Ballot Length in Instant Runoff Voting

AAAI ‘23

Kiran Tomlinson Johan Ugander Jon Kleinberg



Given the preferences of voters, how do we pick a winner?

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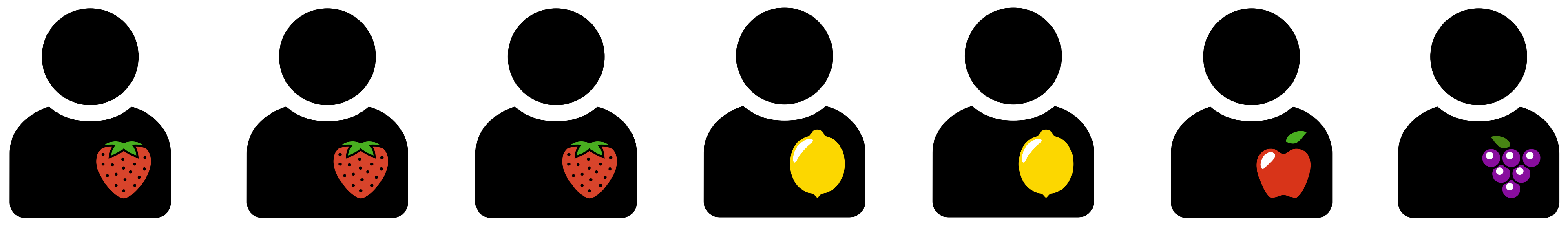

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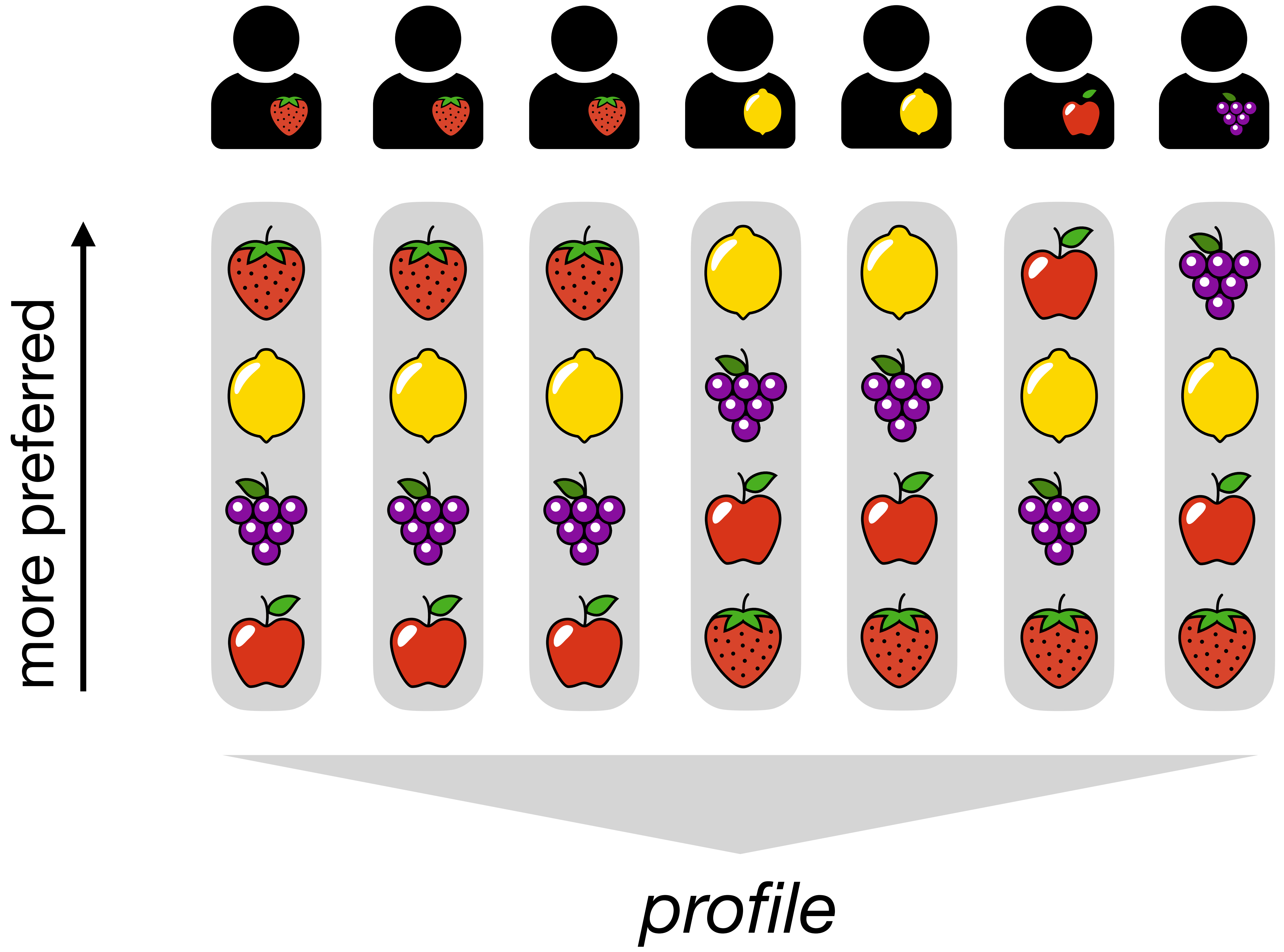

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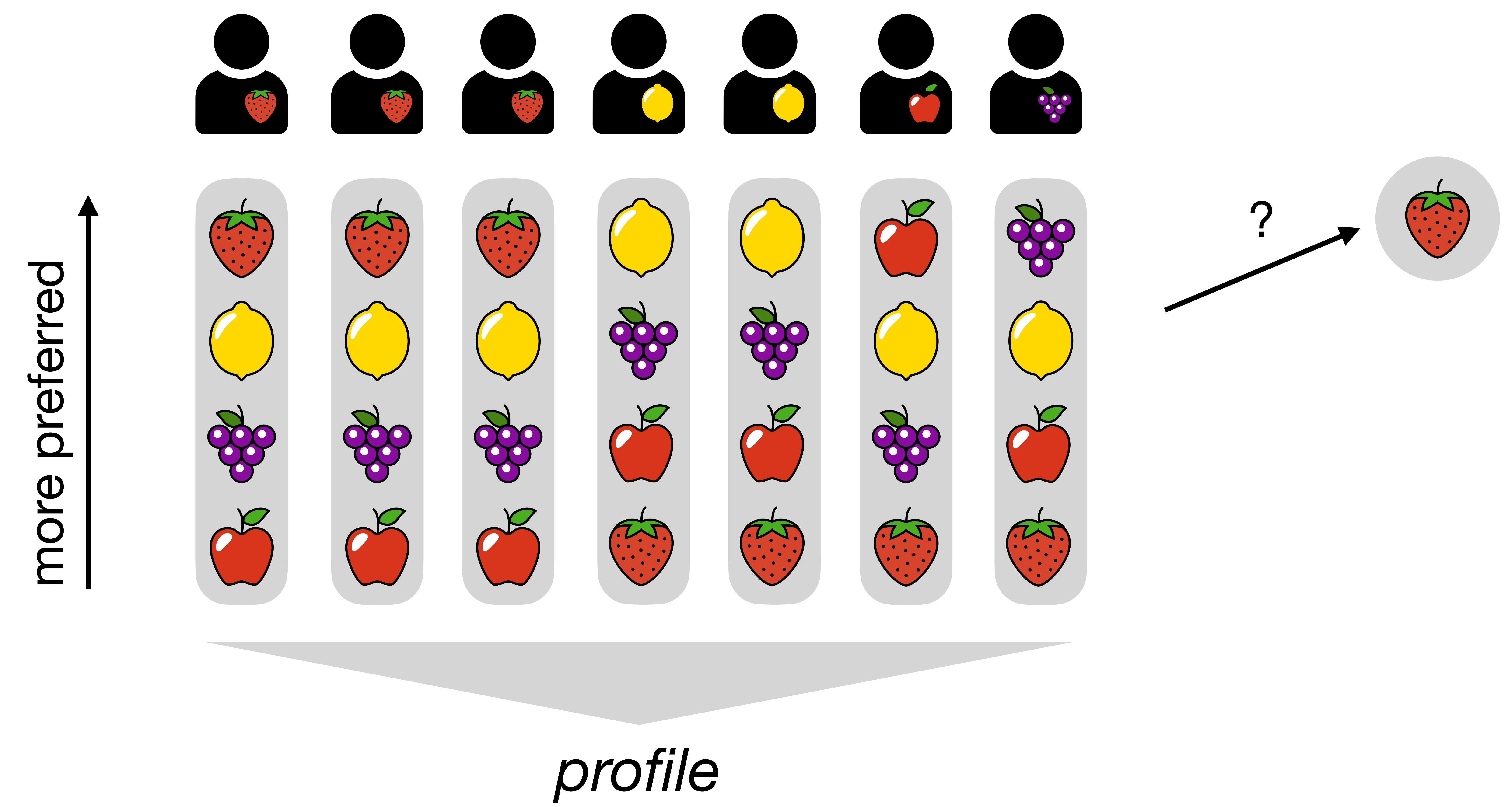

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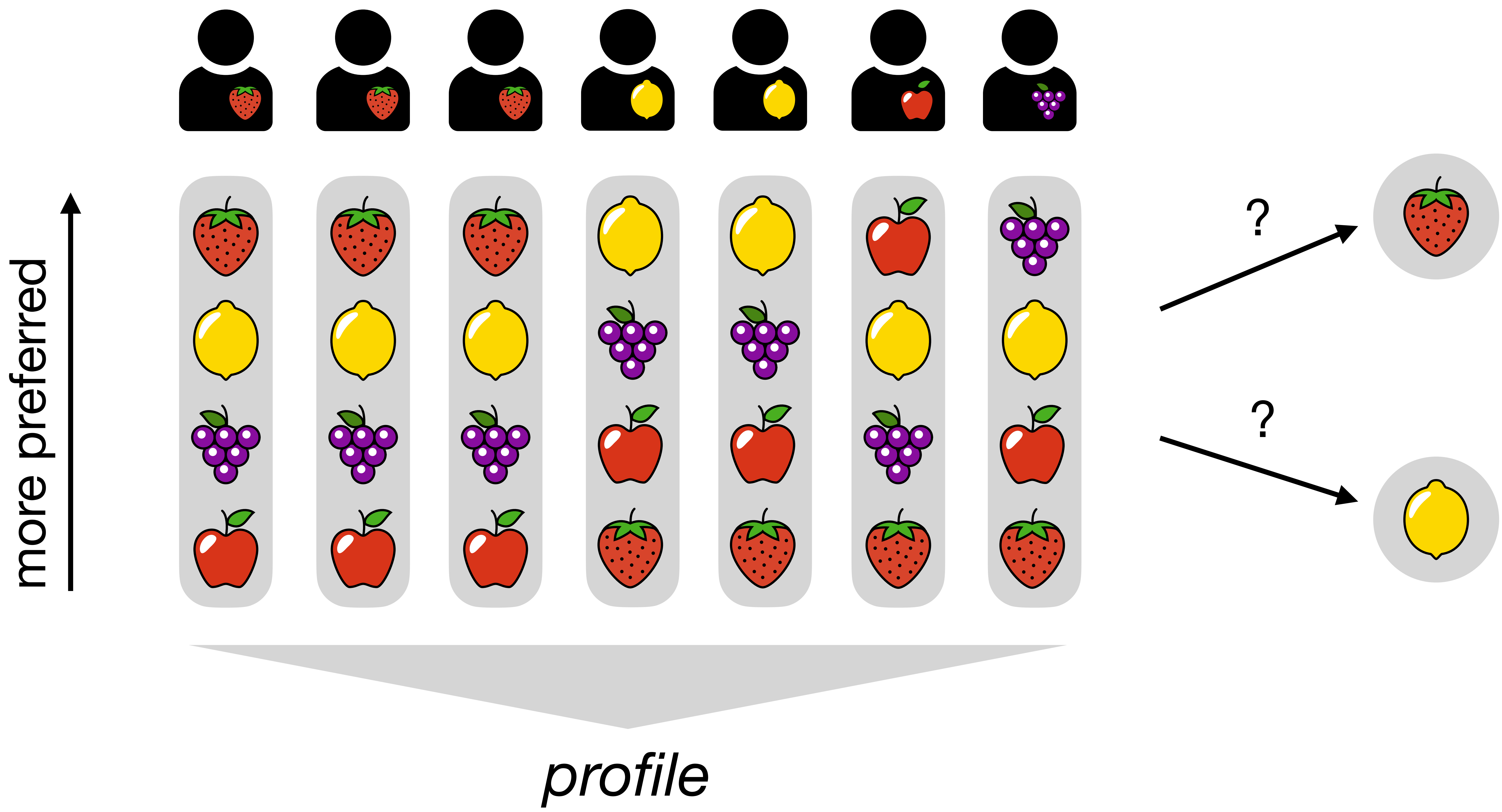
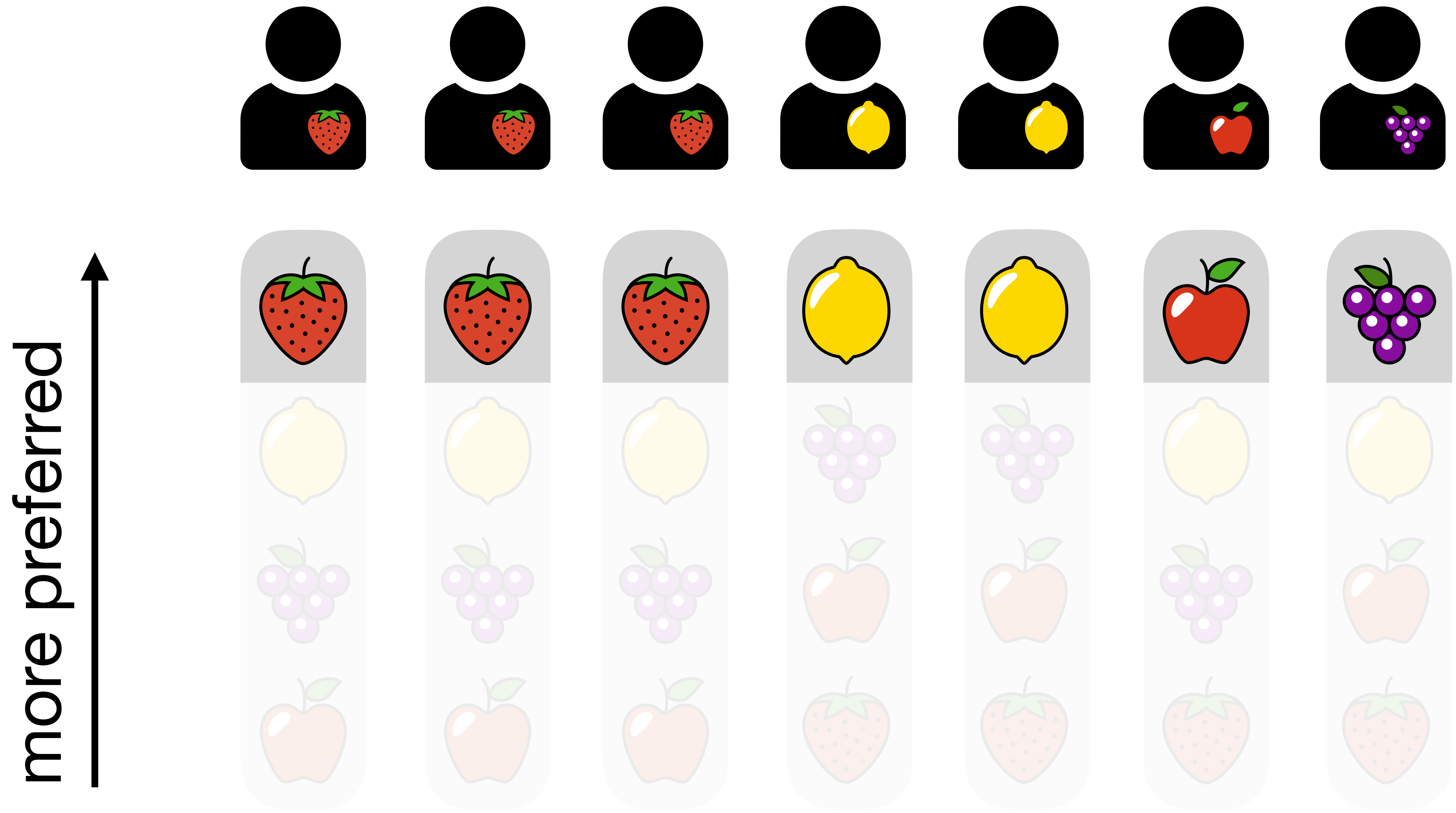


