonathan Bendor
Daniel Diermeier
David A. Siegel
Michael M. Ting



In short, there are good reasons for believing that the basic properties of experiential learning—becoming more likely to use something that has worked in the past and less likely to repeat something that has failed—hold in presidential campaigns.

Bendor, Diermeier, Siegel, and Ting.
A Behavioral Theory of Elections, 2011.

Replicator dynamics for candidate positioning

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Population of *k*-candidate (plurality) elections proceeding in rounds

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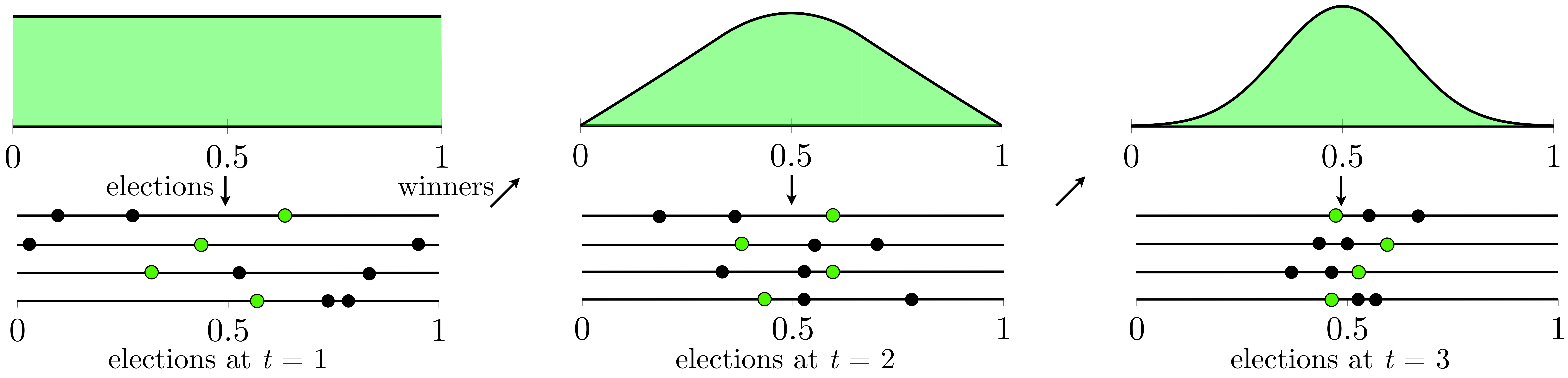

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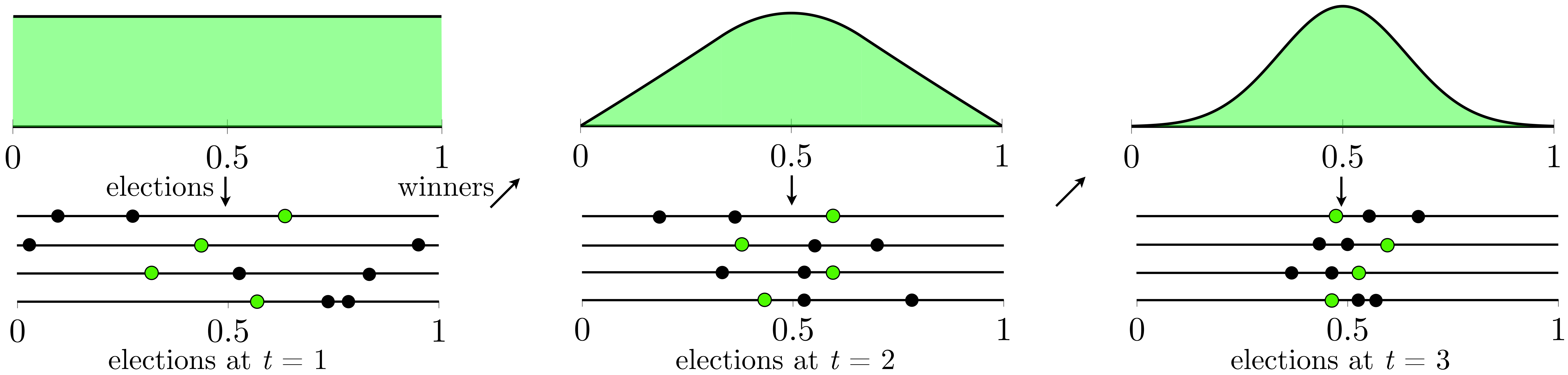

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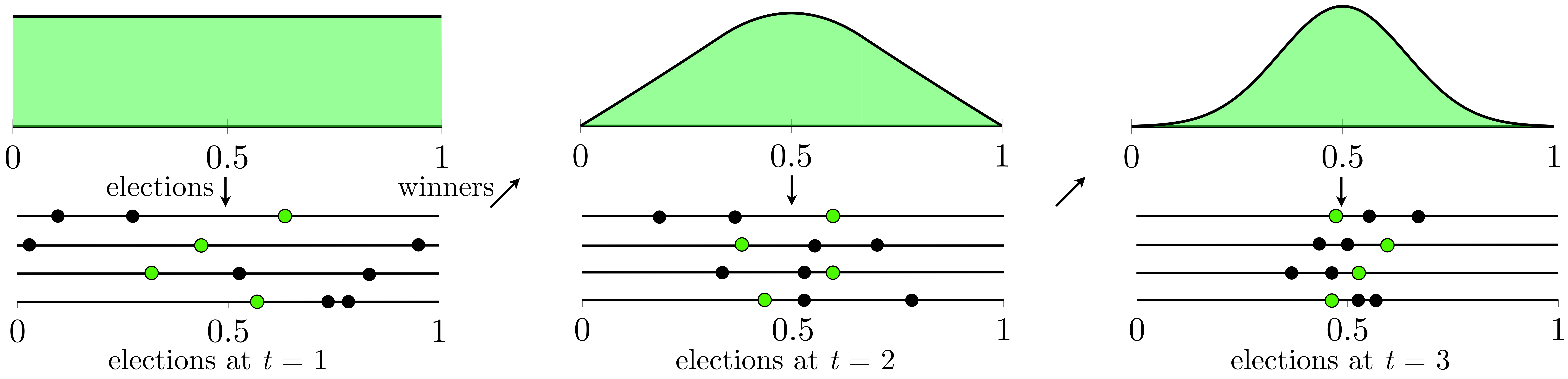

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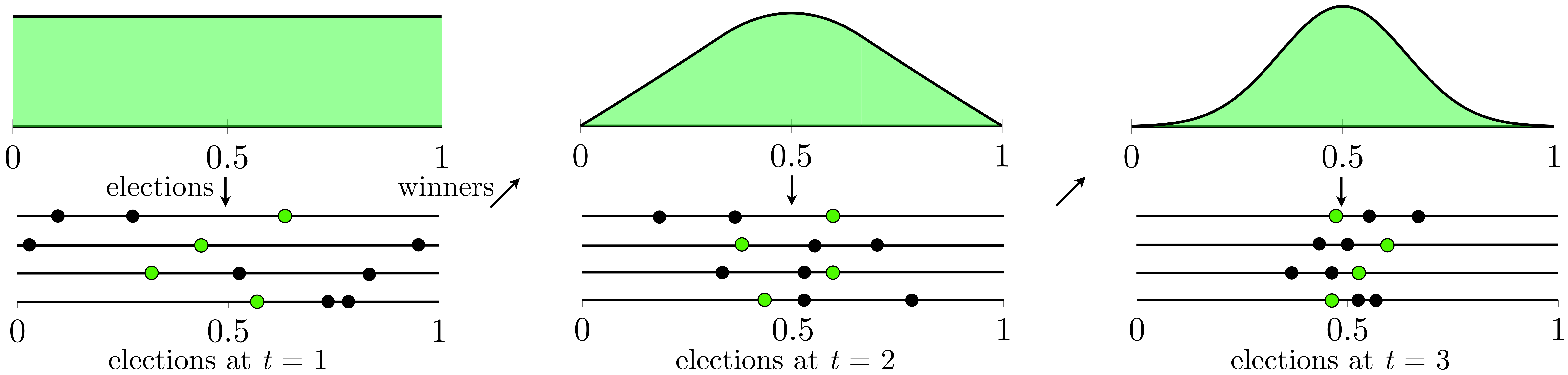

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≡ evolutionary replicator dynamics

(Taylor and Jonker 1978)