Plurality voting

choose the candidate with the most first-place votes



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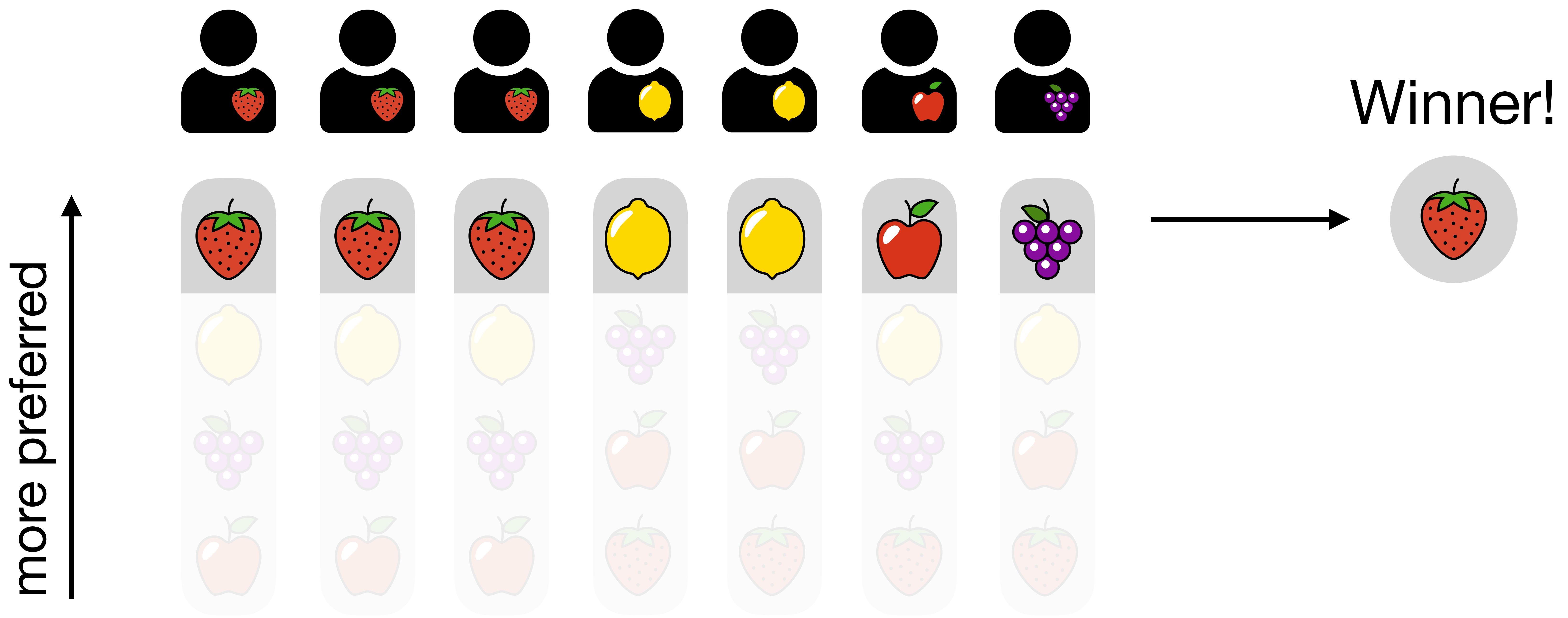
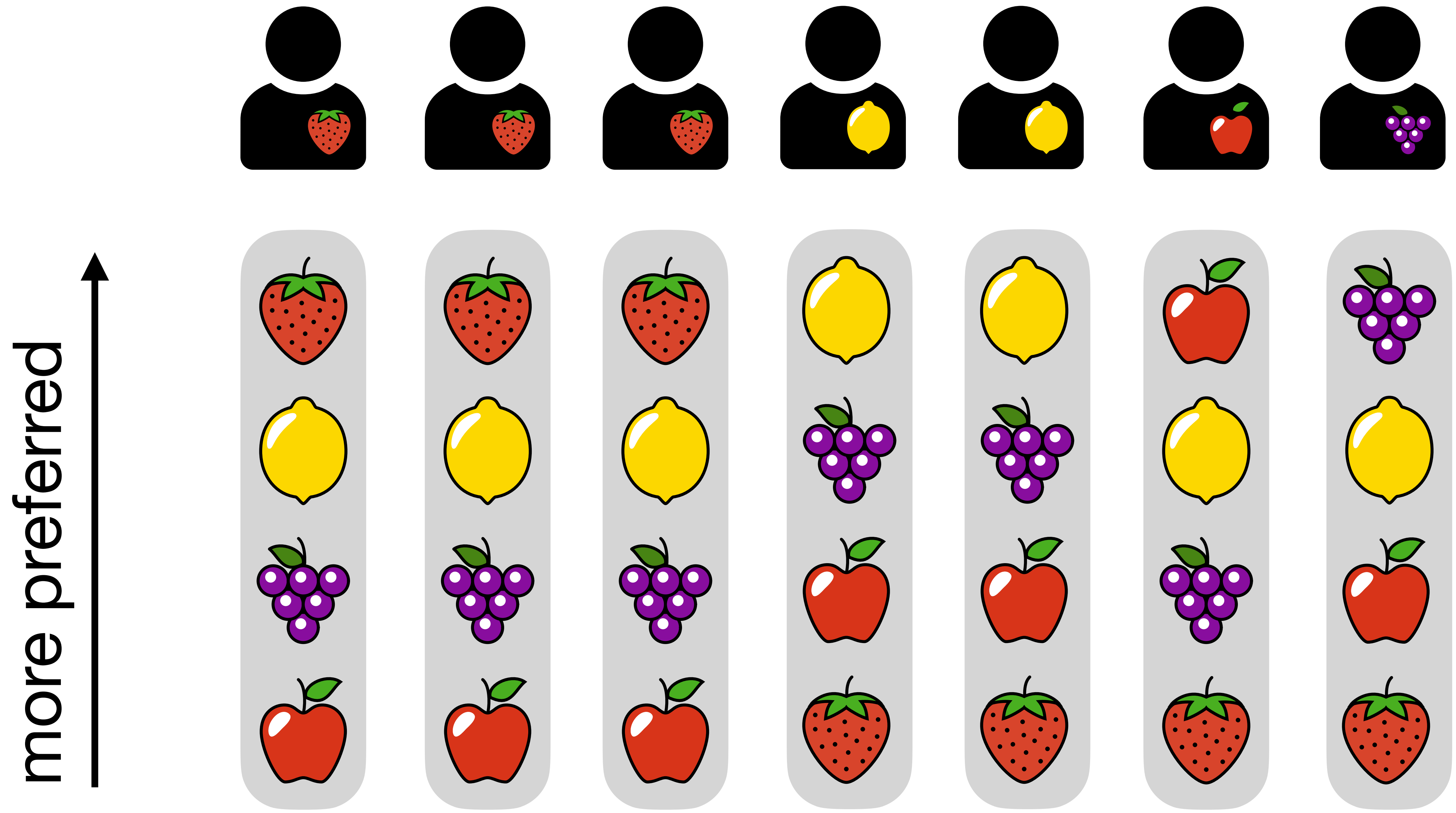


Instant runoff voting (IRV)

repeatedly eliminate the candidate with fewest first-place votes

Elimination order



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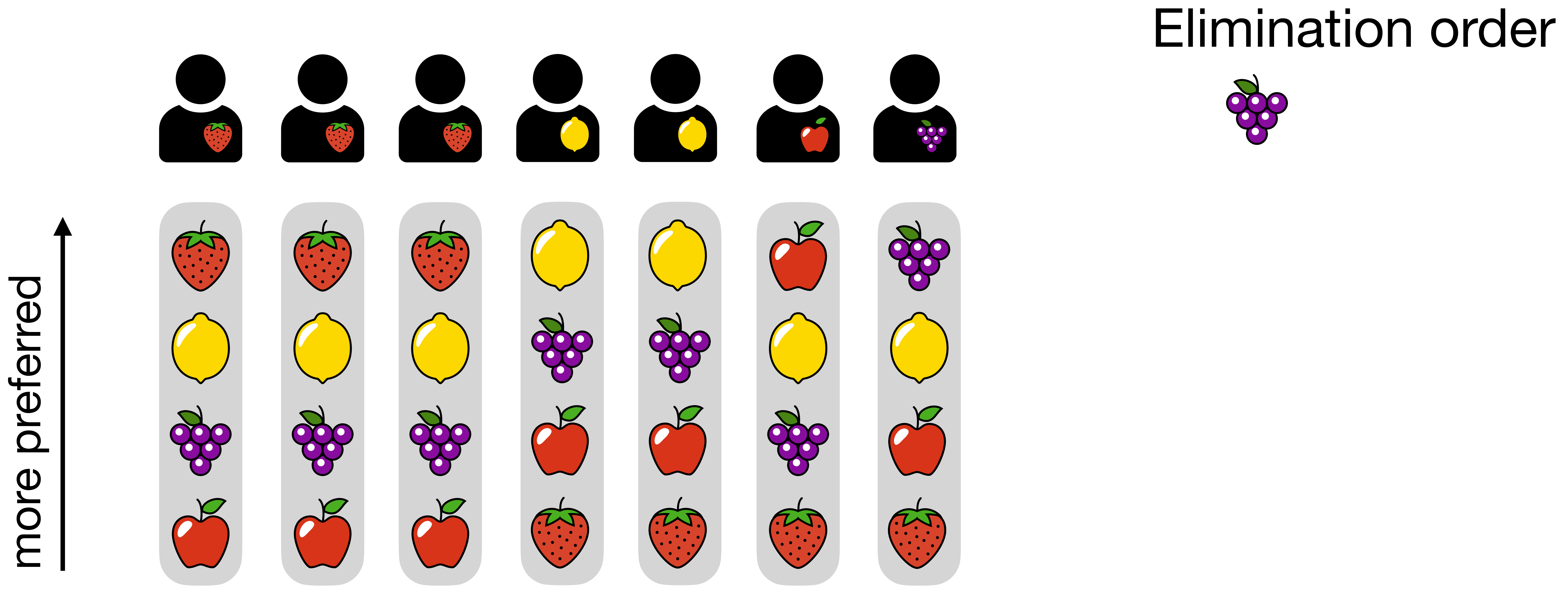

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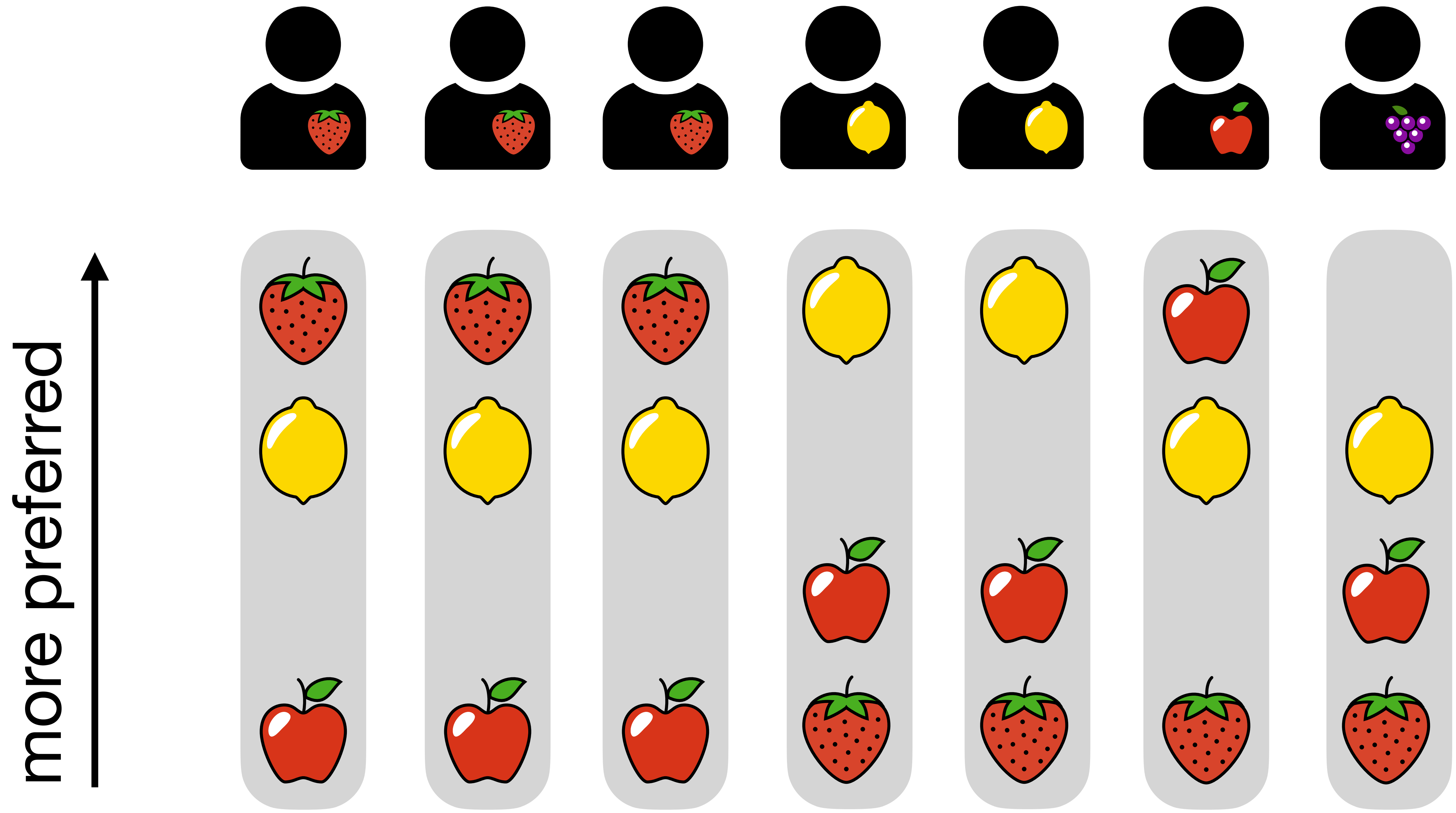

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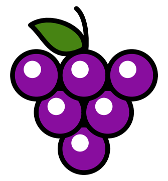

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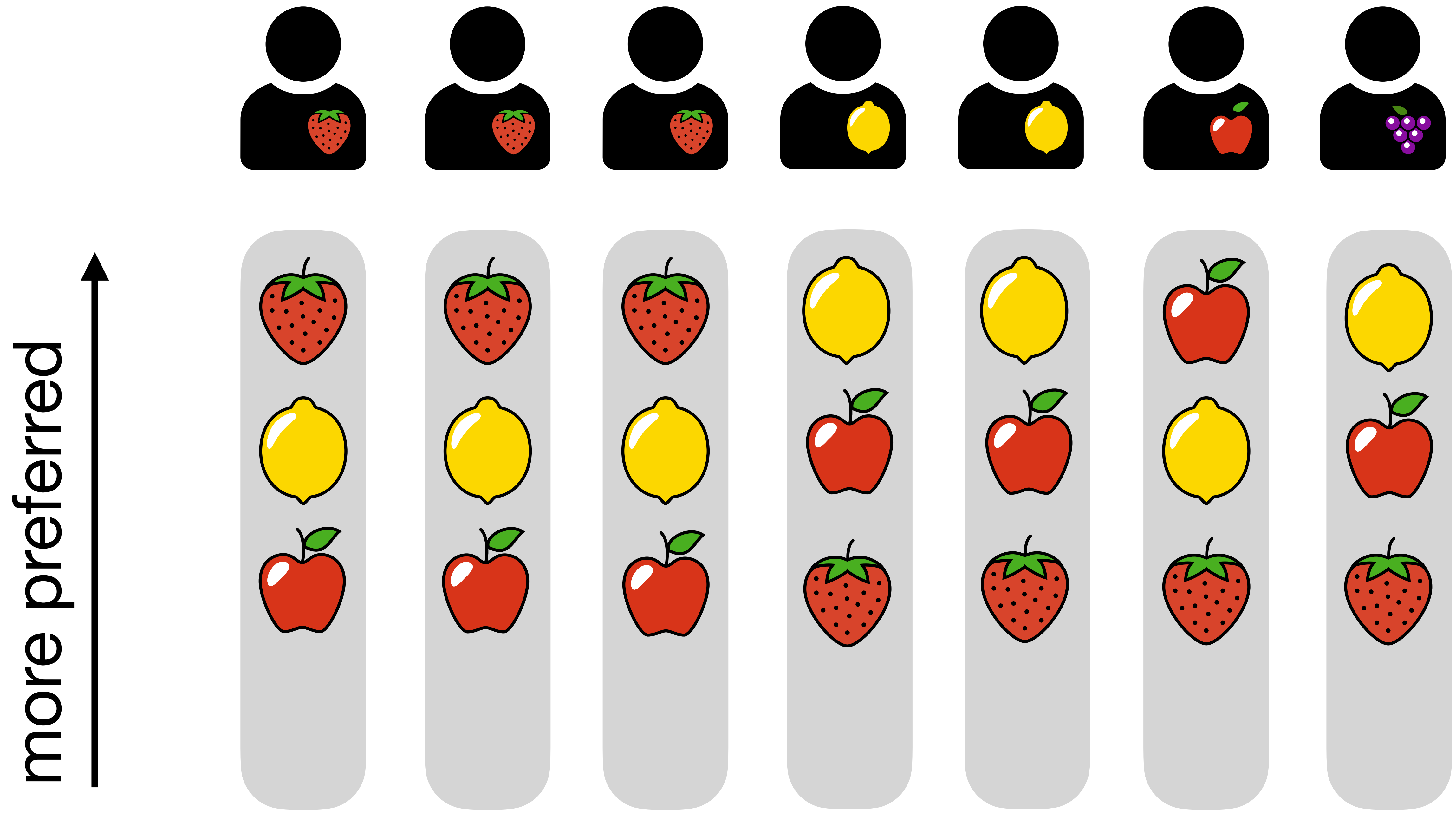

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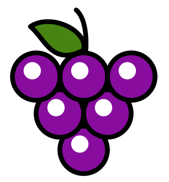

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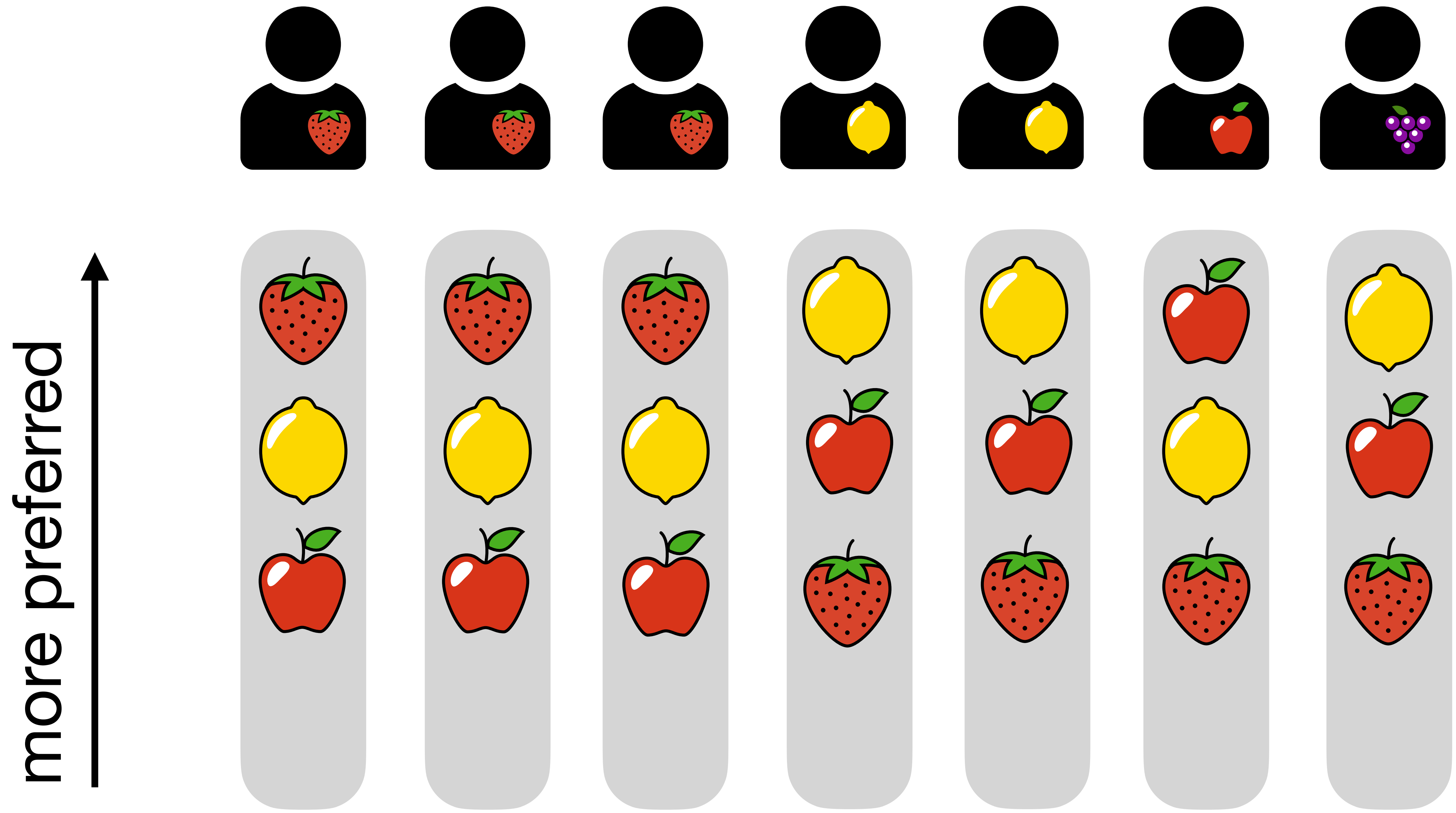

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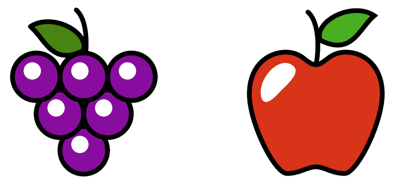

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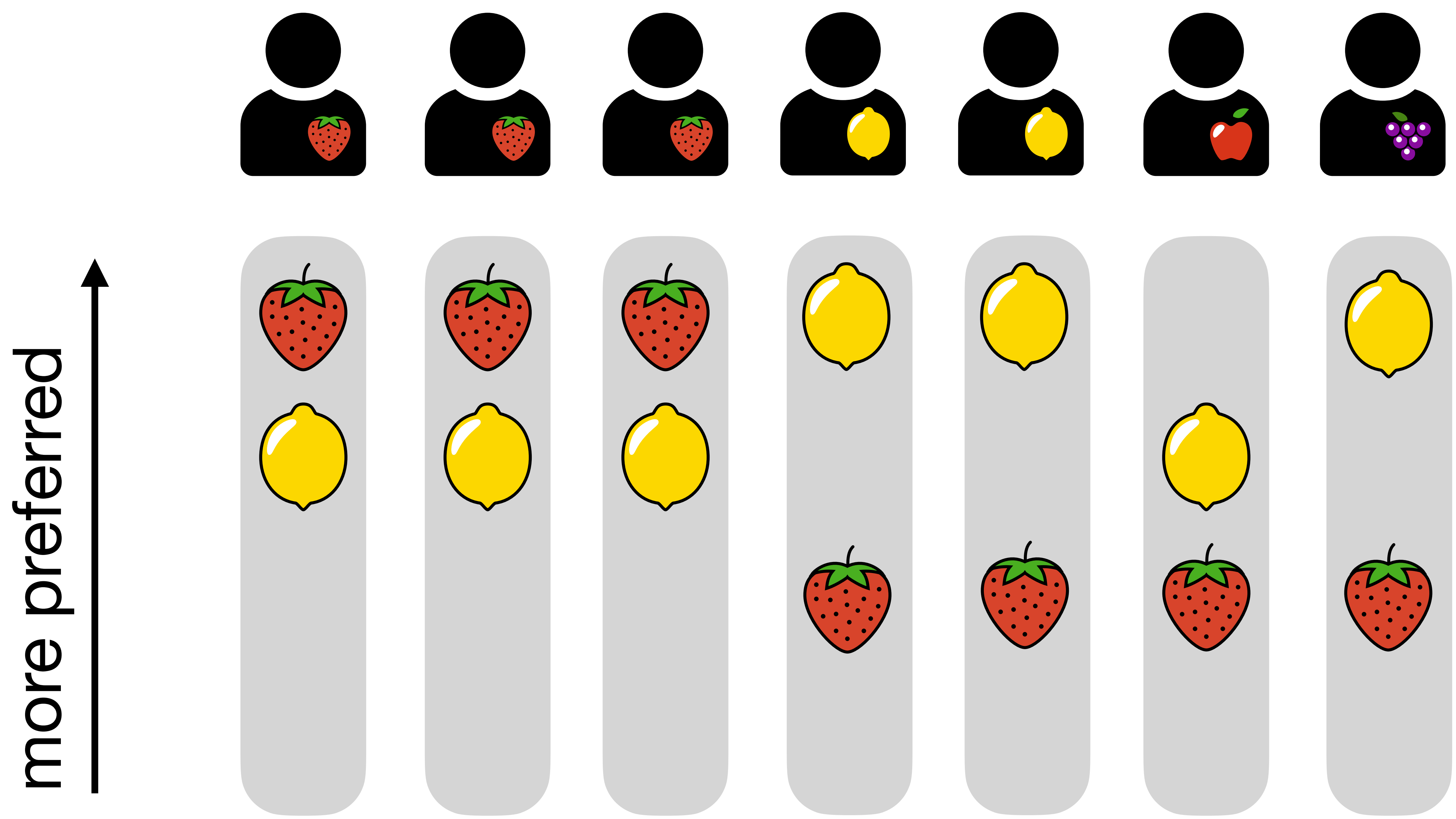

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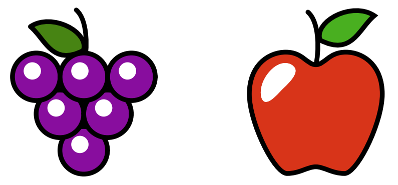

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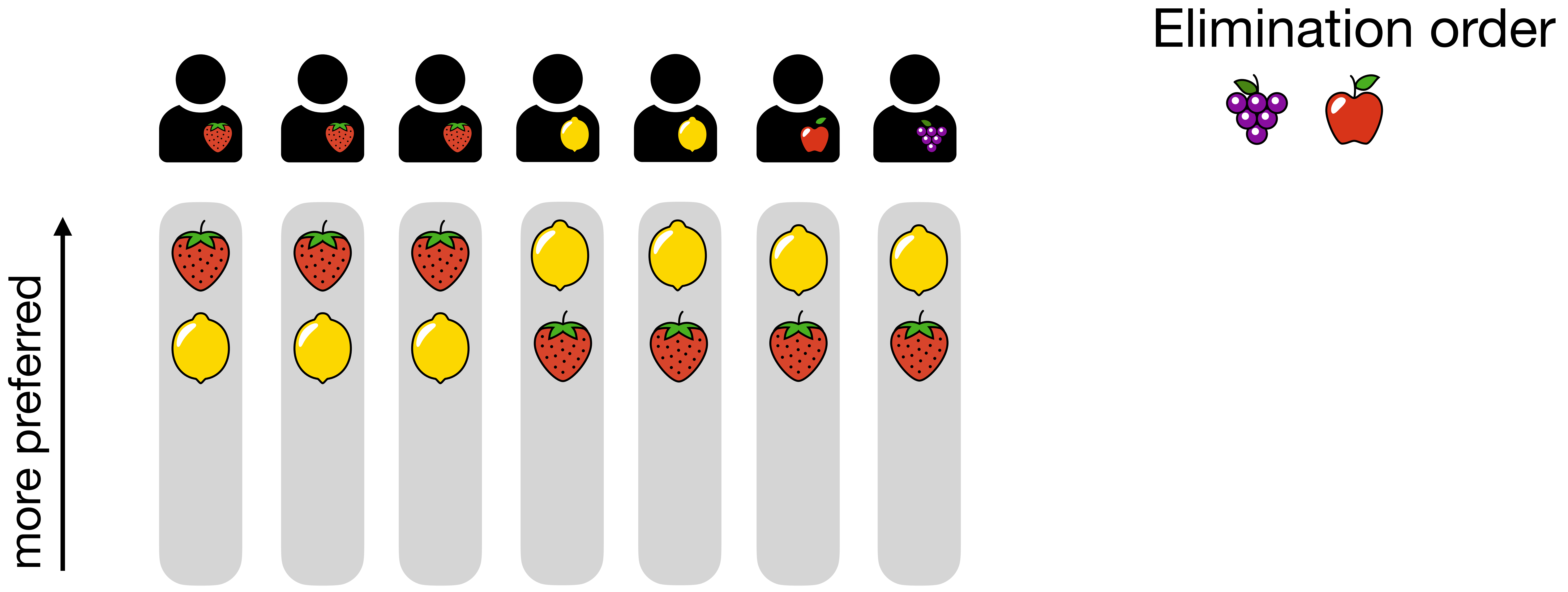

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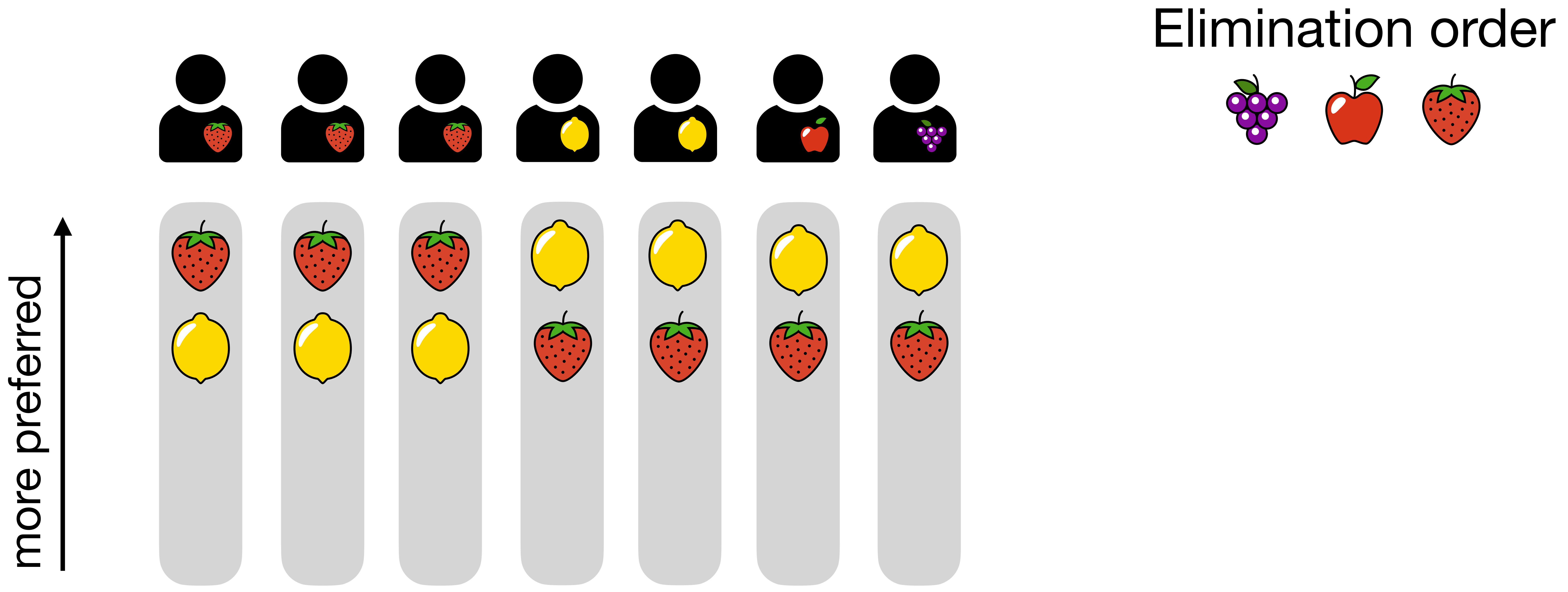

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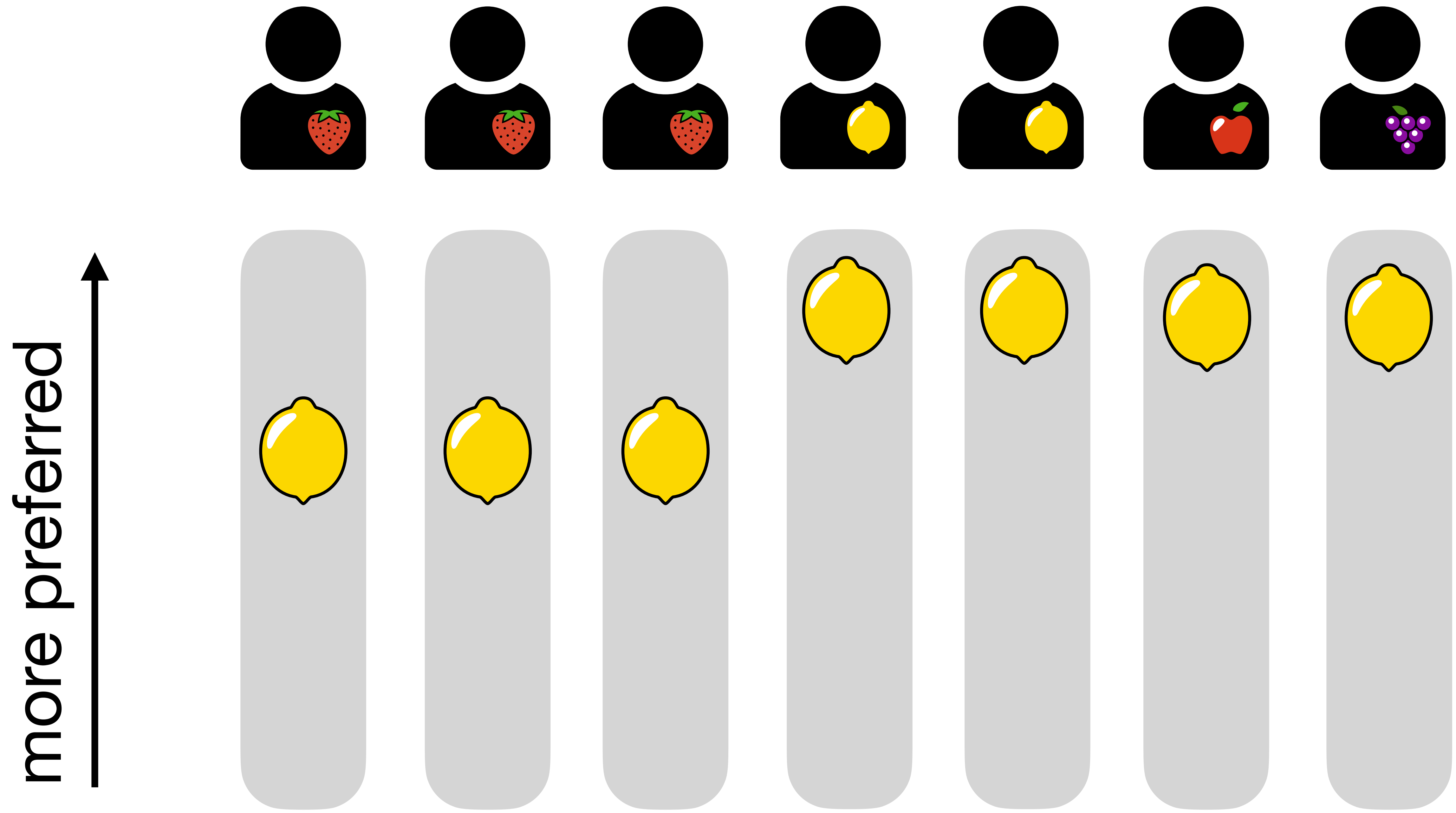

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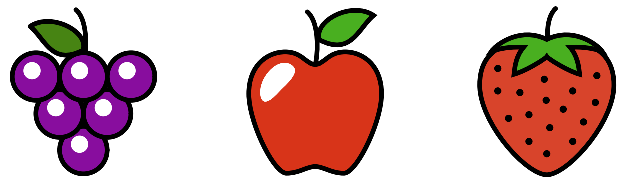