Simulating the dynamics

Initial distribution: uniform

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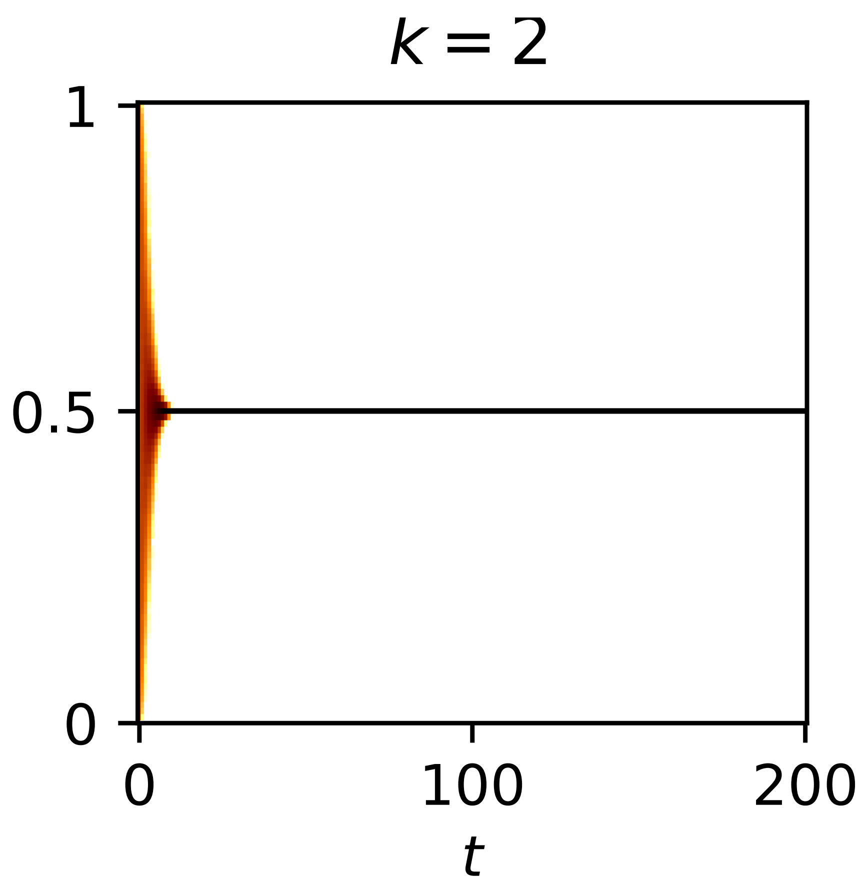

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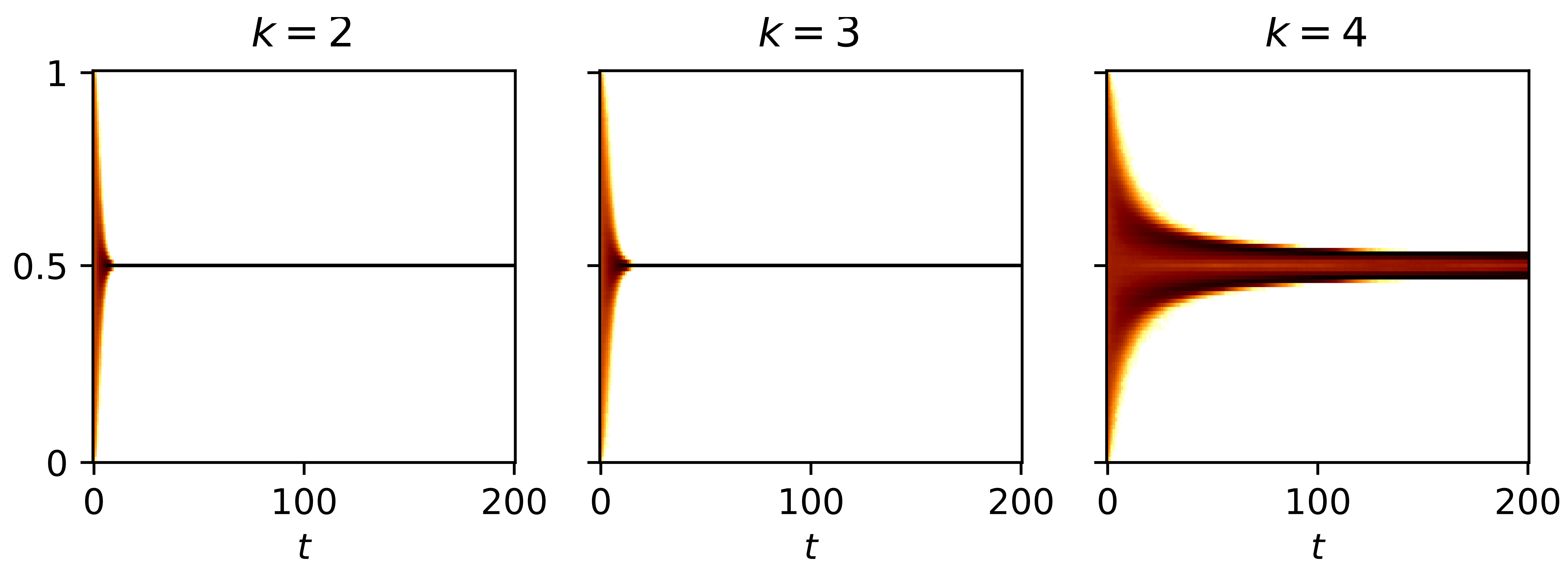

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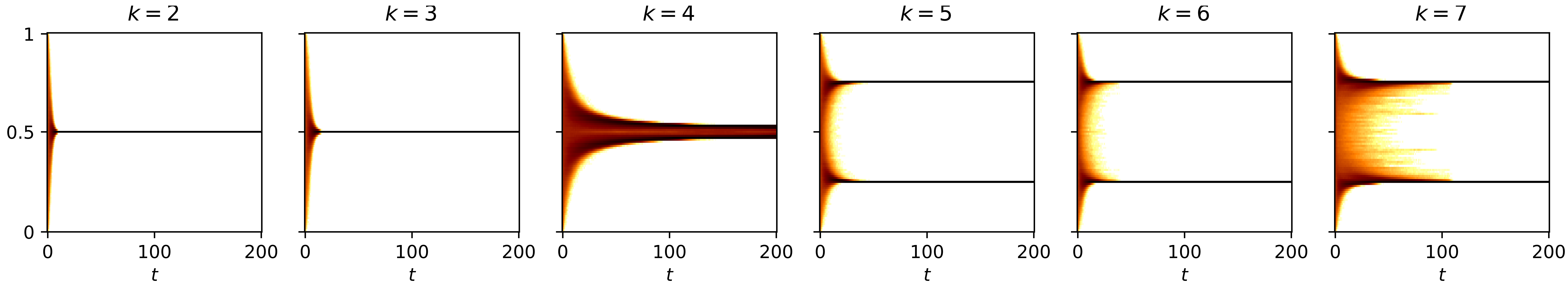

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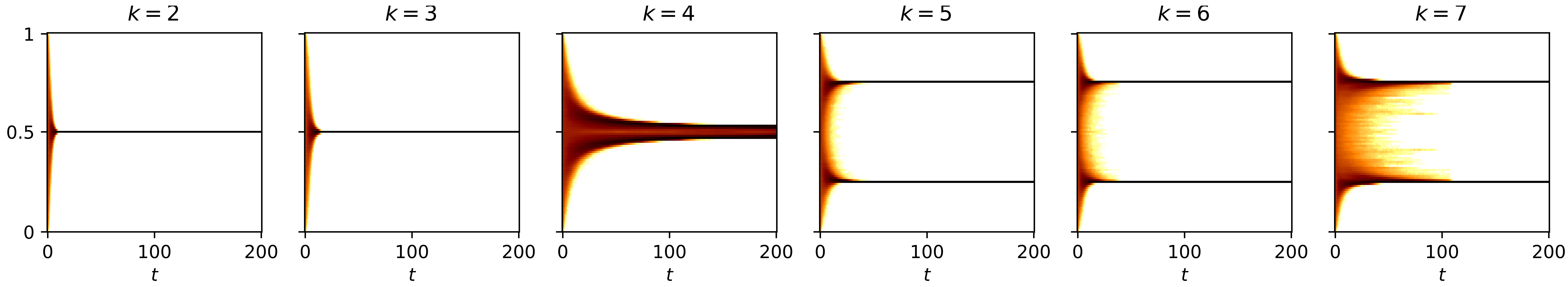
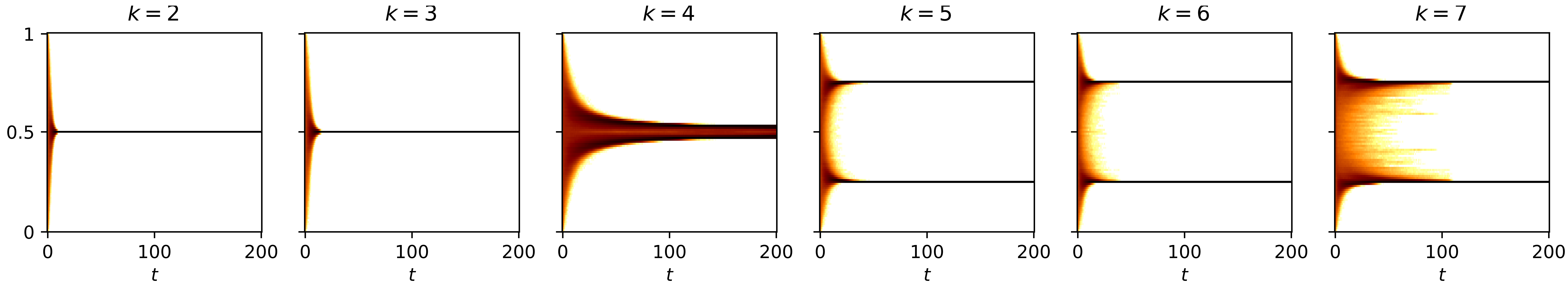


Phase transition?

Simulating the dynamics

Initial distribution: uniform



Phase transition?

Does this only happen when we start from uniform?

Characterizing the dynamics

Theorem
With $k \leq 4$ candidates per election, the candidate distribution converges to 1/2 *for any symmetric initial distribution*.

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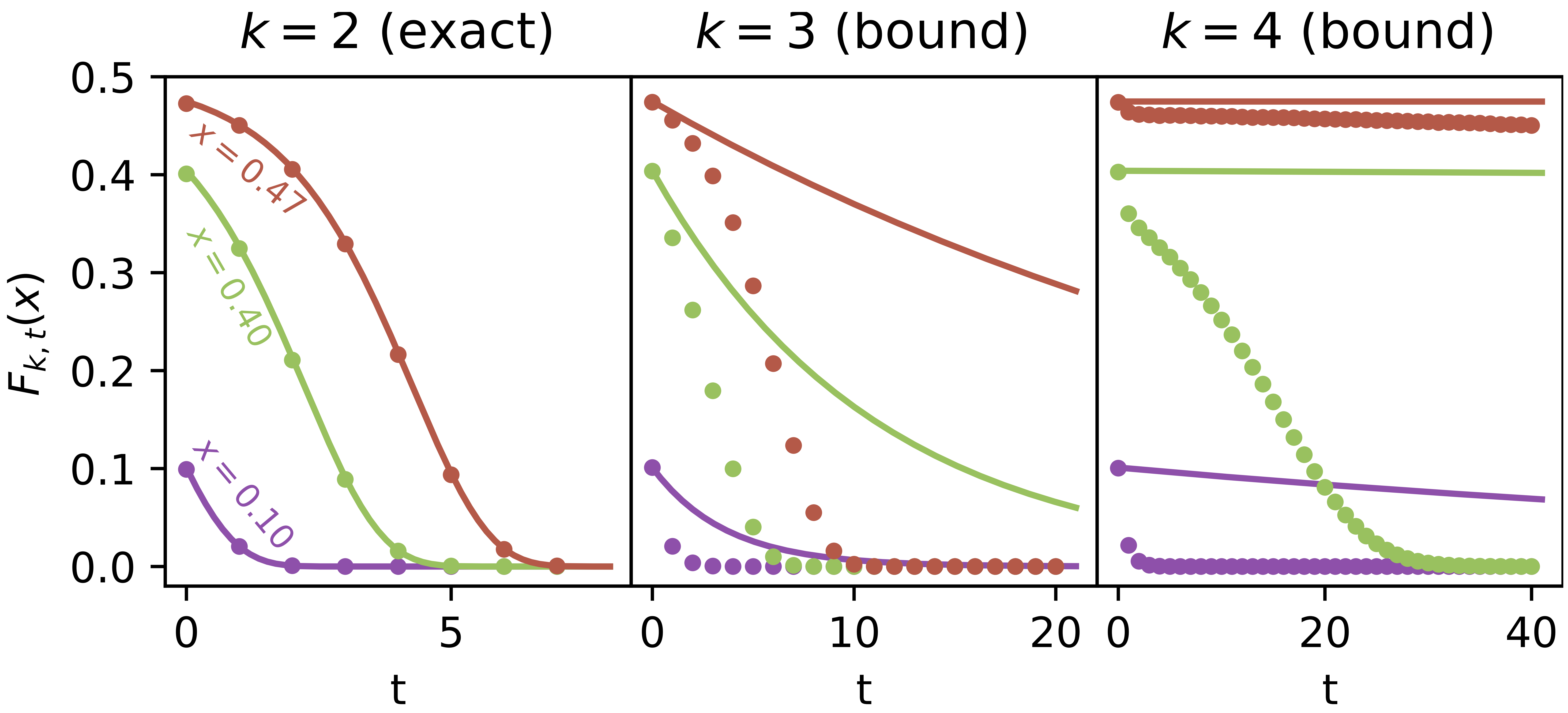
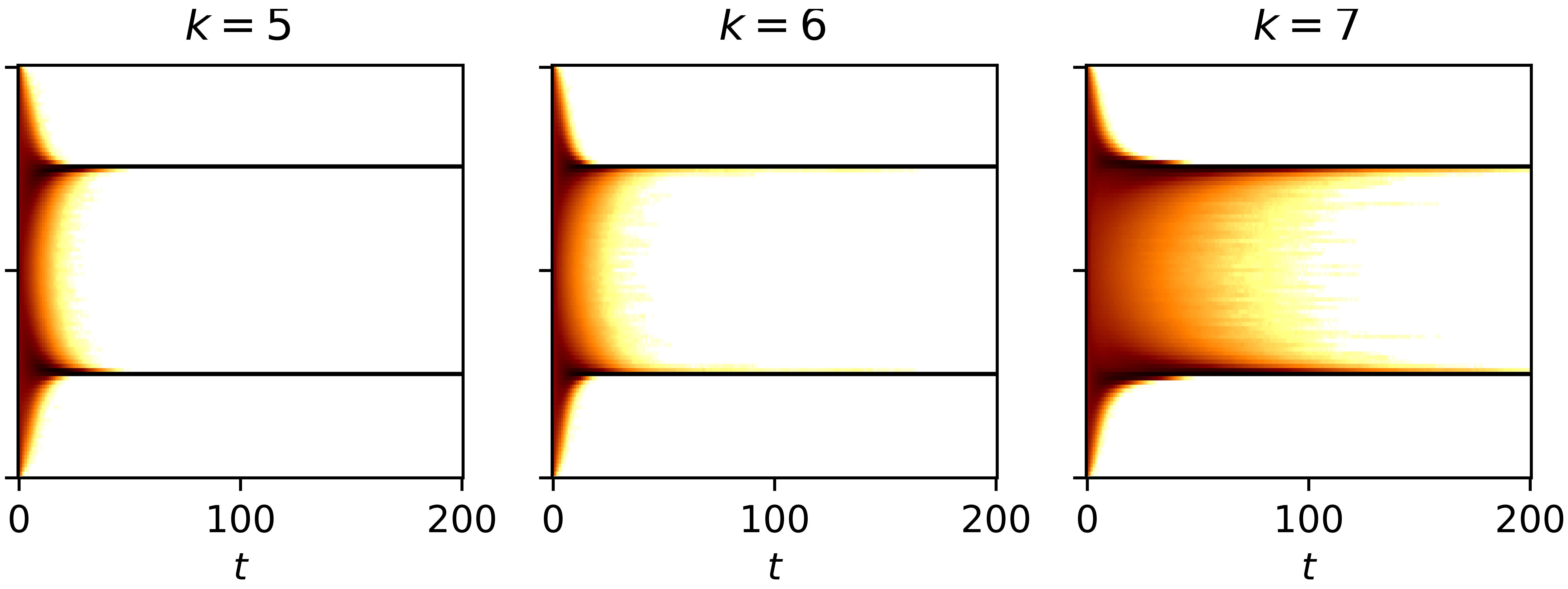


Characterizing the dynamics

Theorem
With $k \geq 5$ candidates per election, the candidate distribution does not converge to 1/2.



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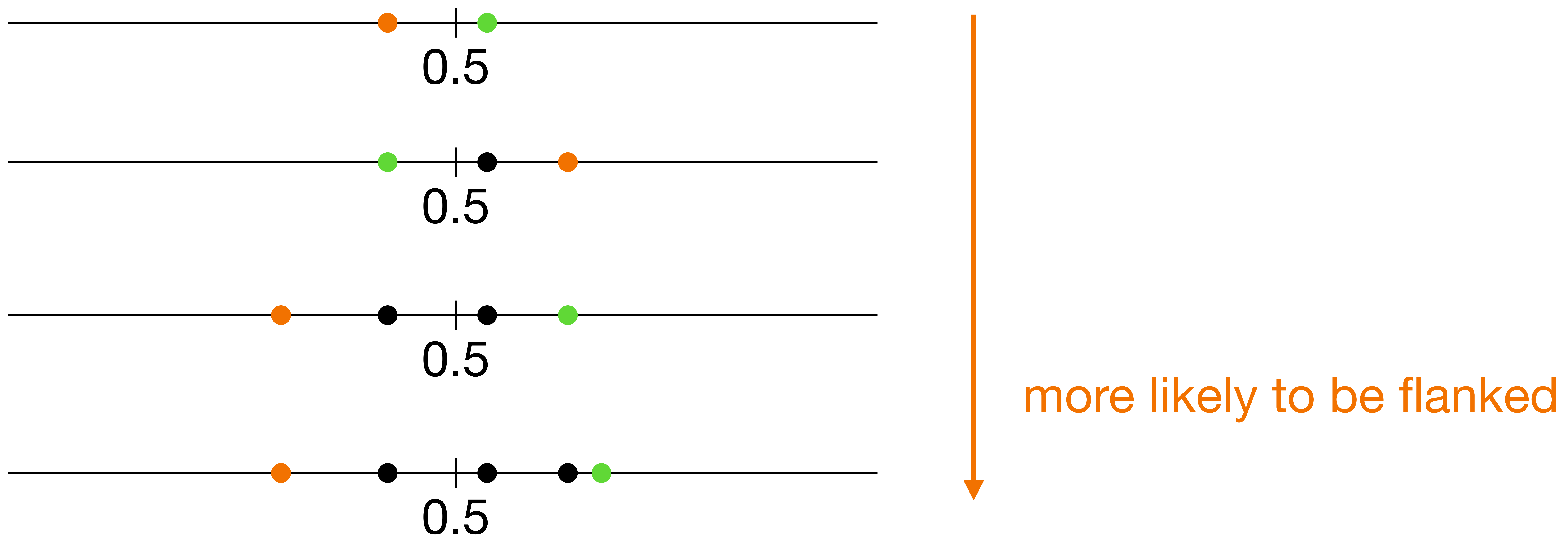