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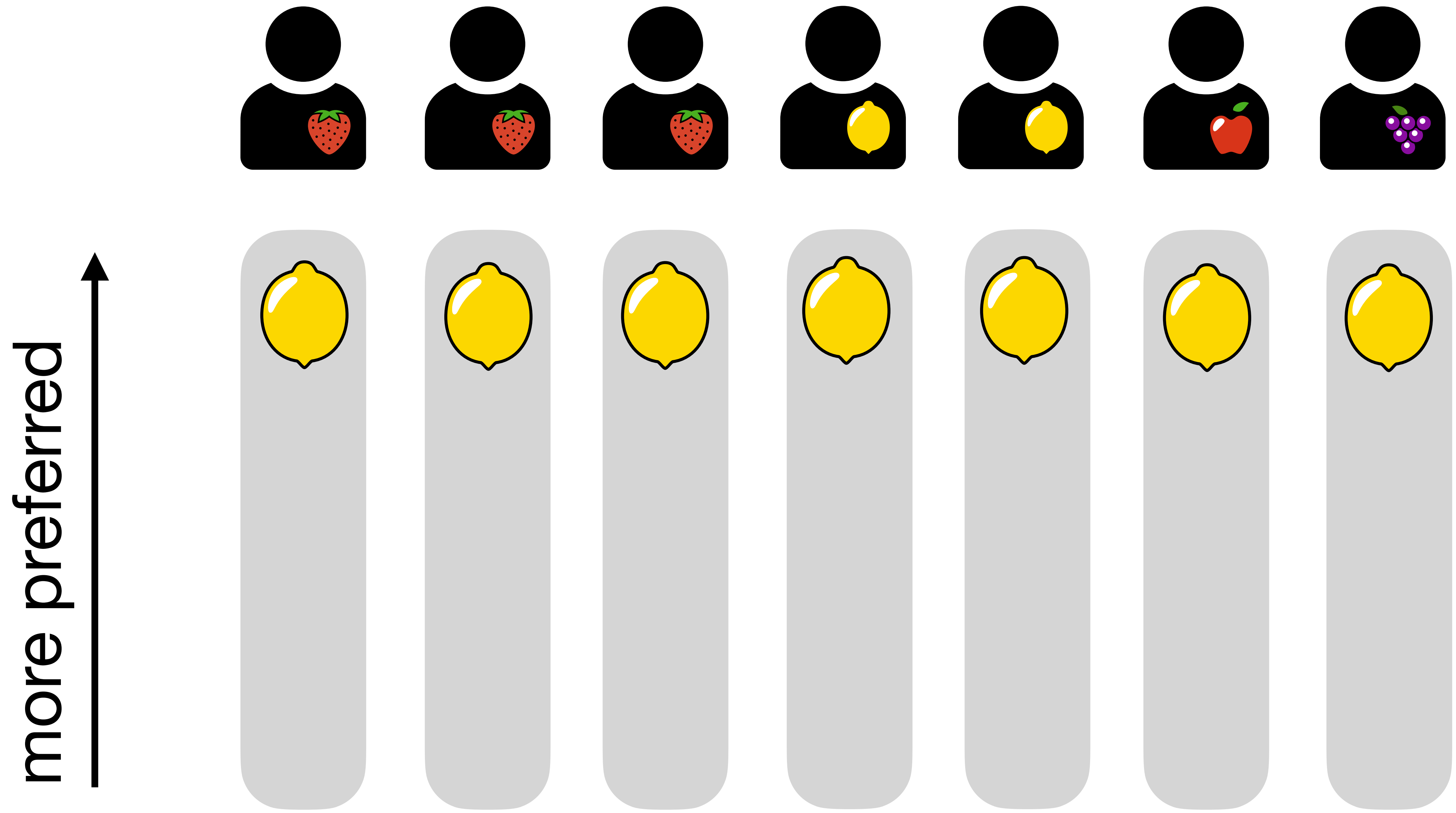

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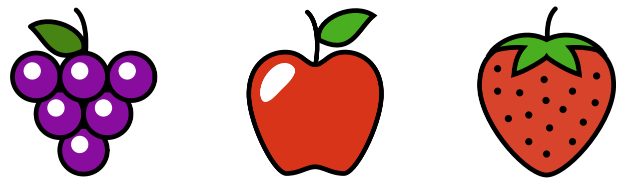

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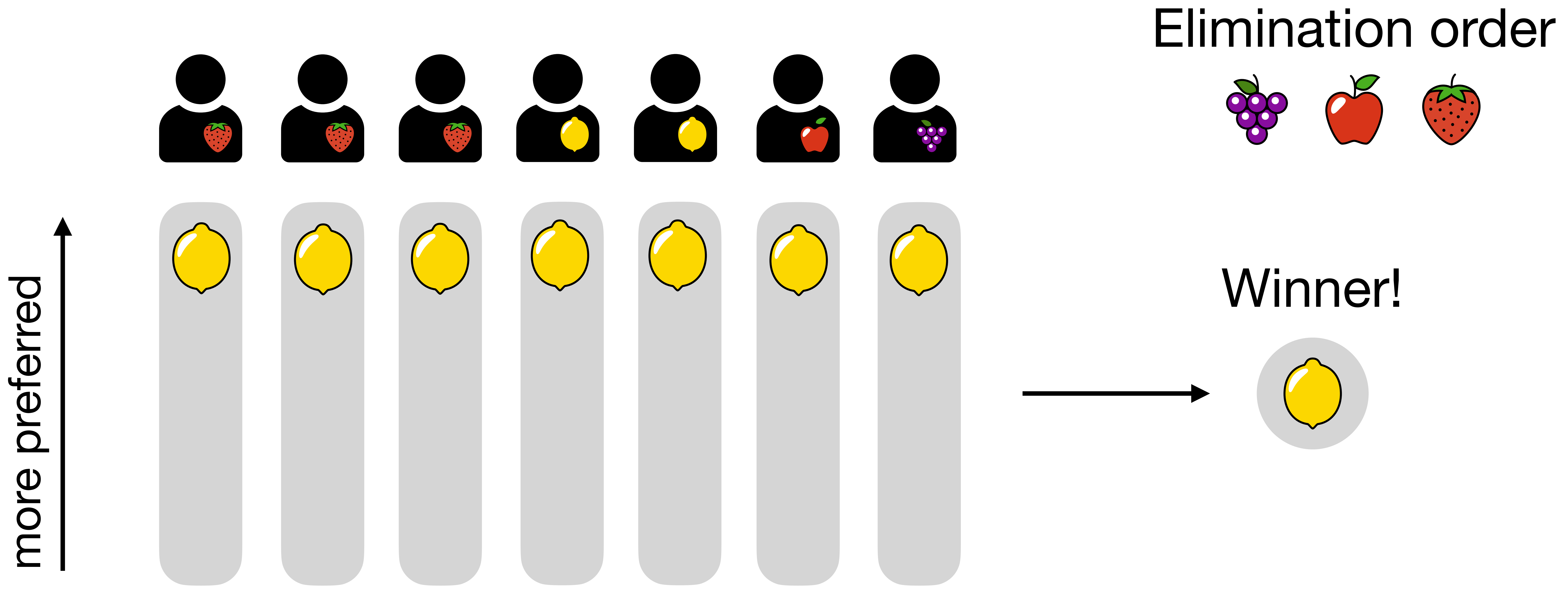

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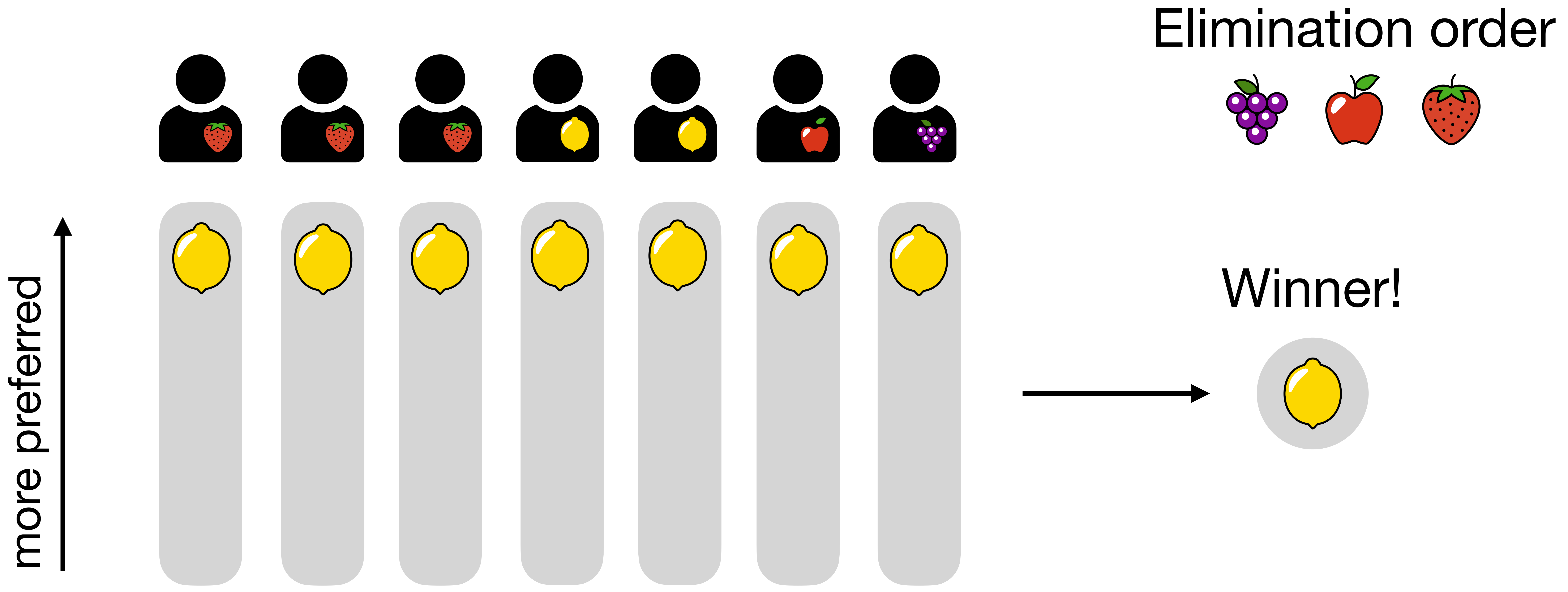
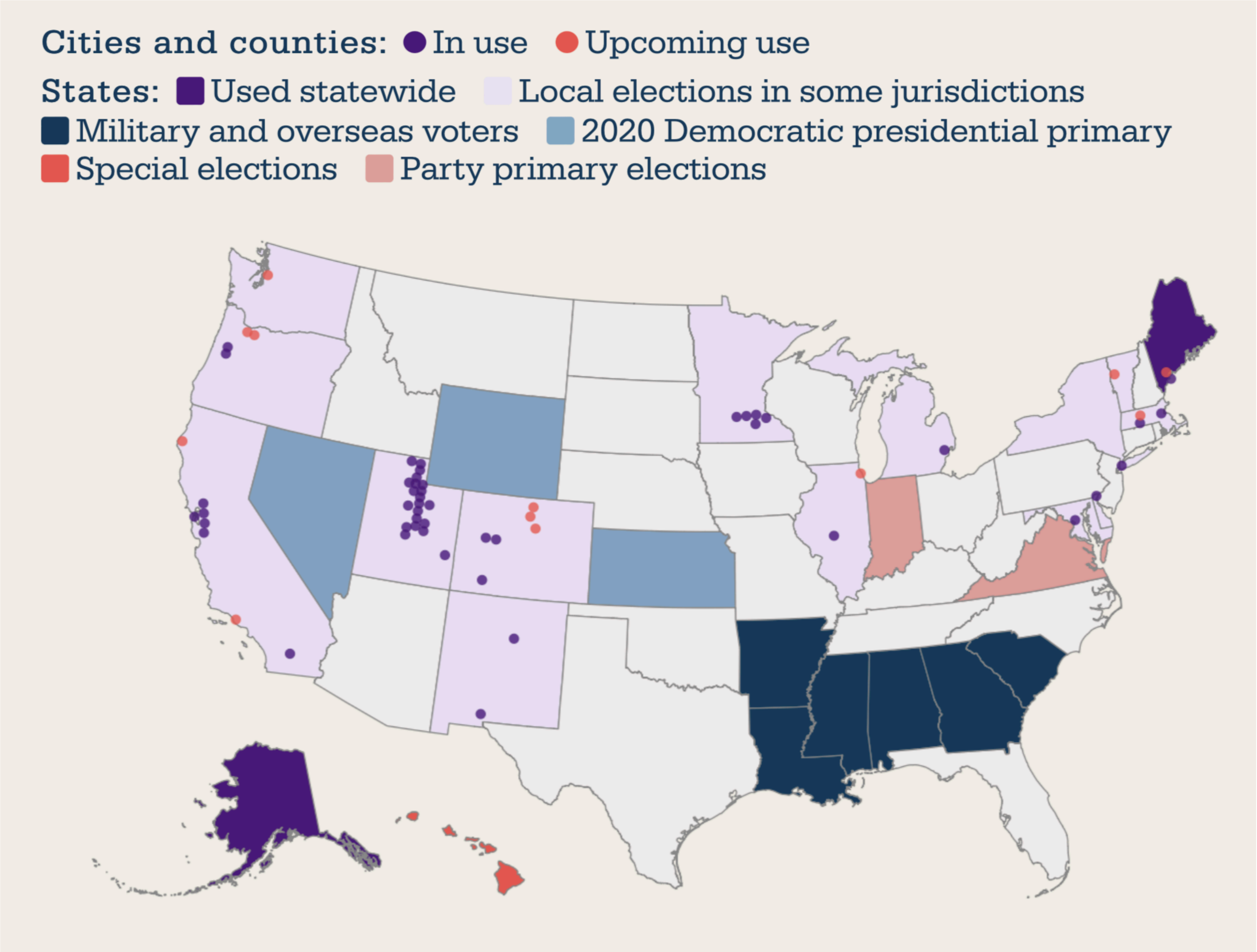


a.k.a. STV, AV, RCV, Hare method, preferential voting

Who uses IRV?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States: Used statewide Local elections in some jurisdictions
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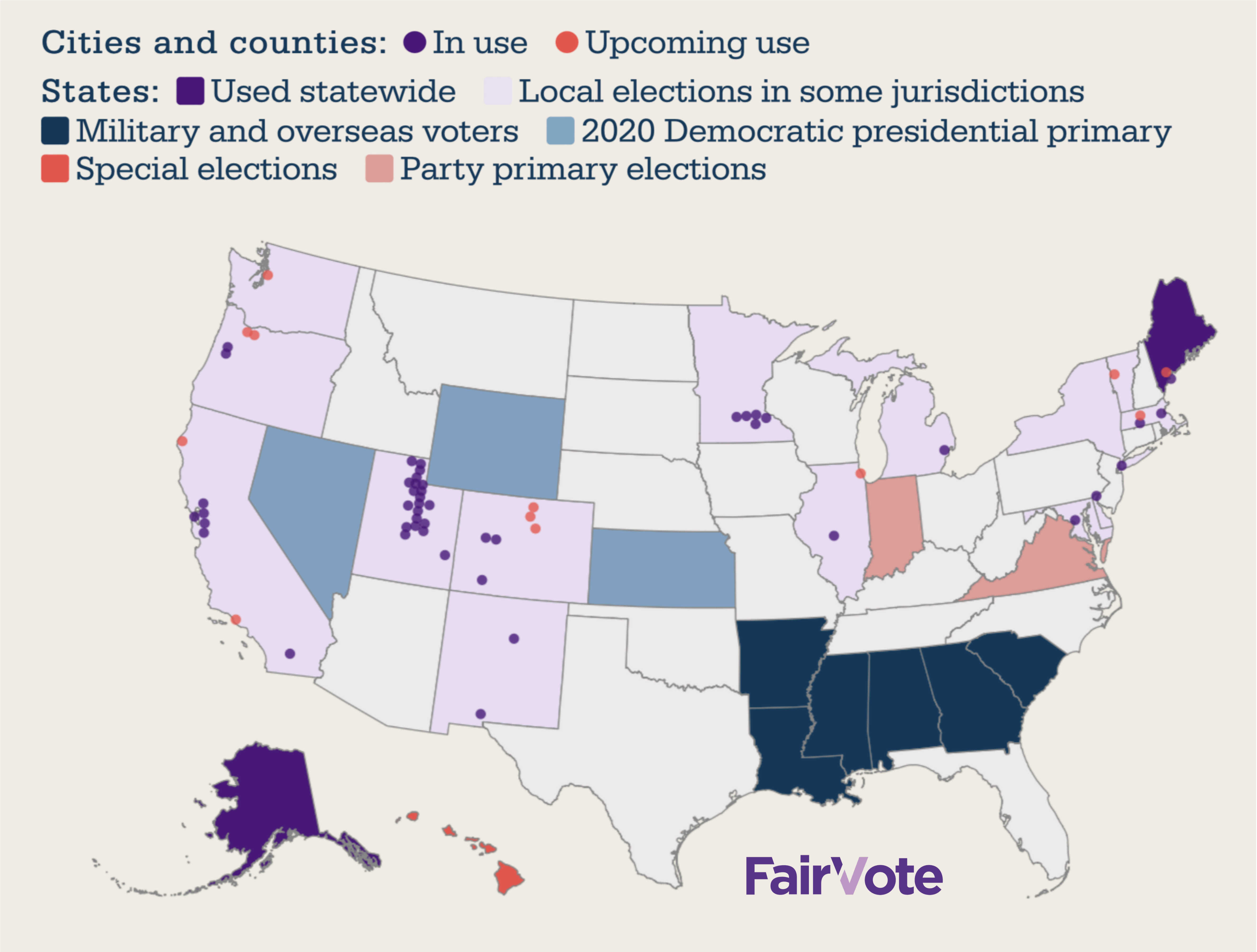