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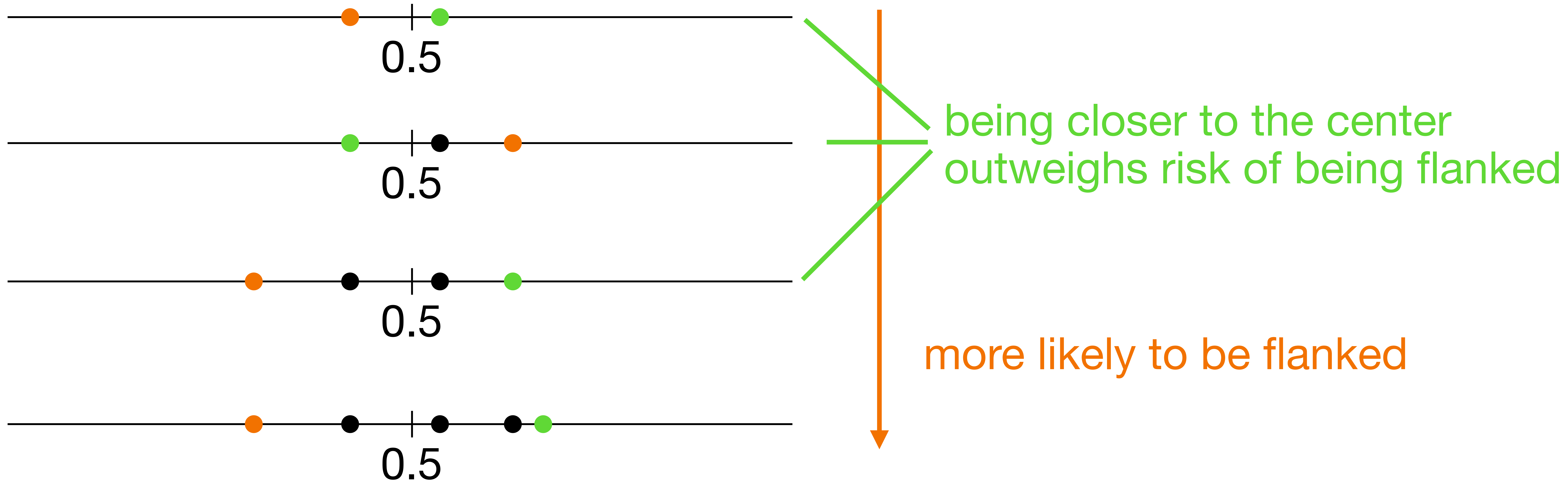

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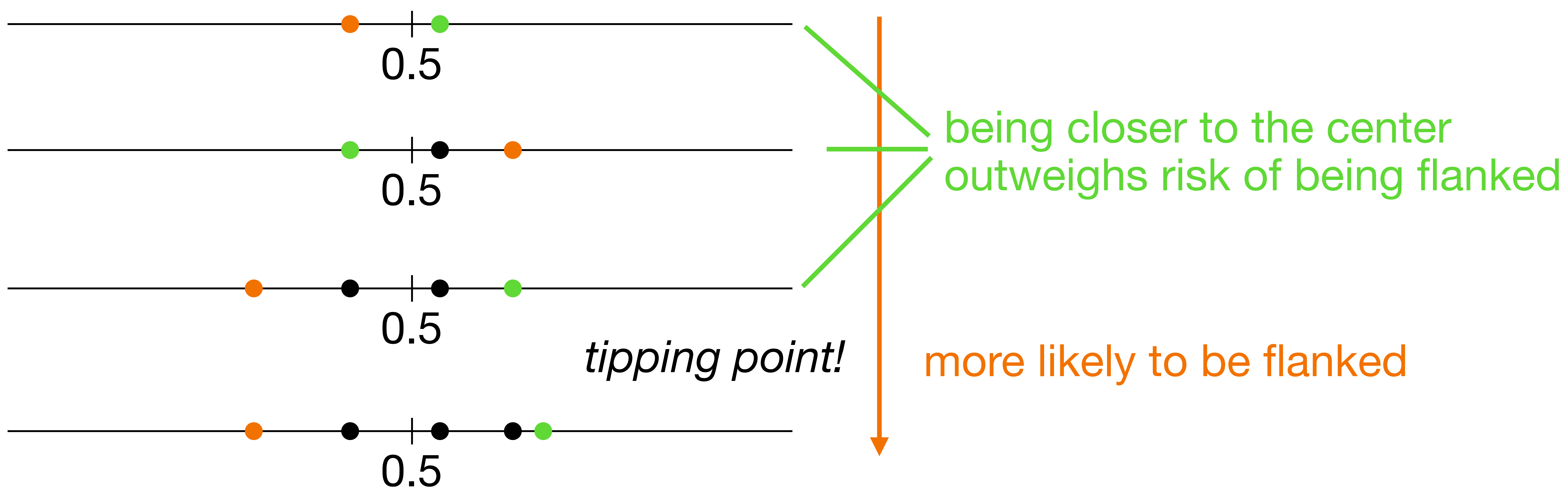

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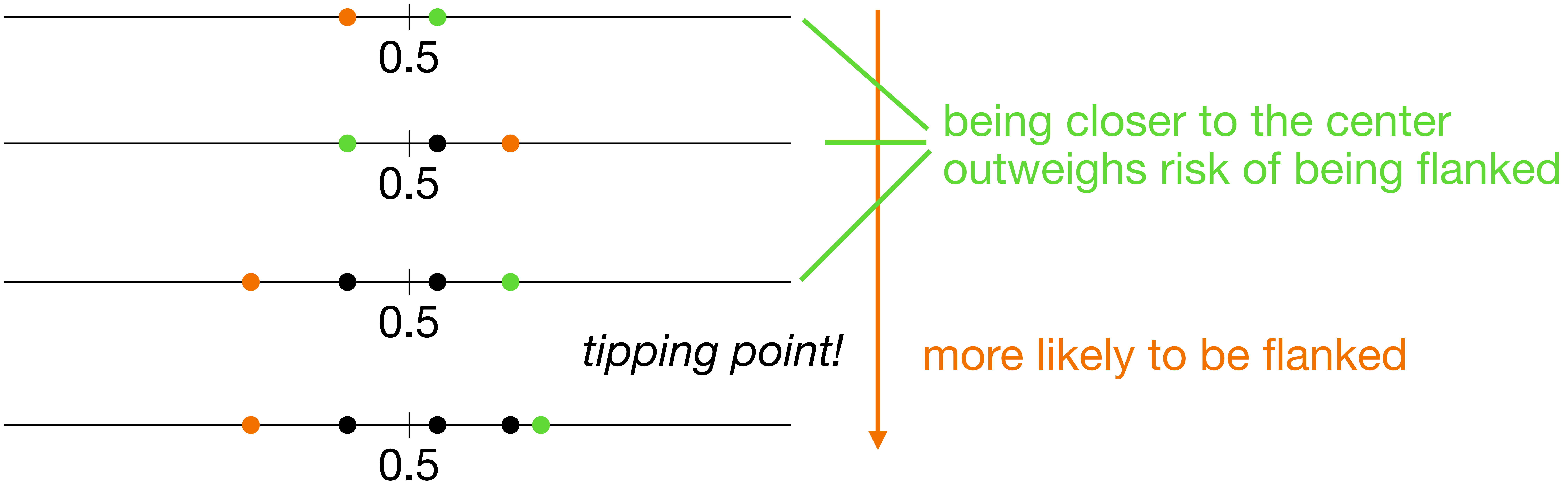
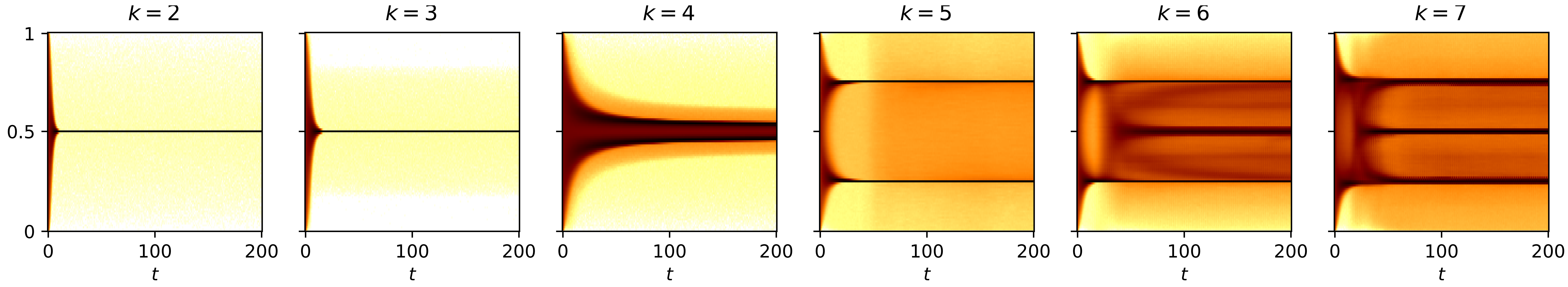


for $k \geq 5$, there are more flanked candidates than flanking candidates

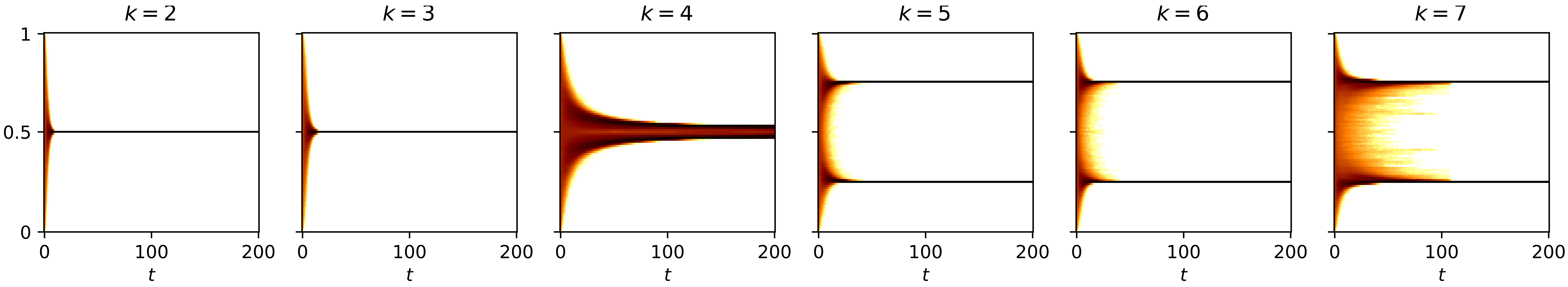
Our results are robust to noise

Theorem
With $k \leq 4$ candidates per election, ***some of which are positioned at random,*** the candidate distribution ***approximately*** converges to 1/2, but not when $k \geq 5$.



Further exploring the dynamics

Recall:



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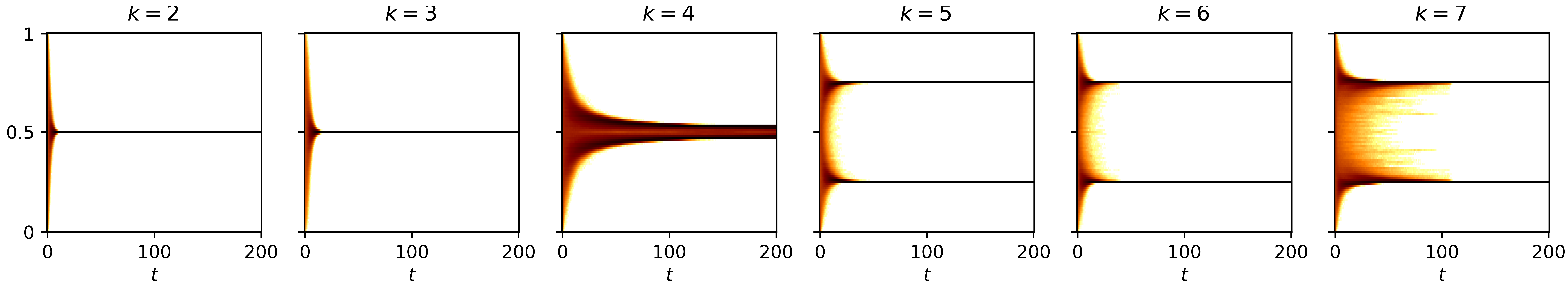
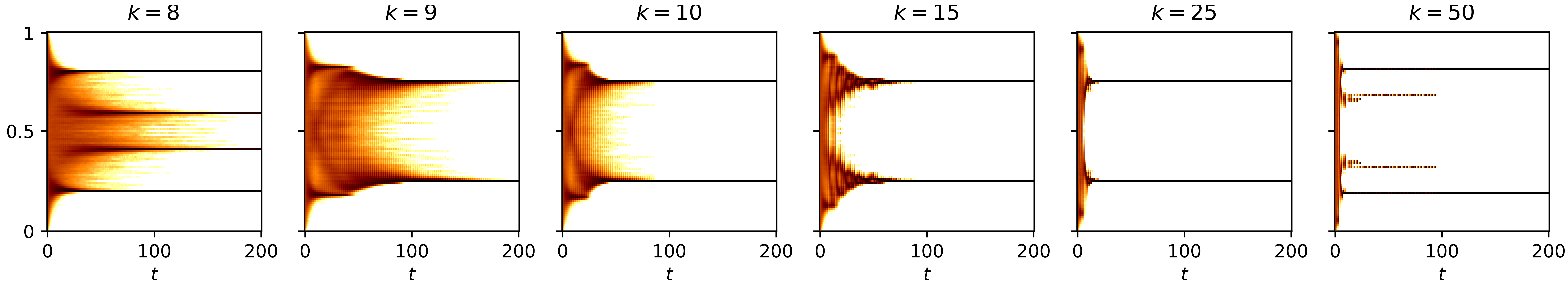
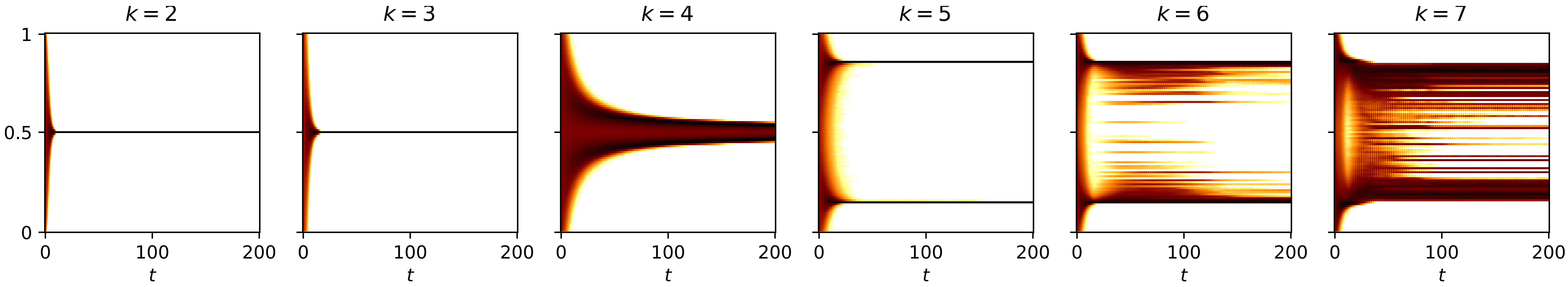


More candidates:



The same pattern occurs with: other voter distributions

Beta(1/2, 1/2)