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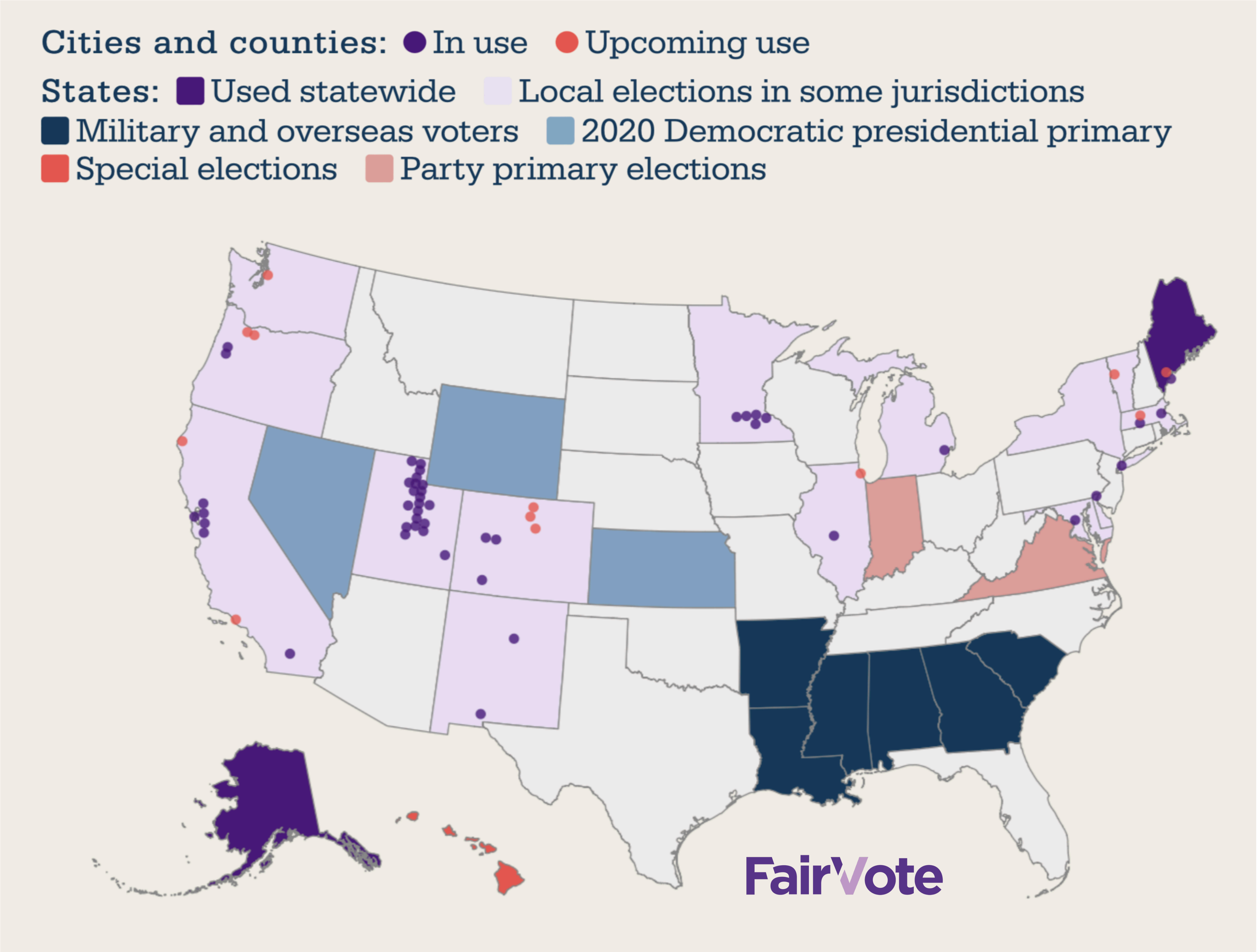
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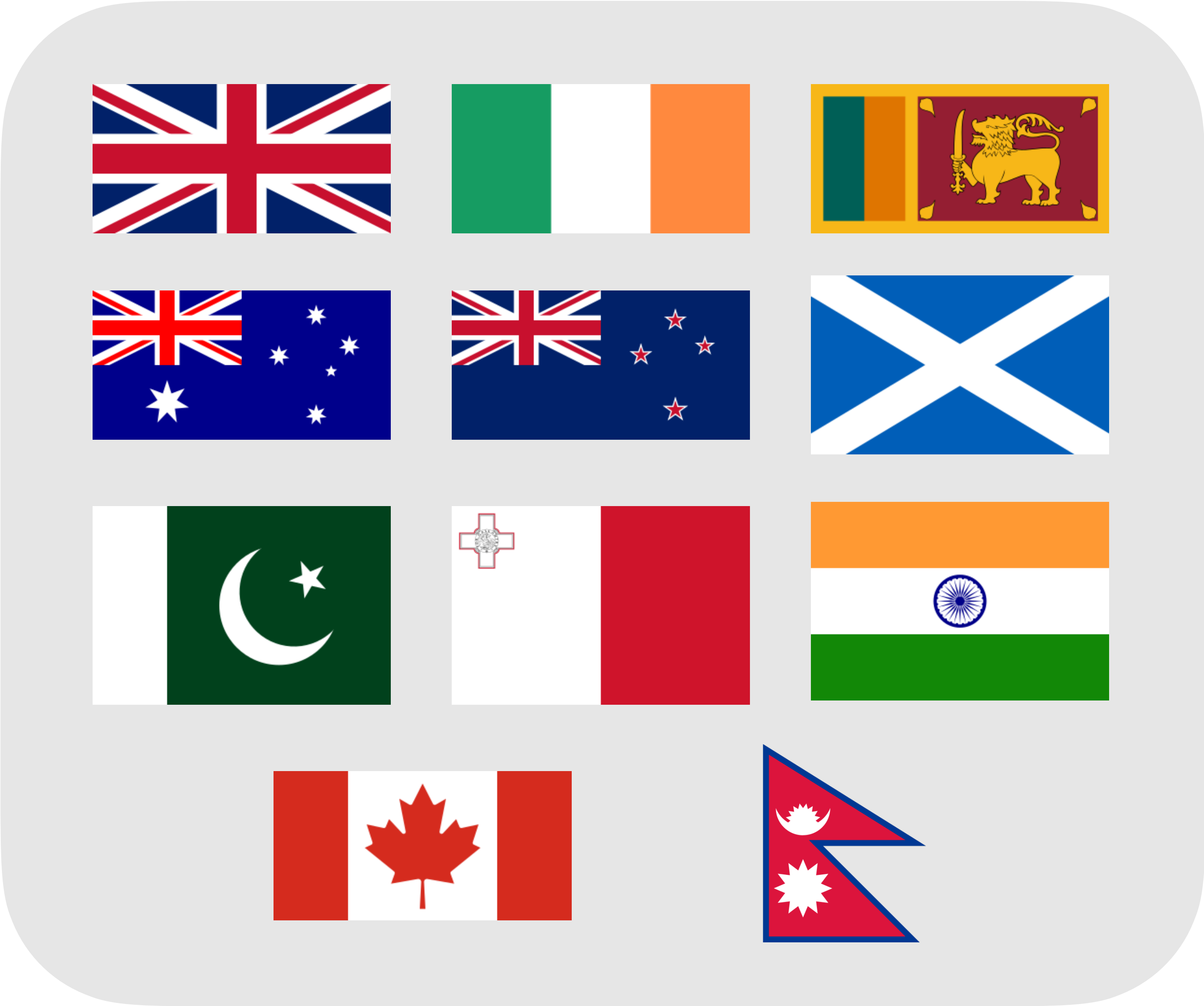
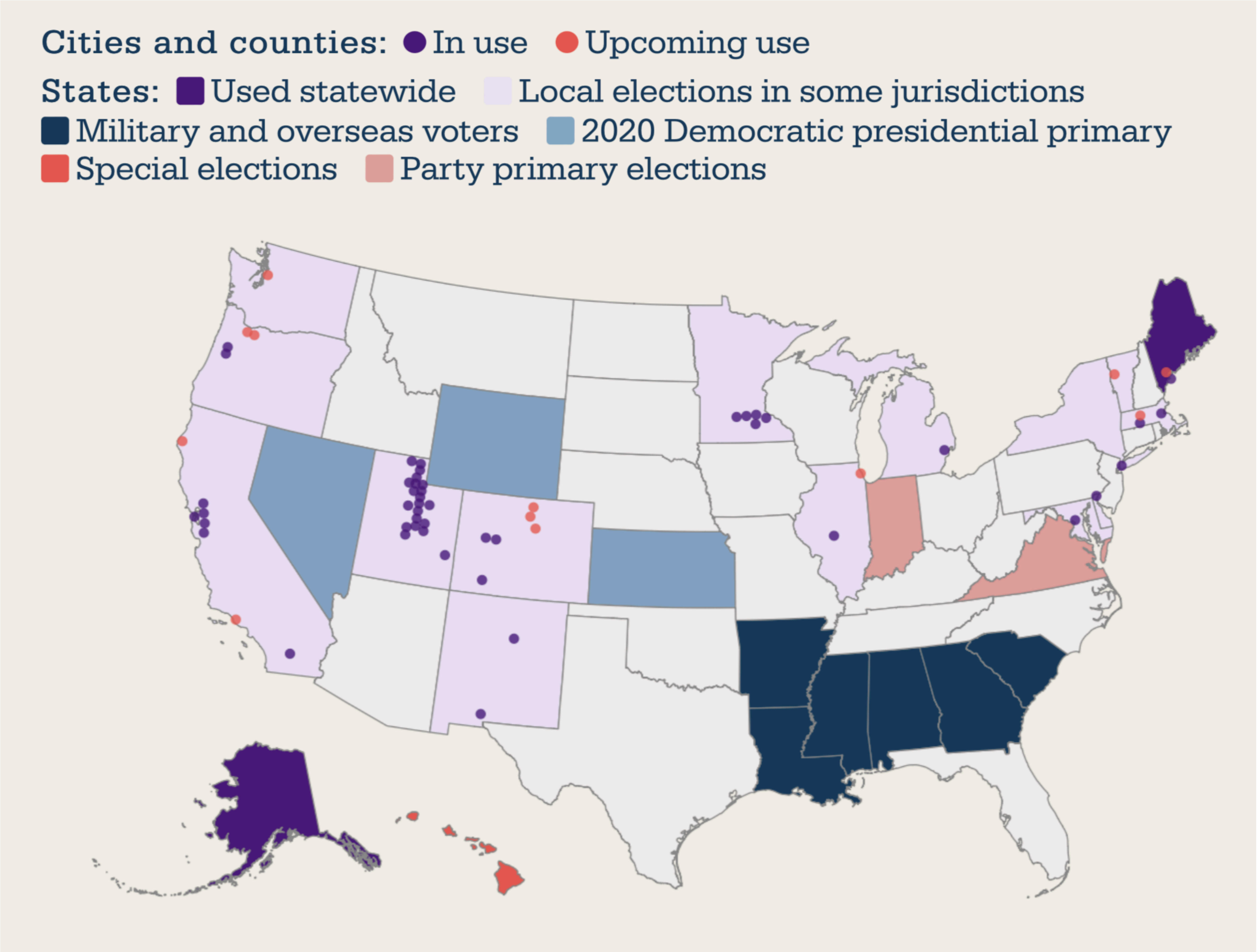


Ballot length: how many candidates can you rank?



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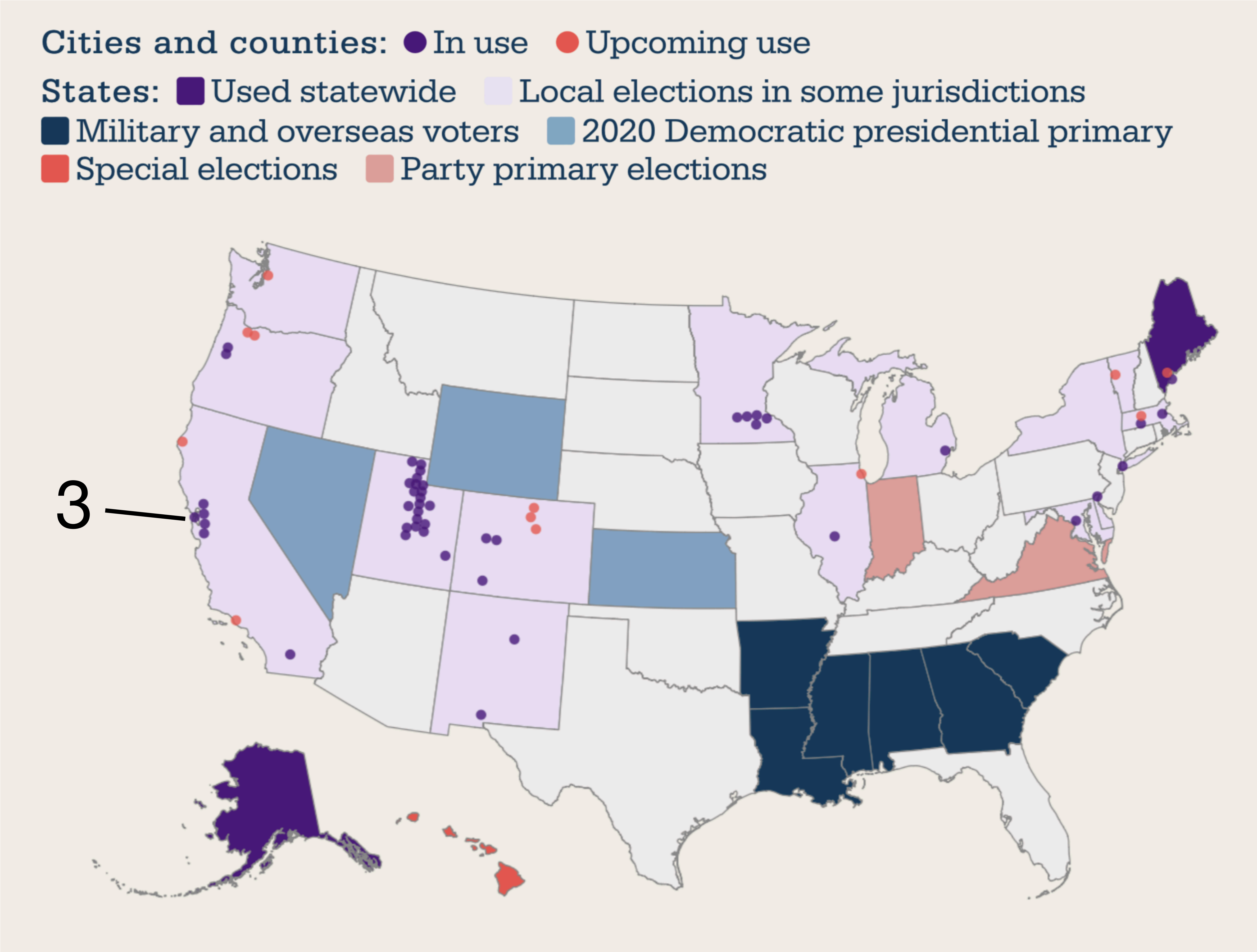
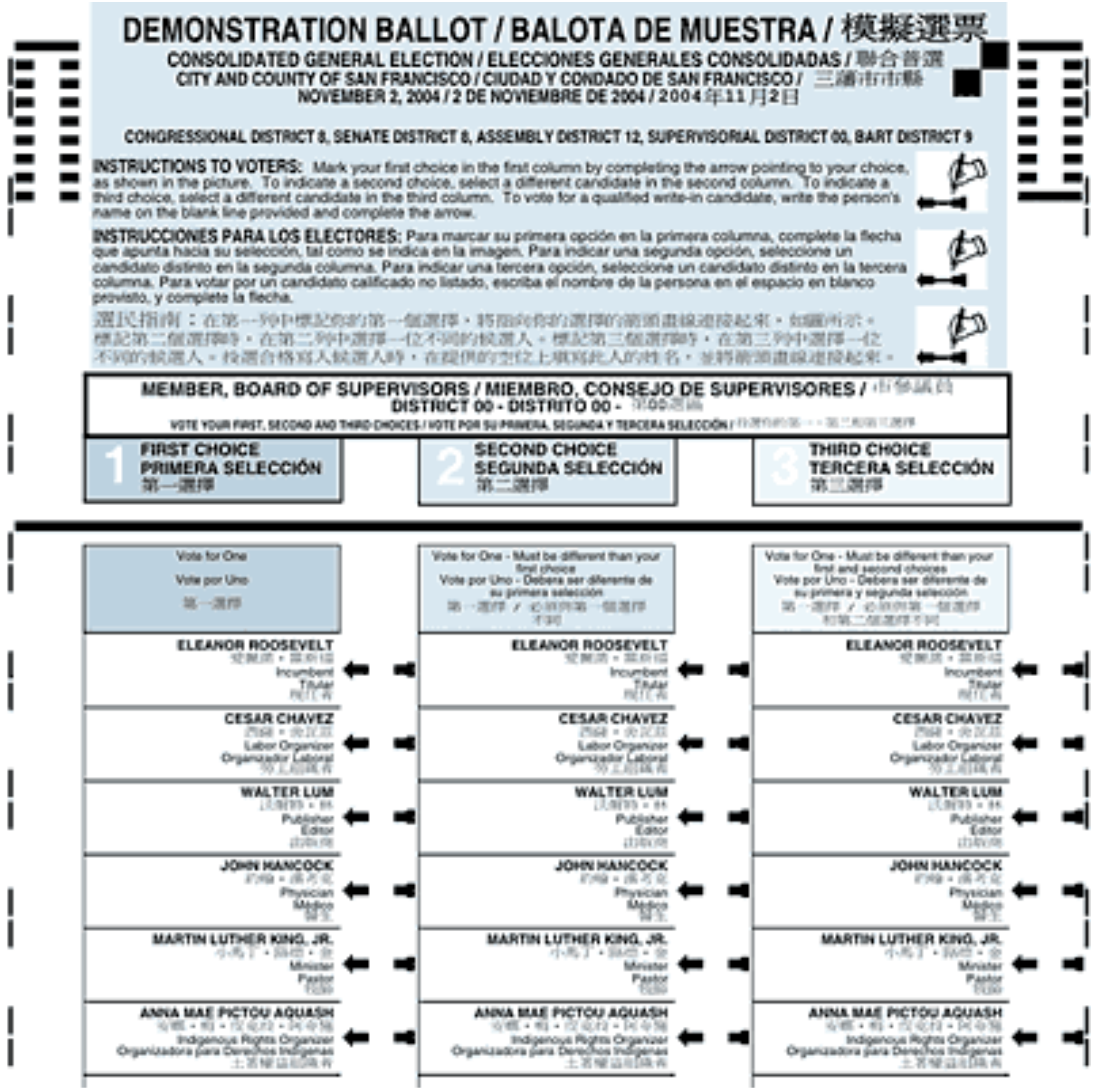


San Fransisco



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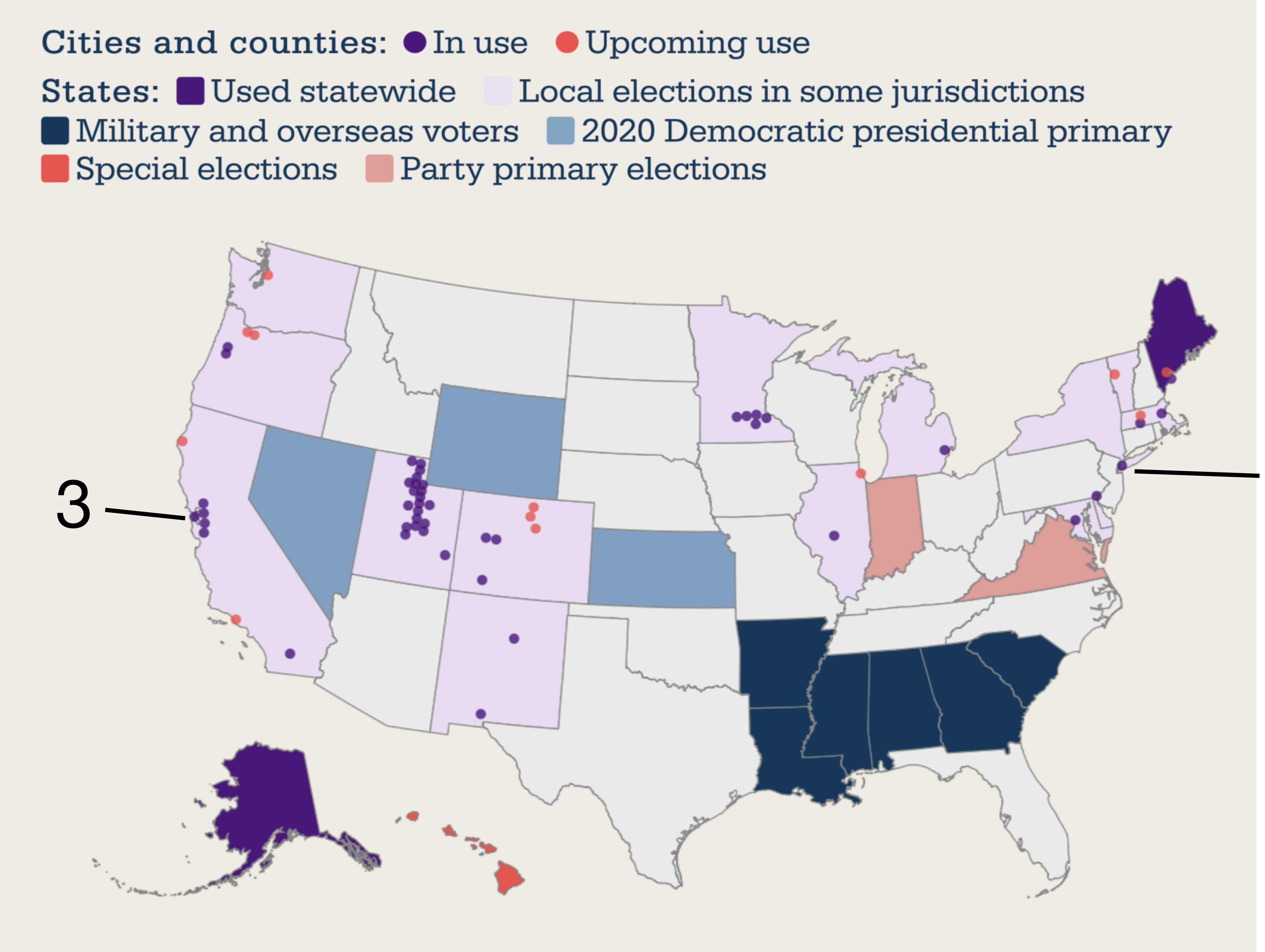
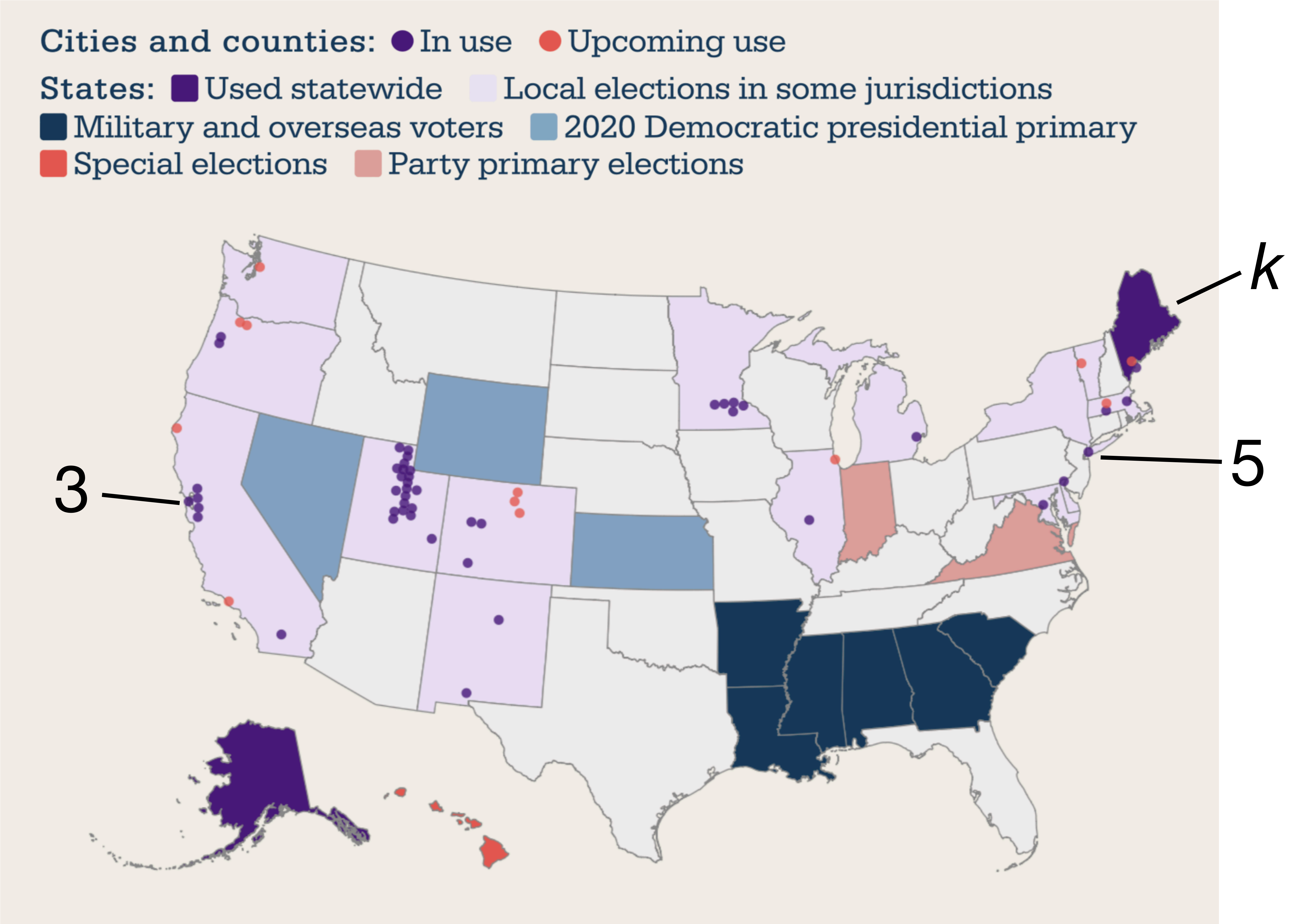


3

5

New York City

Ballot length: how many candidates can you rank?



Maine

State of Maine Sample Ballot
Democratic Primary Election, June 12, 2018
for

Instructions to Voters

To vote, fill in the oval like this ●

To rank your candidate choices, fill in the oval:

- In the 1st column for your 1st choice candidate.
- In the 2nd column for your 2nd choice candidate, and so on.

Continue until you have ranked as many or as few candidates as you like.

Fill in no more than one oval for each candidate or column.

To rank a write-in candidate, write the person's name in the write-in space and fill in the oval for the ranking of your choice.

Governor	1st Choice	2nd Choice	3rd Choice	4th Choice	5th Choice	6th Choice	7th Choice	8th Choice
Cote, Adam Roland Sanford	○	○	○	○	○	○	○	○
Dion, Donna J. Biddeford	○	○	○	○	○	○	○	○
Dion, Mark N. Portland	○	○	○	○	○	○	○	○
Eves, Mark W. North Berwick	○	○	○	○	○	○	○	○
Mills, Janet T. Farmington	○	○	○	○	○	○	○	○
Russell, Diane Marie Portland	○	○	○	○	○	○	○	○
Sweet, Elizabeth A. Hallowell	○	○	○	○	○	○	○	○
Write-in	○	○	○	○	○	○	○	○

SOURCE: Maine Secretary of State Office

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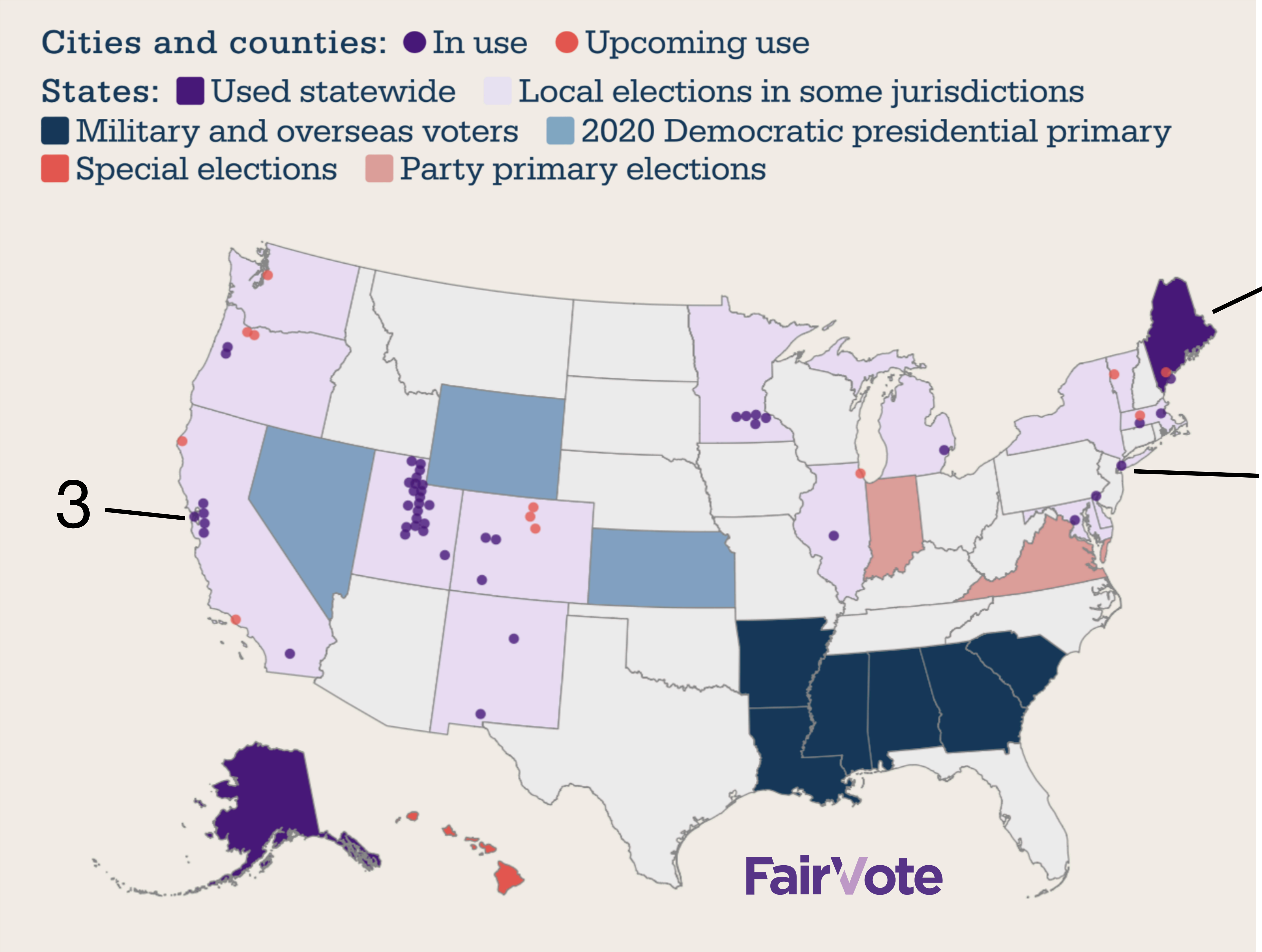
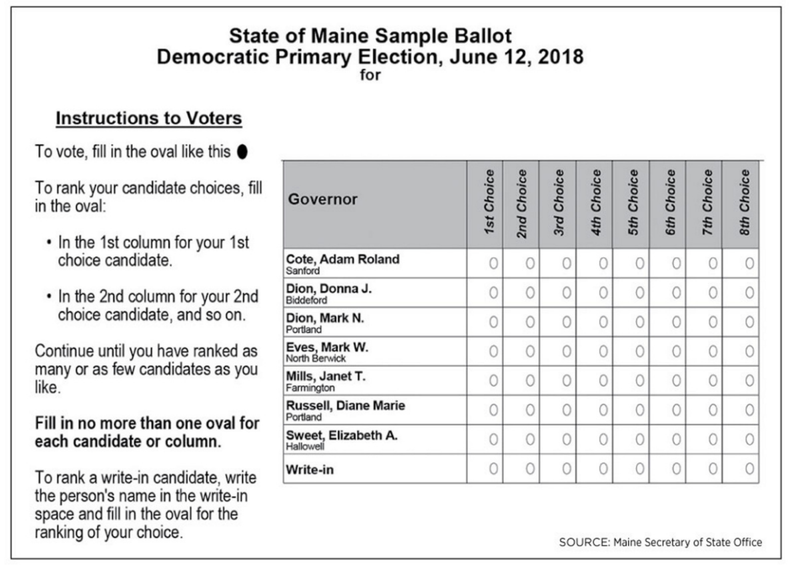
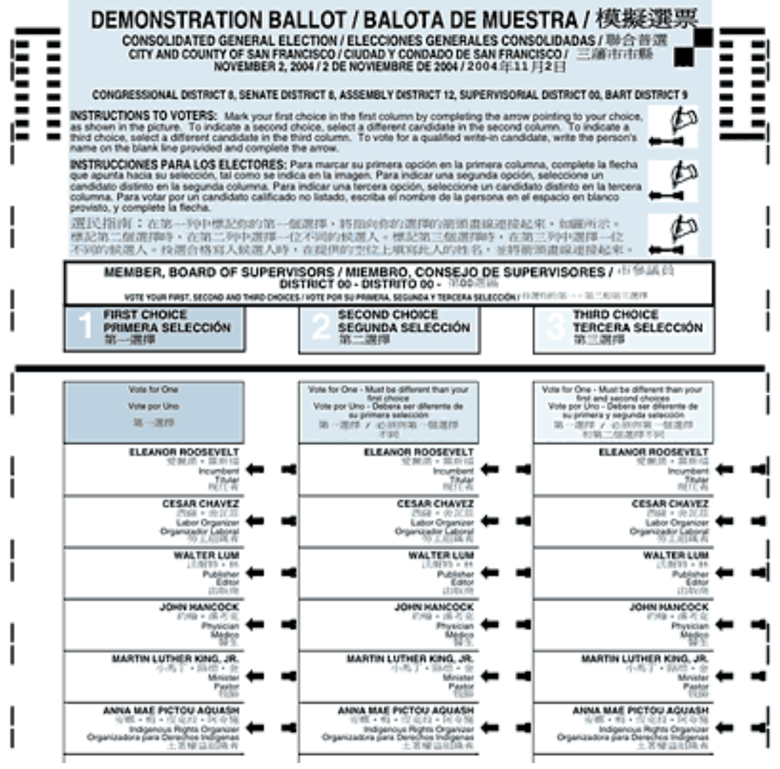


3

5

k



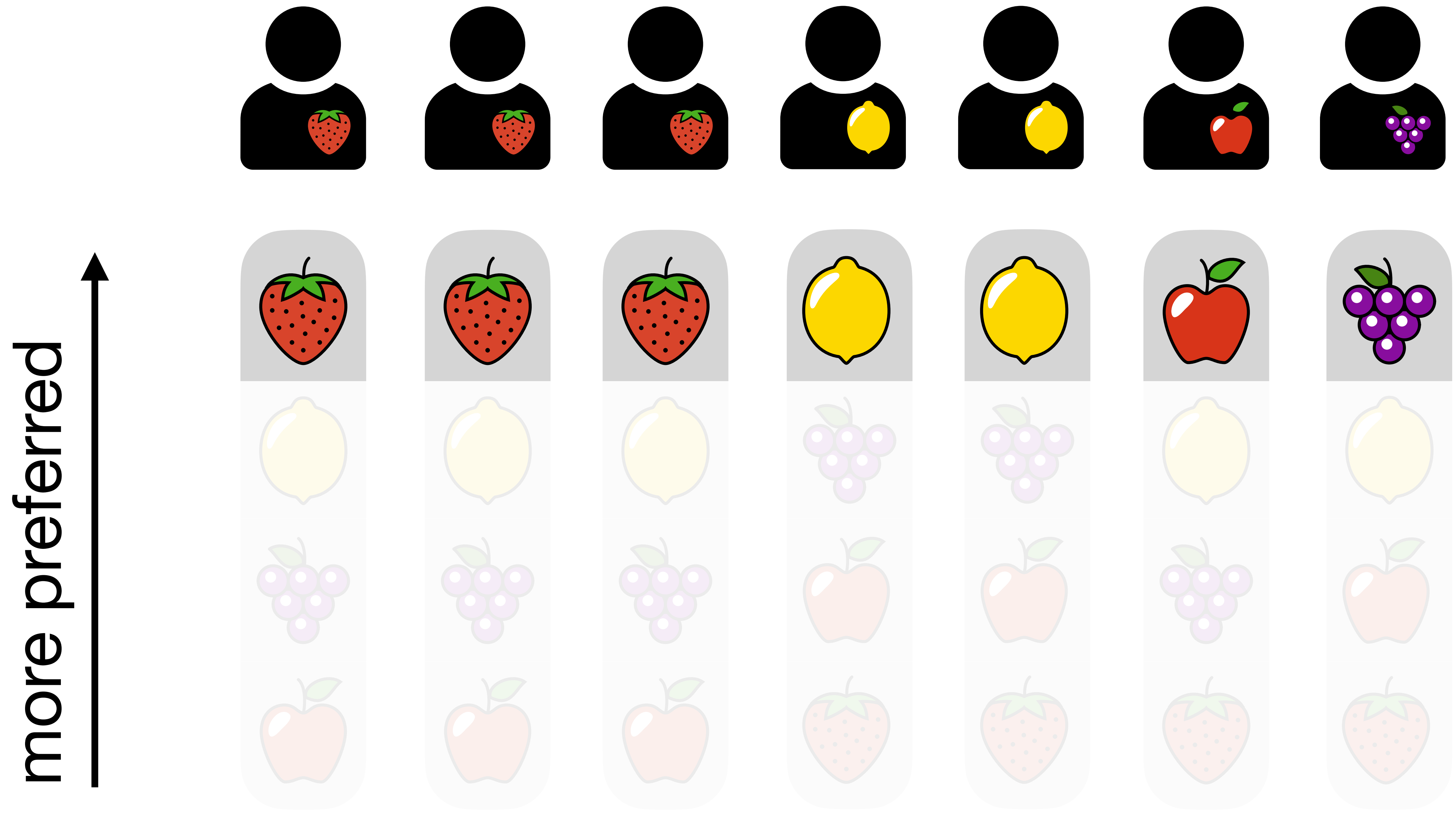


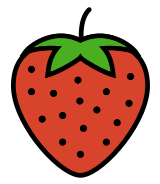
How much does ballot length matter?