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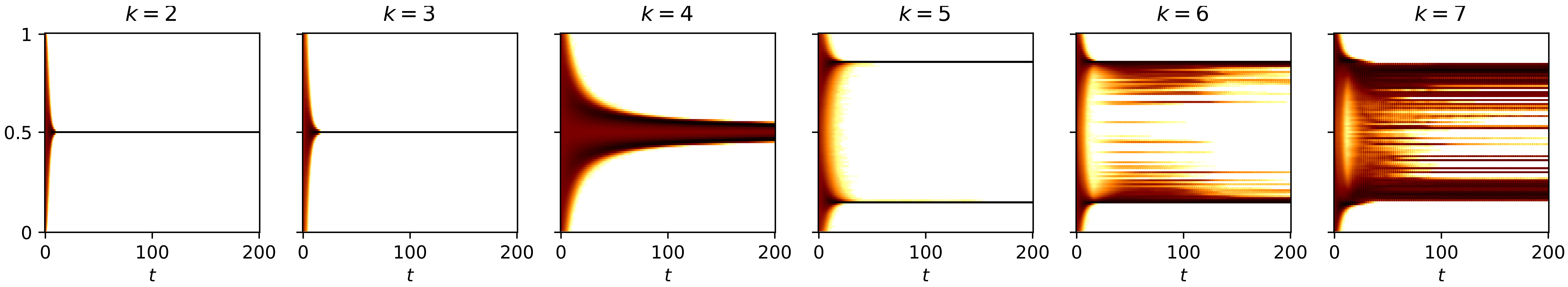
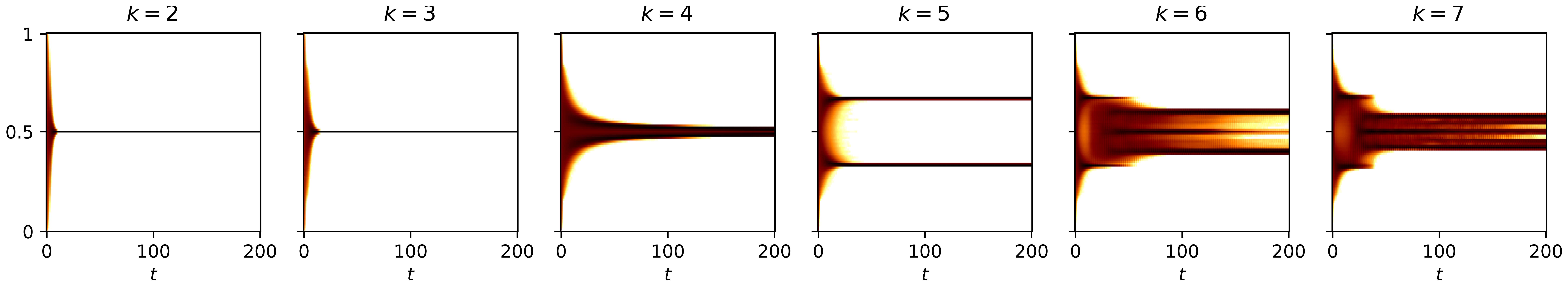
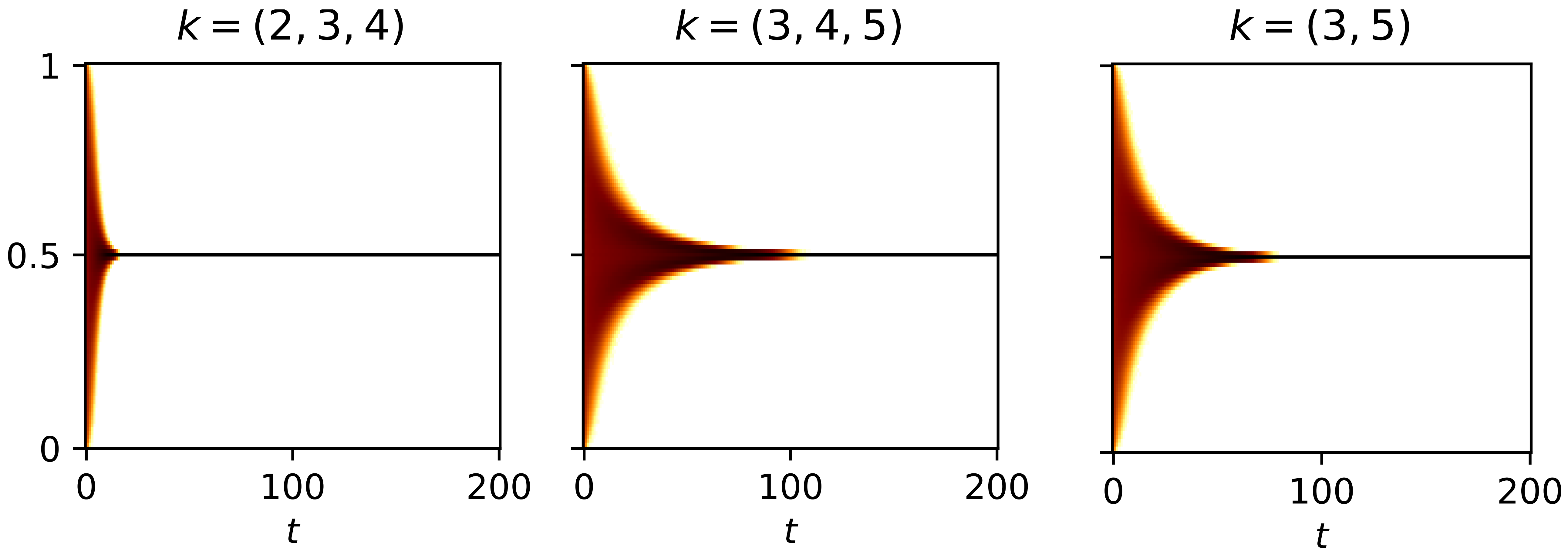


Beta(2, 2)



The same pattern occurs with: a mixture of candidate counts

Mostly < 5



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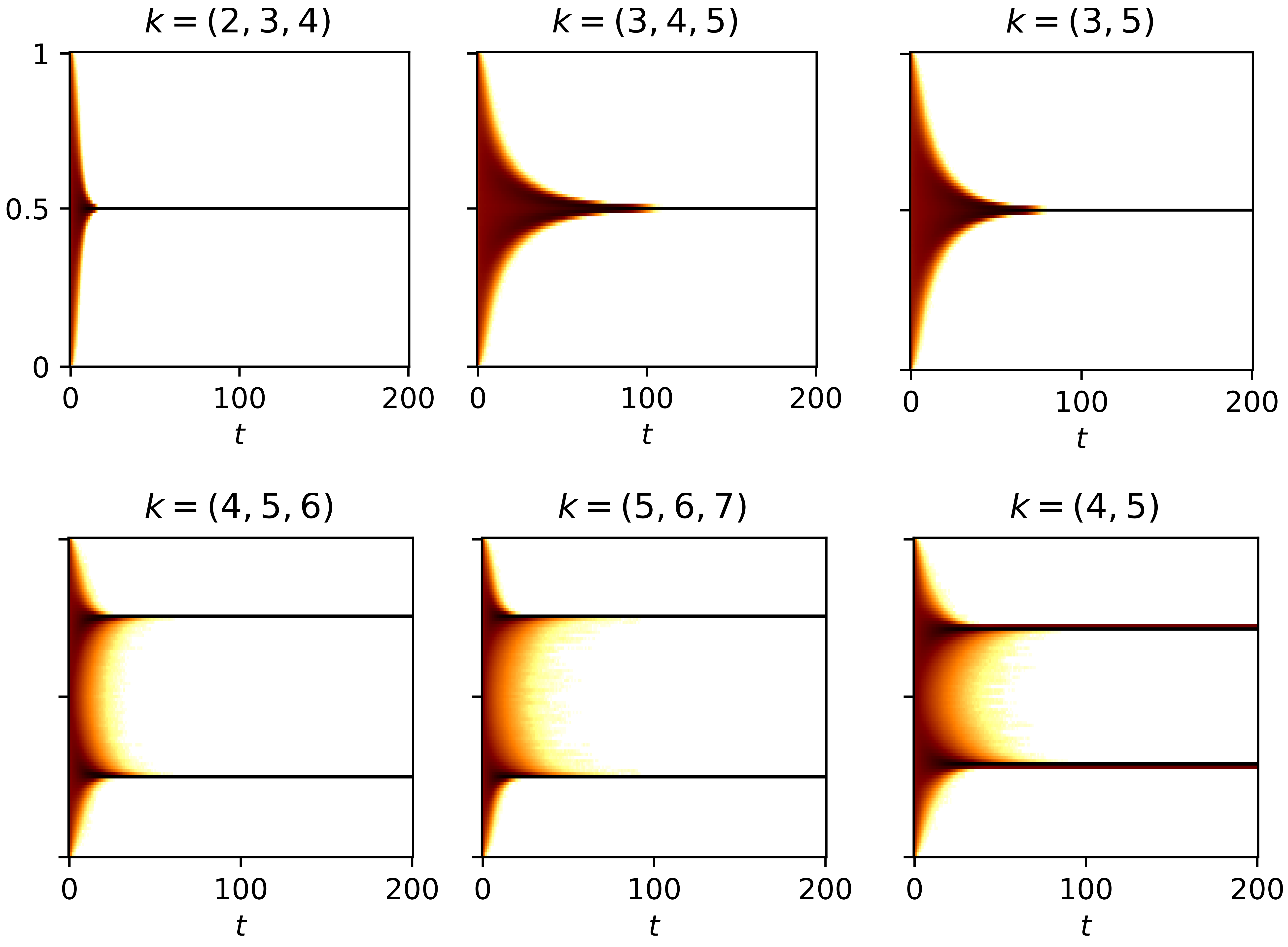
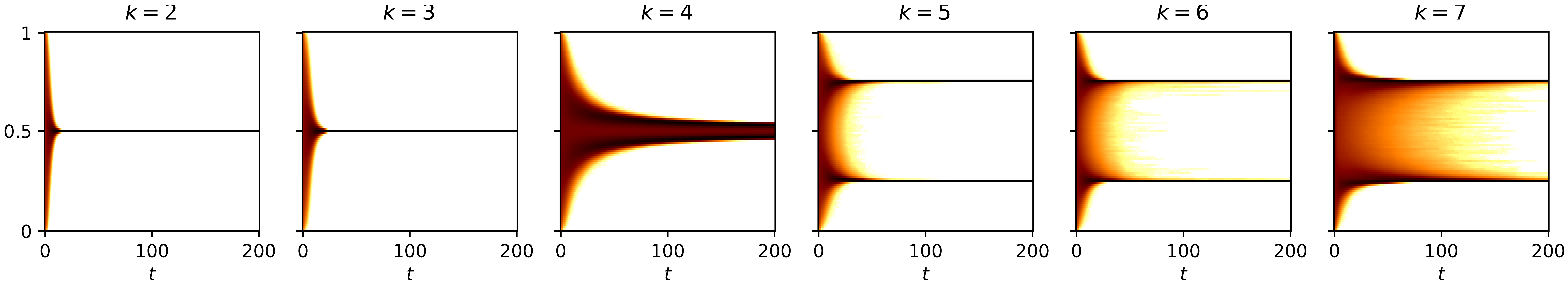
Mostly ≥ 5



The same pattern occurs with: memory of prior rounds

2 round memory



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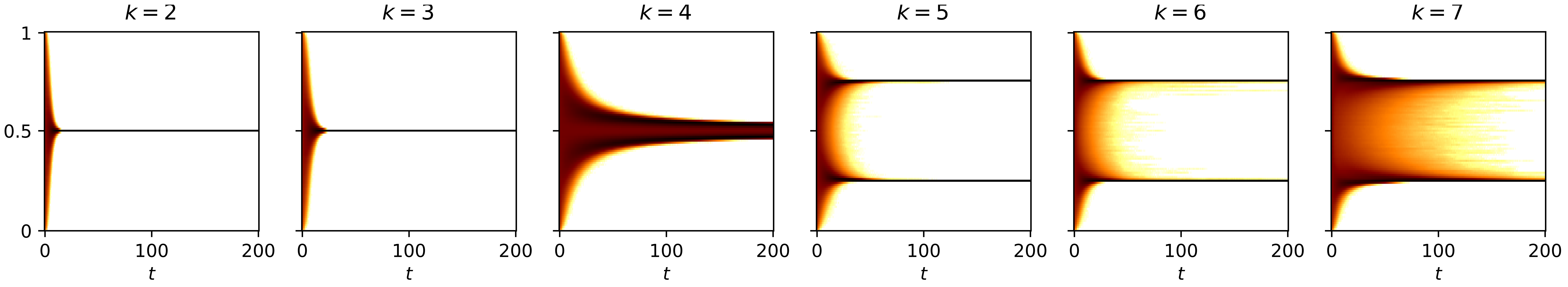
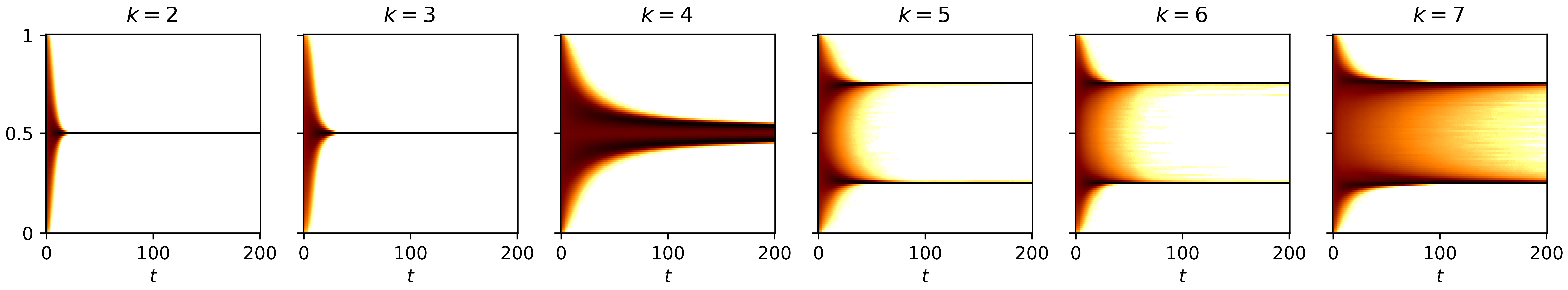
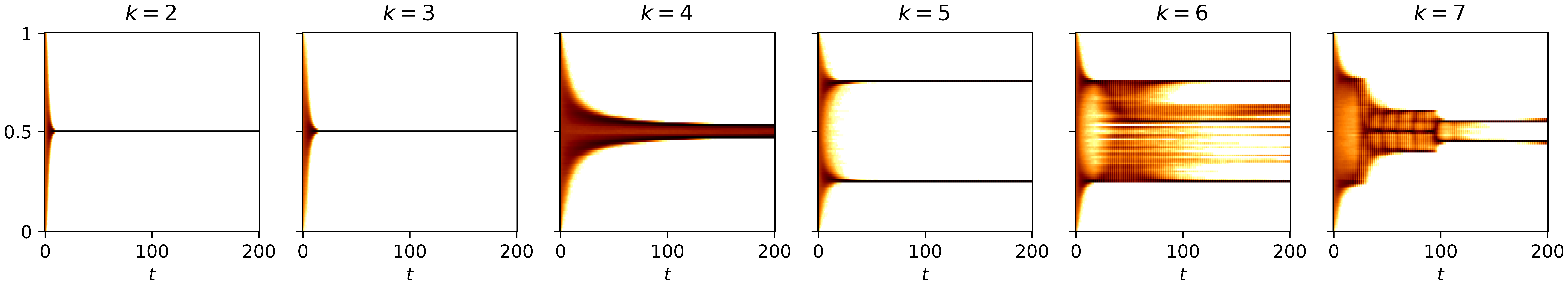


3 round memory



With imperfect imitation (copy + noise), chaos!



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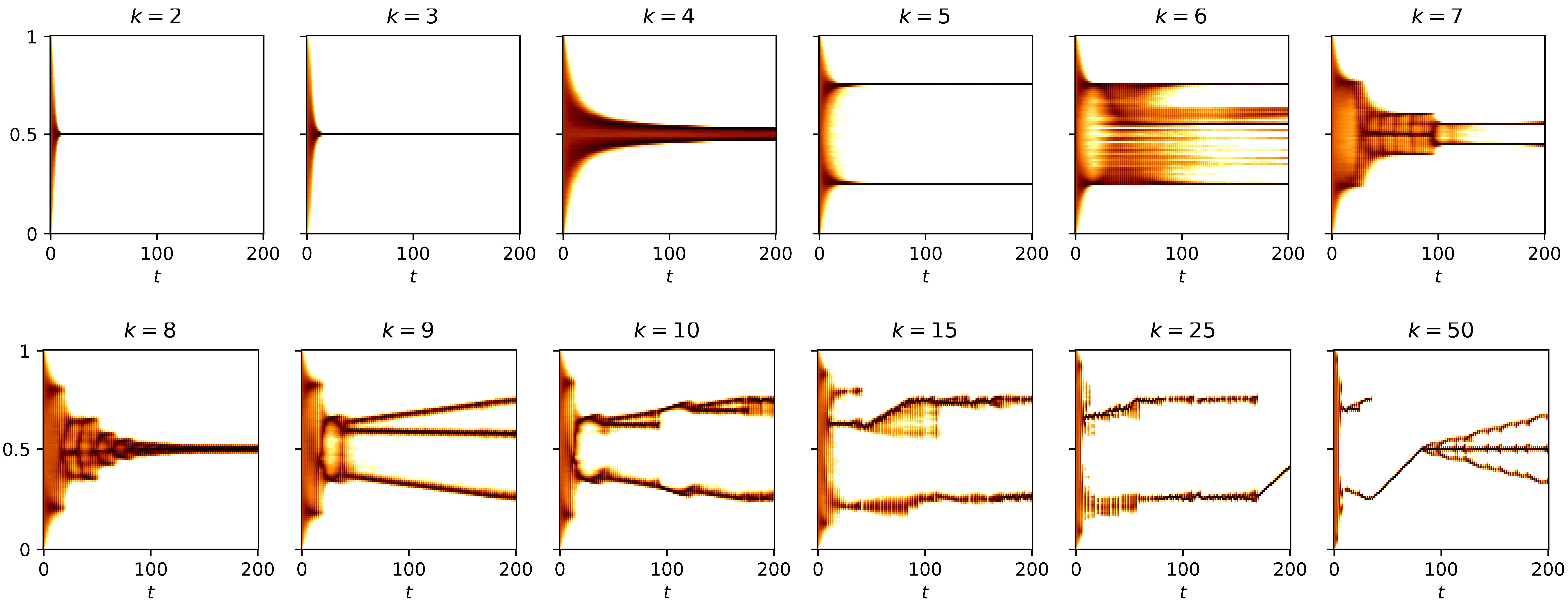
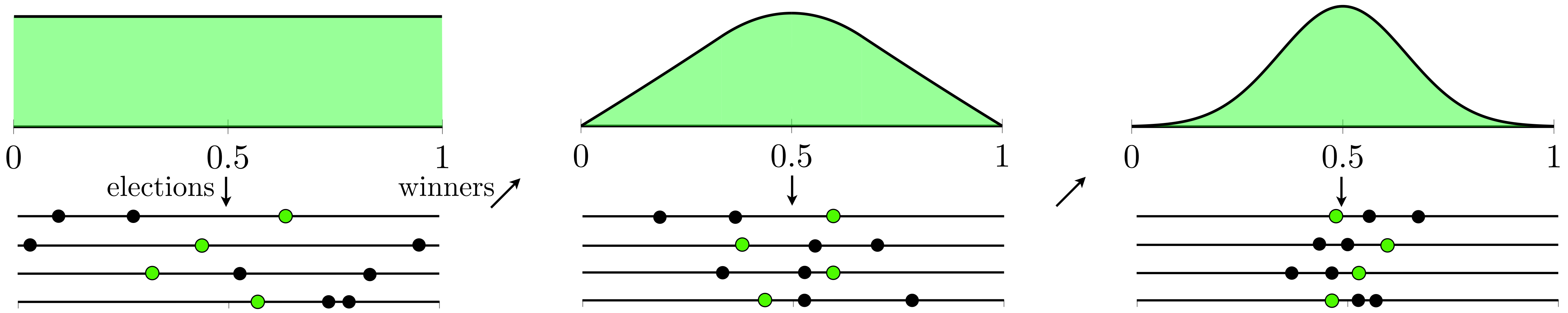