Ballot length can change the winner

fix the profile, truncate all rankings

h = ballot length
k = # candidates



h	winner
1	

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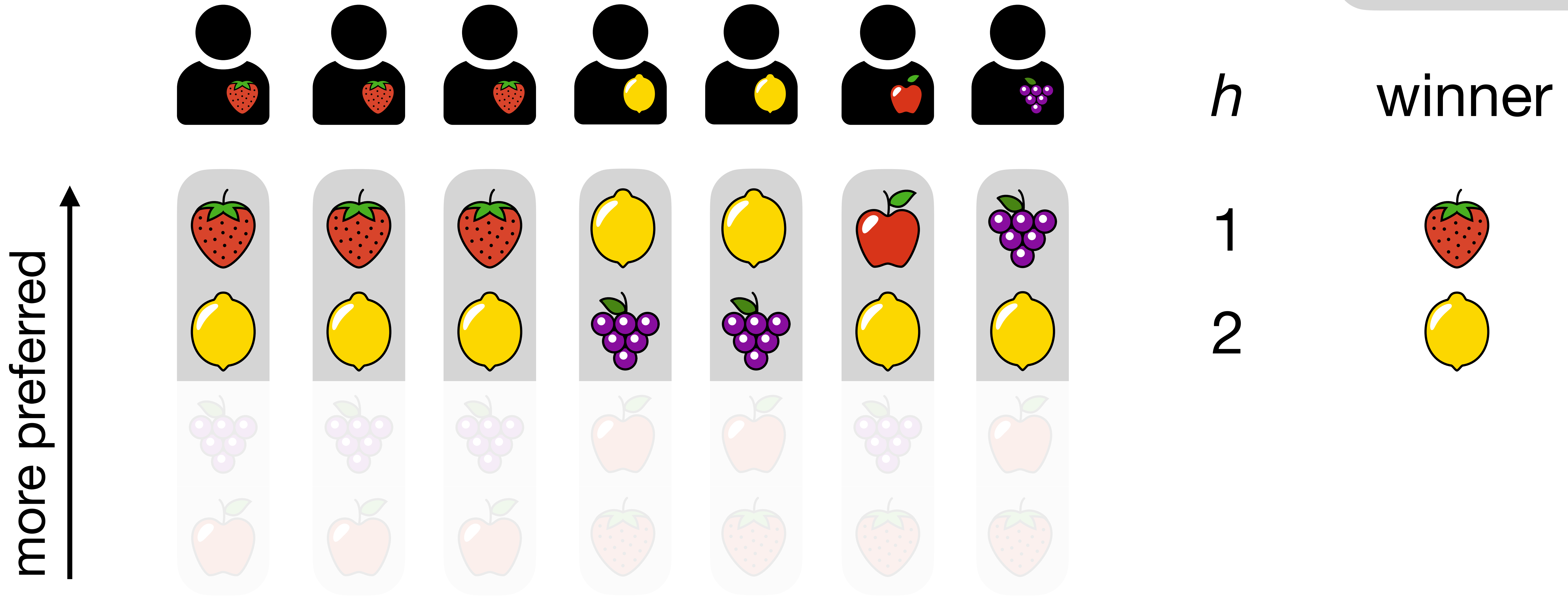

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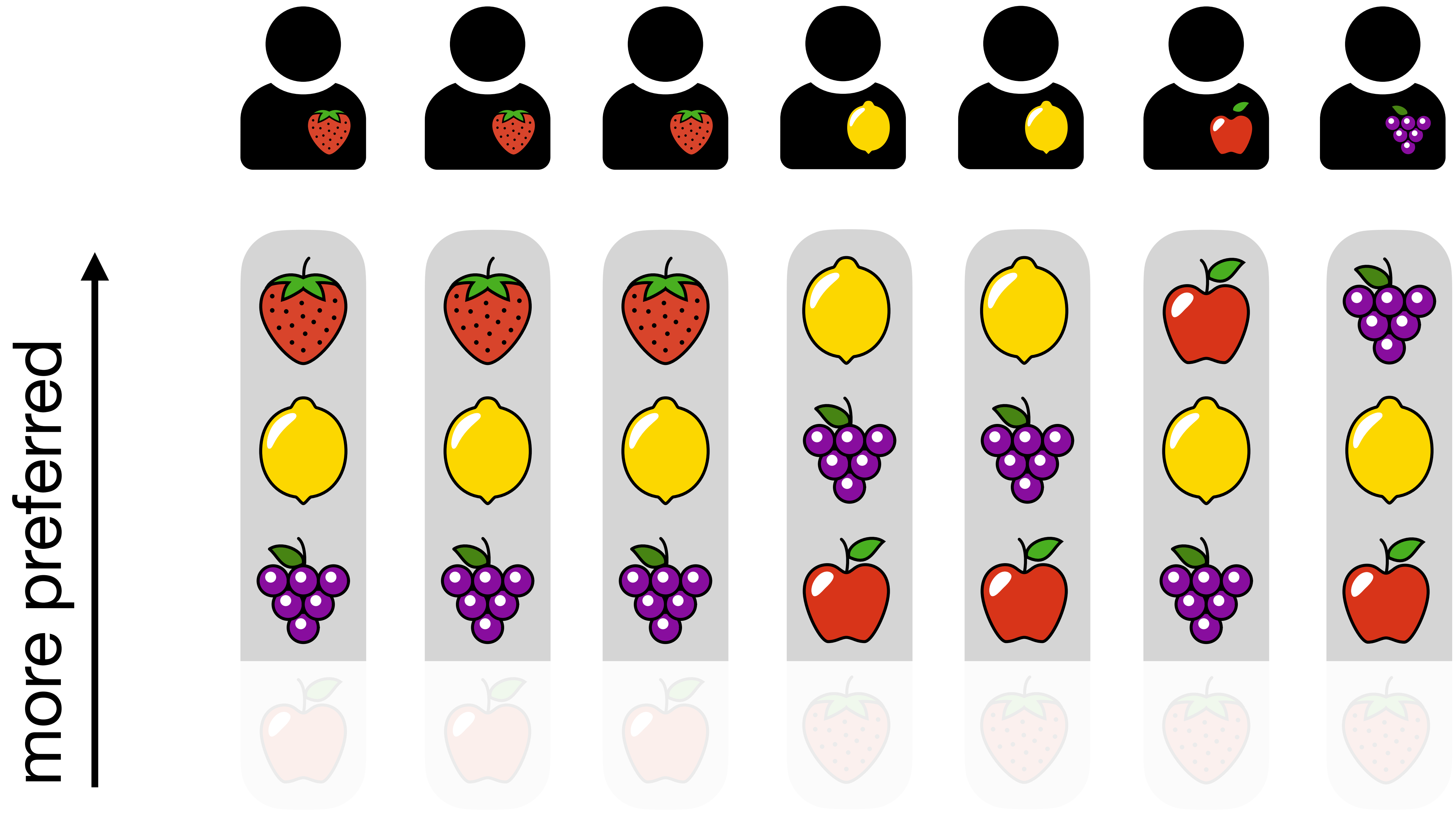

h	winner
1	
2	
3	

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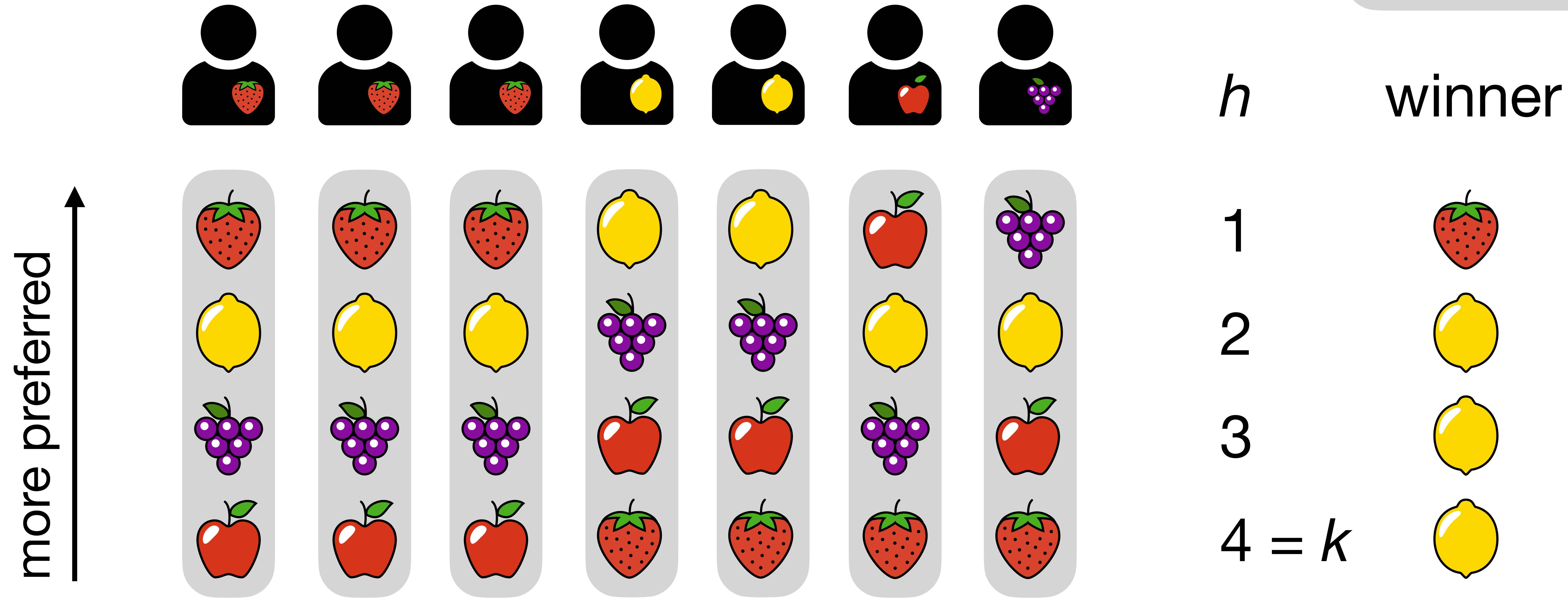

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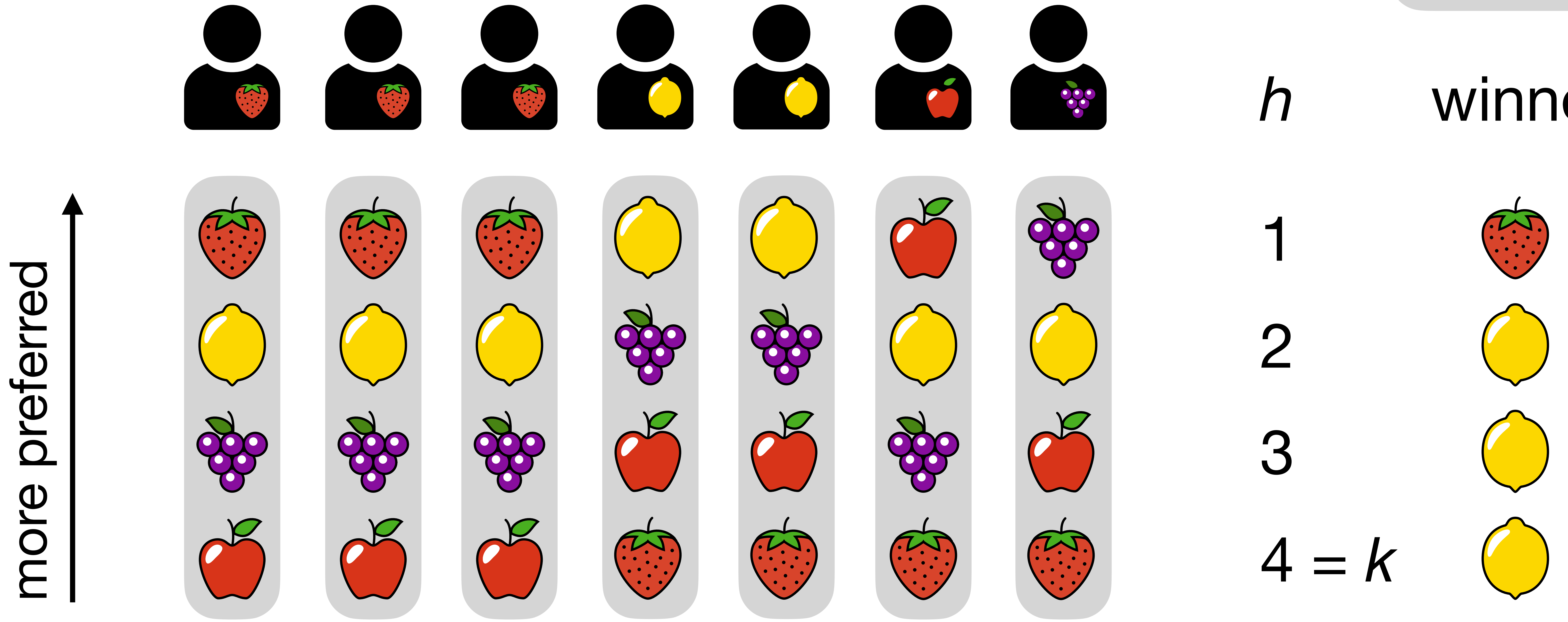

$k \equiv k - 1$

Prior work

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voluntary truncation

[Saari & Newenhizen, *Public Choice* 1988]
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Public Choice (2020) 184:197–218

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"A natural question […] is whether the outcome of the election stays the same as the extent of truncation increases from 0 (complete ballots) to k – 1. If not, how many different winners are possible?"

"In thousands of simulations involving k = 4, 5, and 6 candidates, we found instances of up to k – 2 different winners."

A k-1 winner construction for $k = 4$

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voter count	2	5	6	6	3	2
	A	A	B	C	D	D
	D		D		B	C
	C		A			

A *k*-1 winner construction for *k* = 4

voter count	2	5	6	6	3	2
	A	A	B	C	D	D
	D		D		B	C
	C		A			

ballot length h:	1	2	3
winner:	A	B	C

A *k*-1 winner construction for *k* = 4

voter count	2	5	6	6	3	2
	A	A	B	C	D	D
	D		D		B	C
	C		A			

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no smaller 3-winner k = 4 profile exists

A k-1 winner construction for k = 4

voter count	2	5	6	6	3	2
	A	A	B	C	D	D
	D		D		B	C
	C		A			

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no smaller 3-winner k = 4 profile exists

we generalize this construction to any *k*

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Theorem 2

For every $k > 3$, there are consequential-tie-free profiles with $2k^2 - 2k$ voters and $k - 1$ truncation winners.

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Theorem 2

For every $k > 3$, there are consequential-tie-free profiles with $2k^2 - 2k$ voters and $k - 1$ truncation winners.

Theorem 1

For every $k > 3$, a consequential-tie-free profile needs at least $2k^2 - 2k$ voters to have $k - 1$ truncation winners.

Actually, it’s even worse….