Takeaways



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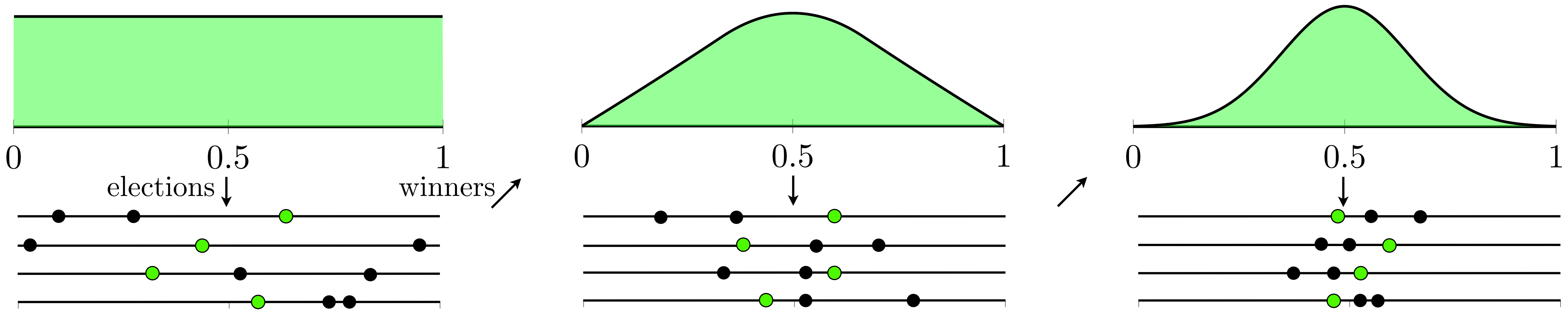
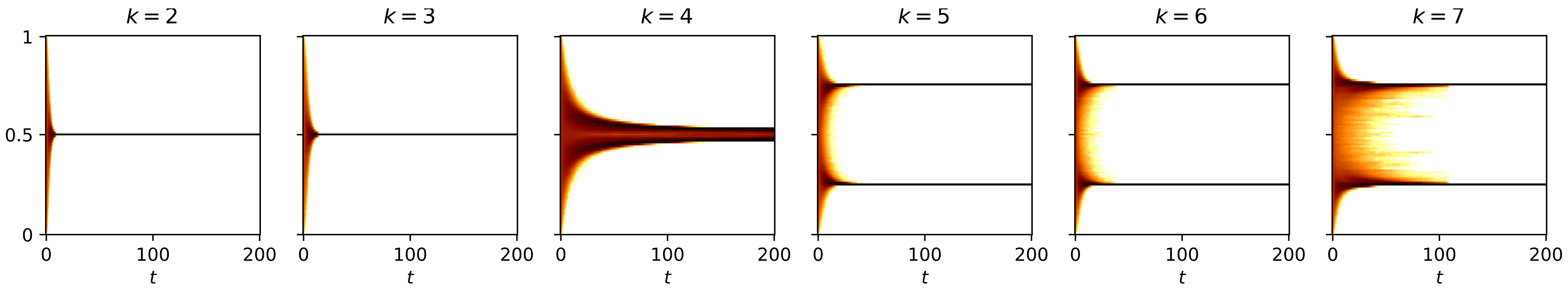


1. Imitation of successful policies can produce two divergent clusters as in Duverger's Law



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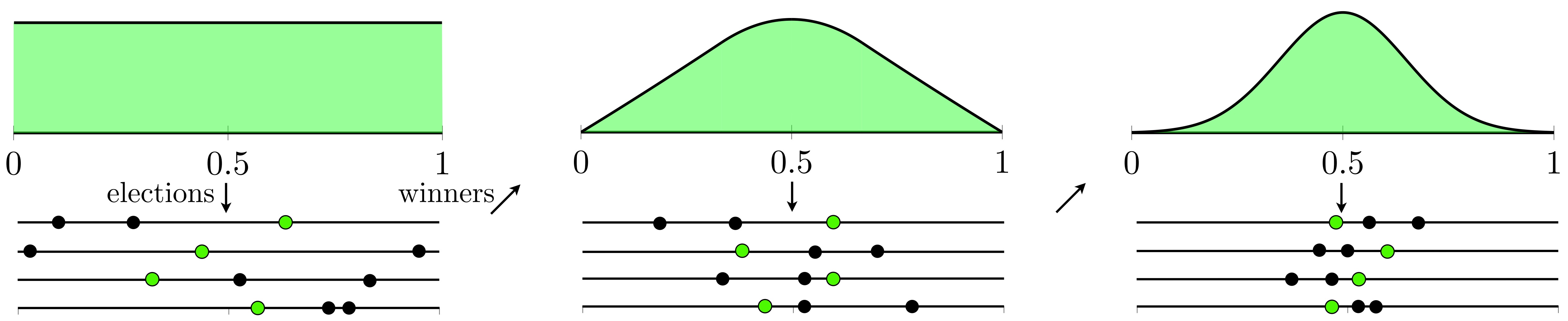

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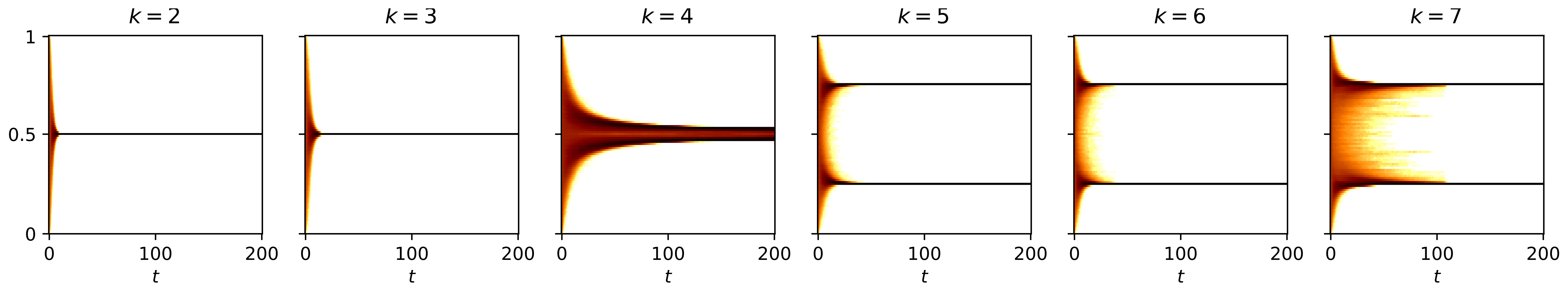
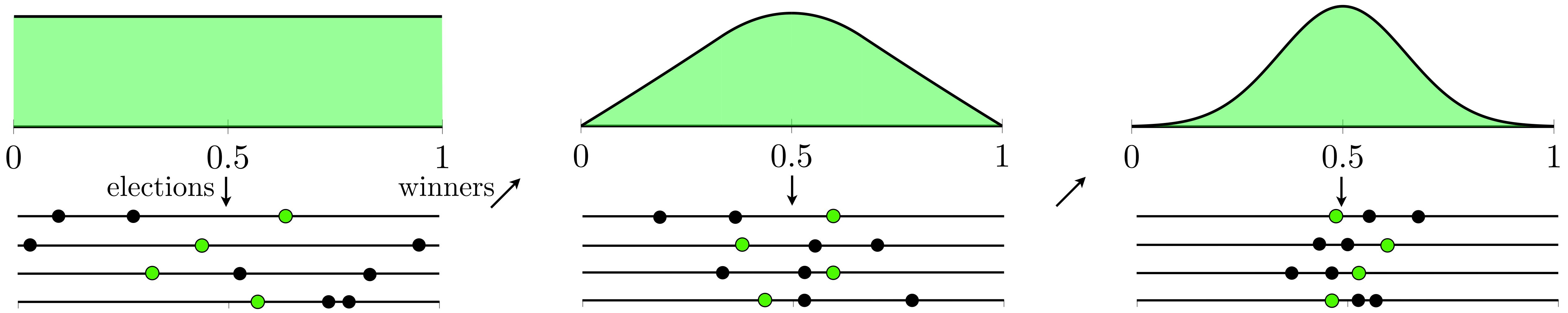


2. The replicator dynamics are very robust to variants, in contrast with Nash equilibria (more in paper). We get stable theoretical results with > 2 candidates!

Thank you!



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📝 arxiv.org/abs/2402.17109

🧑‍💻 github.com/tomlinsonk/plurality-replicator-dynamics

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