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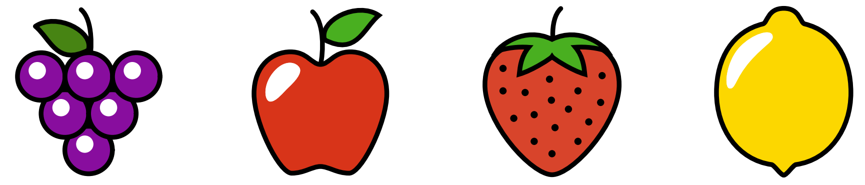
label candidates in IRV elimination order:

elimination order

1 2 3 4



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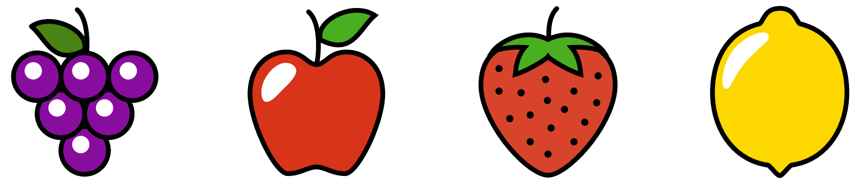
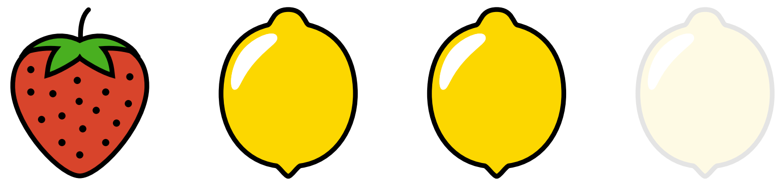


truncation winner sequence

ballot length h 1 2 3 4



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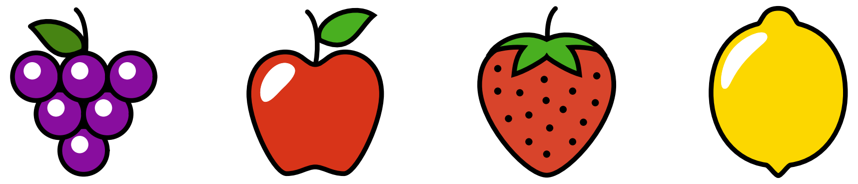

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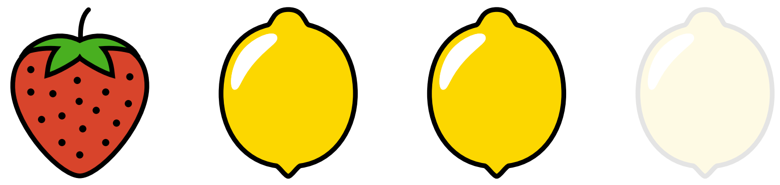

winner 3 4 4

a truncation winner sequence is *feasible* if it's element-wise $> 1, 2, \ldots, k-1$

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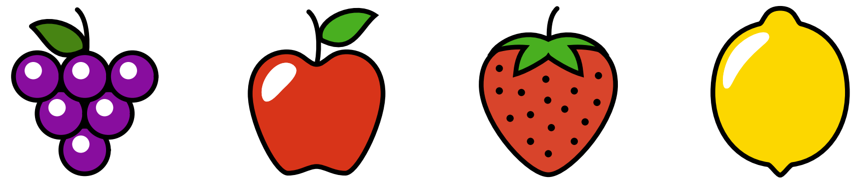

truncation winner sequence

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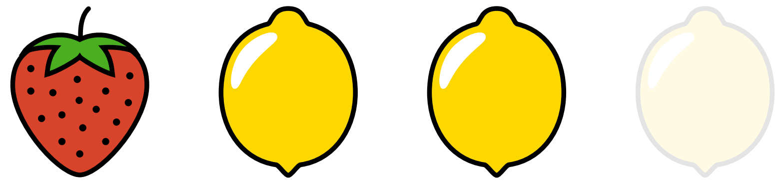

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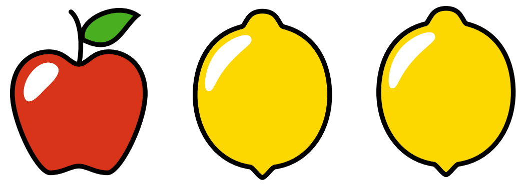

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label candidates in IRV elimination order:

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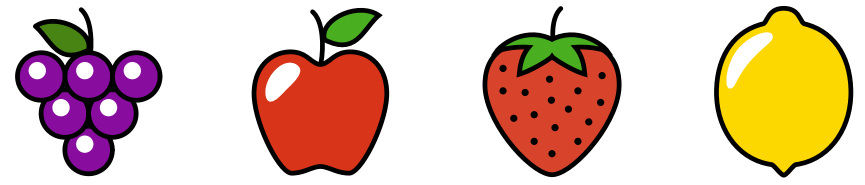

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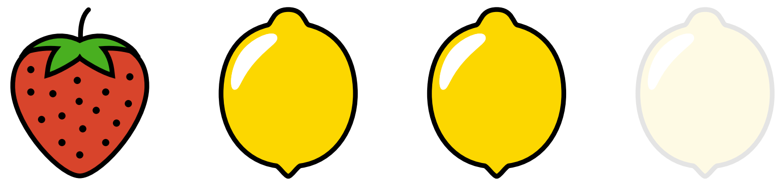

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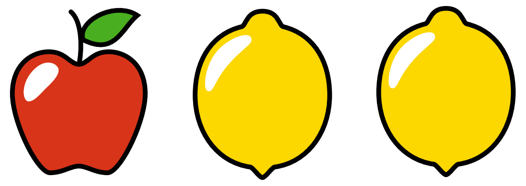

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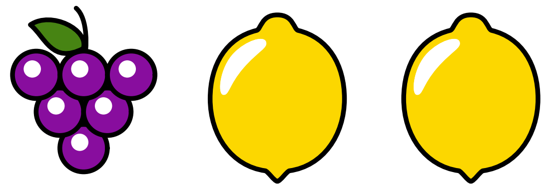

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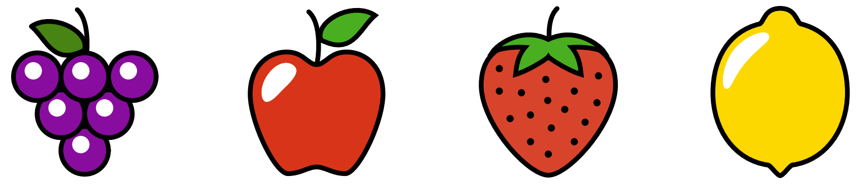

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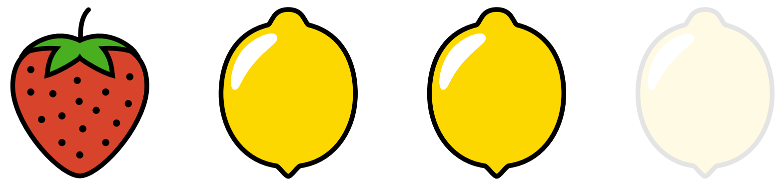

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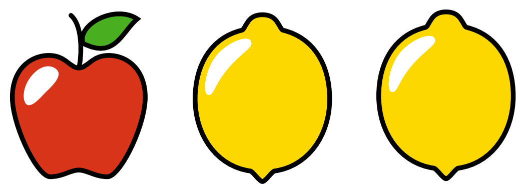

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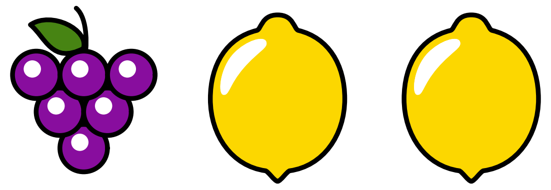

1 4 4

Theorem 2

For every $k > 3$ and **every feasible truncation winner sequence**, there is a consequential-tie-free profile with $2k^2 - 2k$ voters achieving that sequence.

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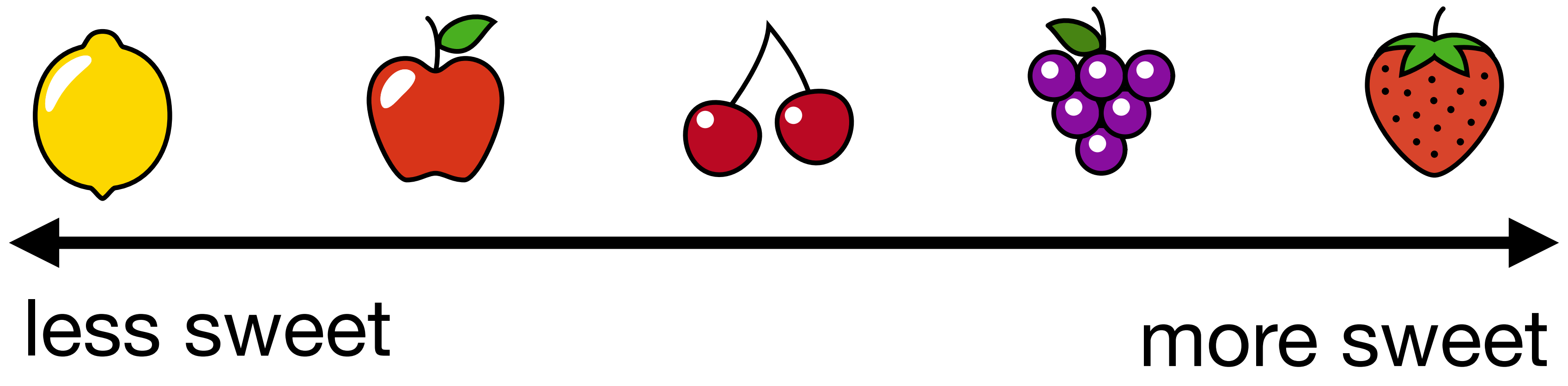

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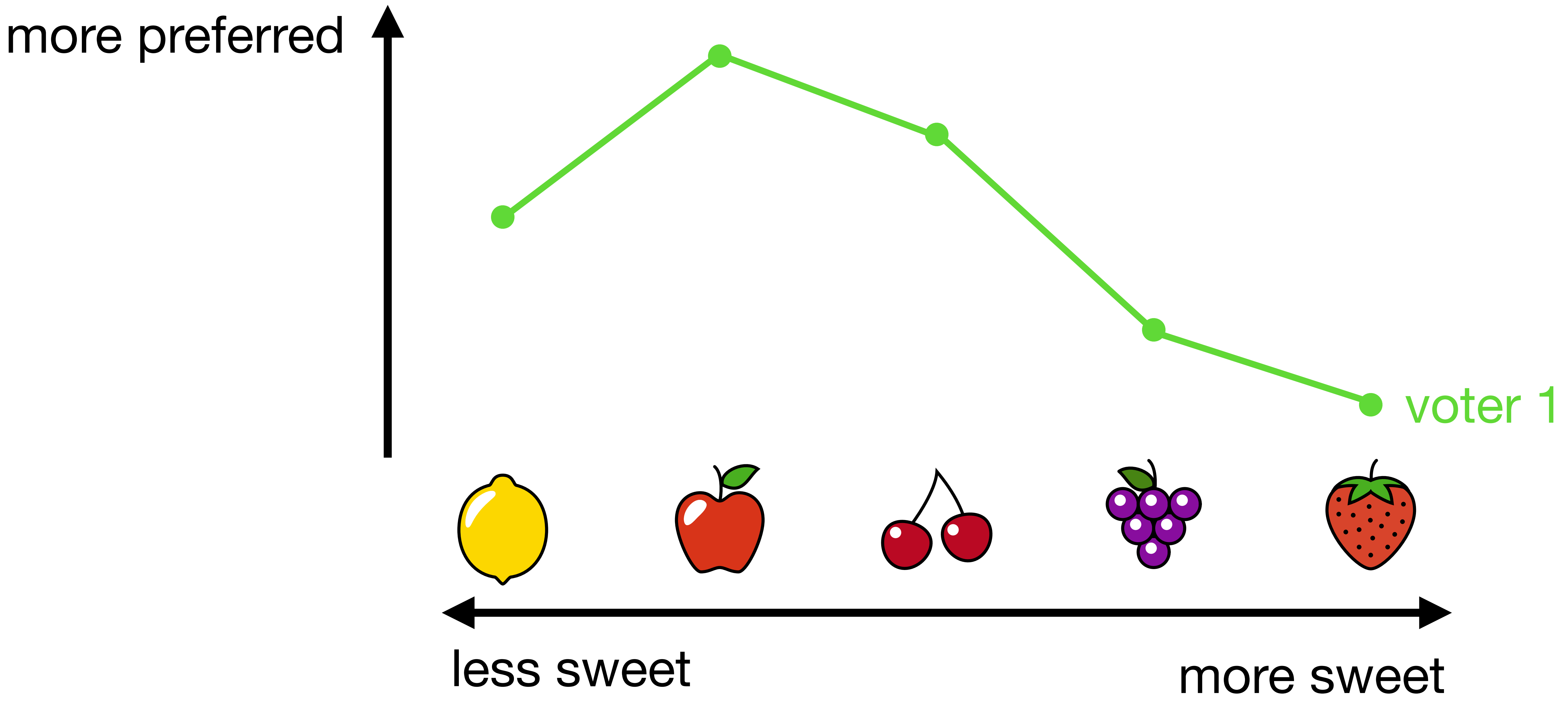

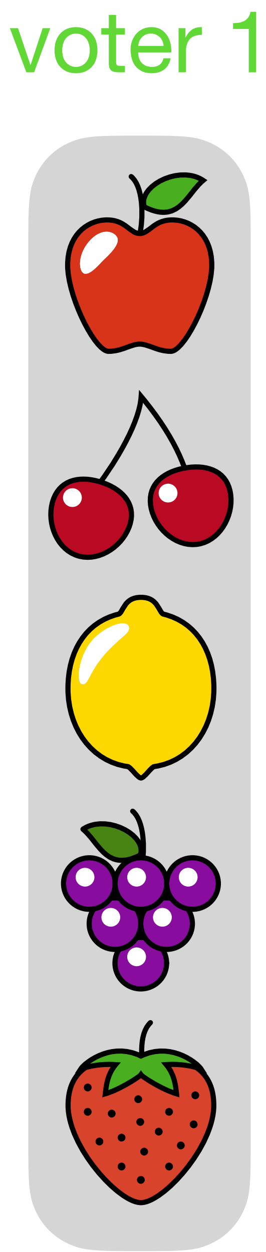

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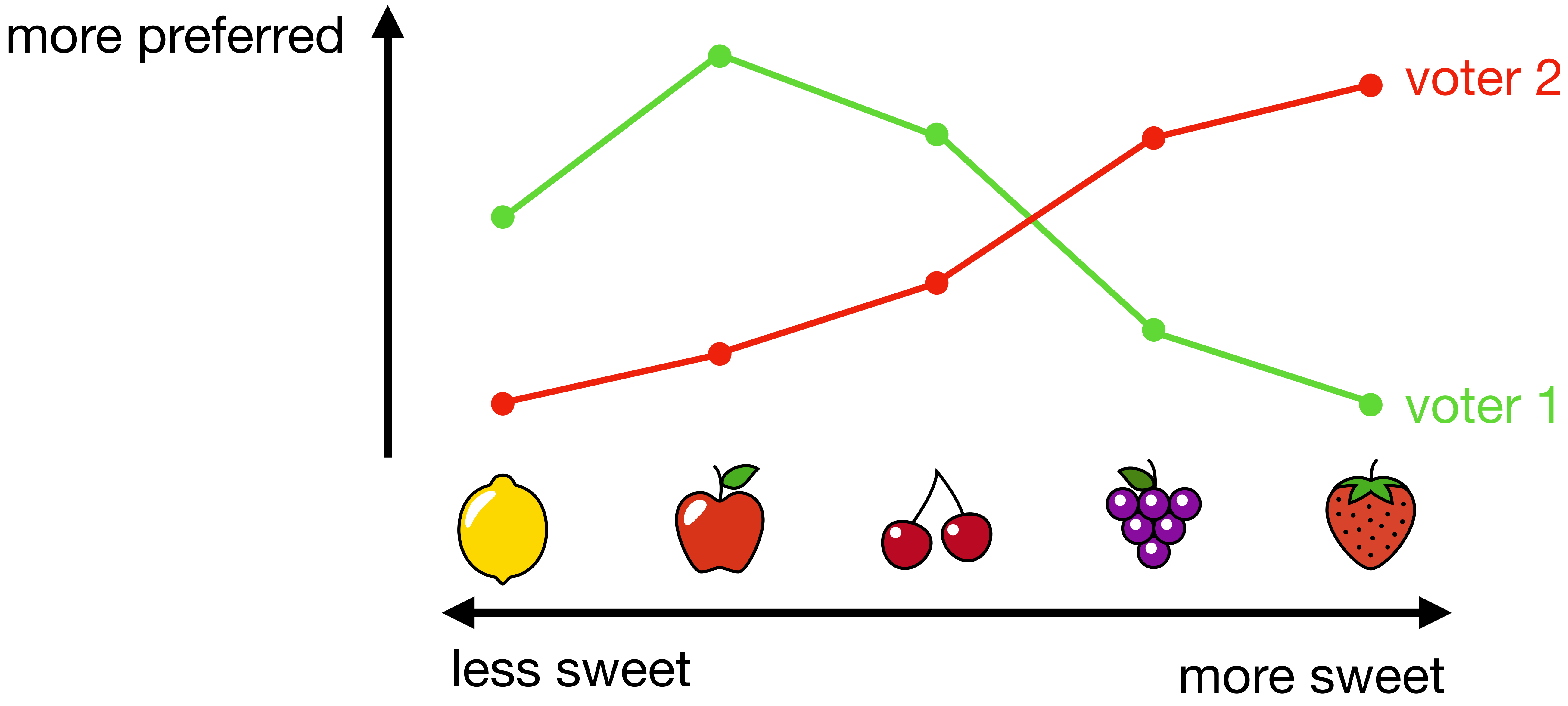

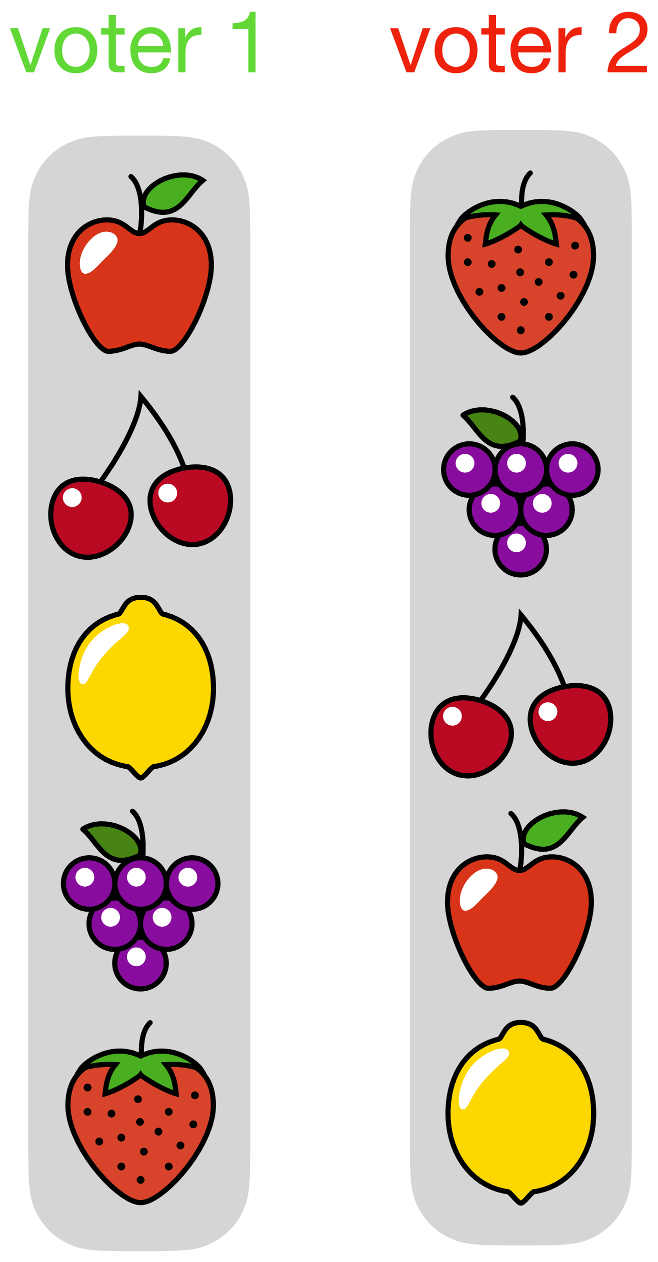

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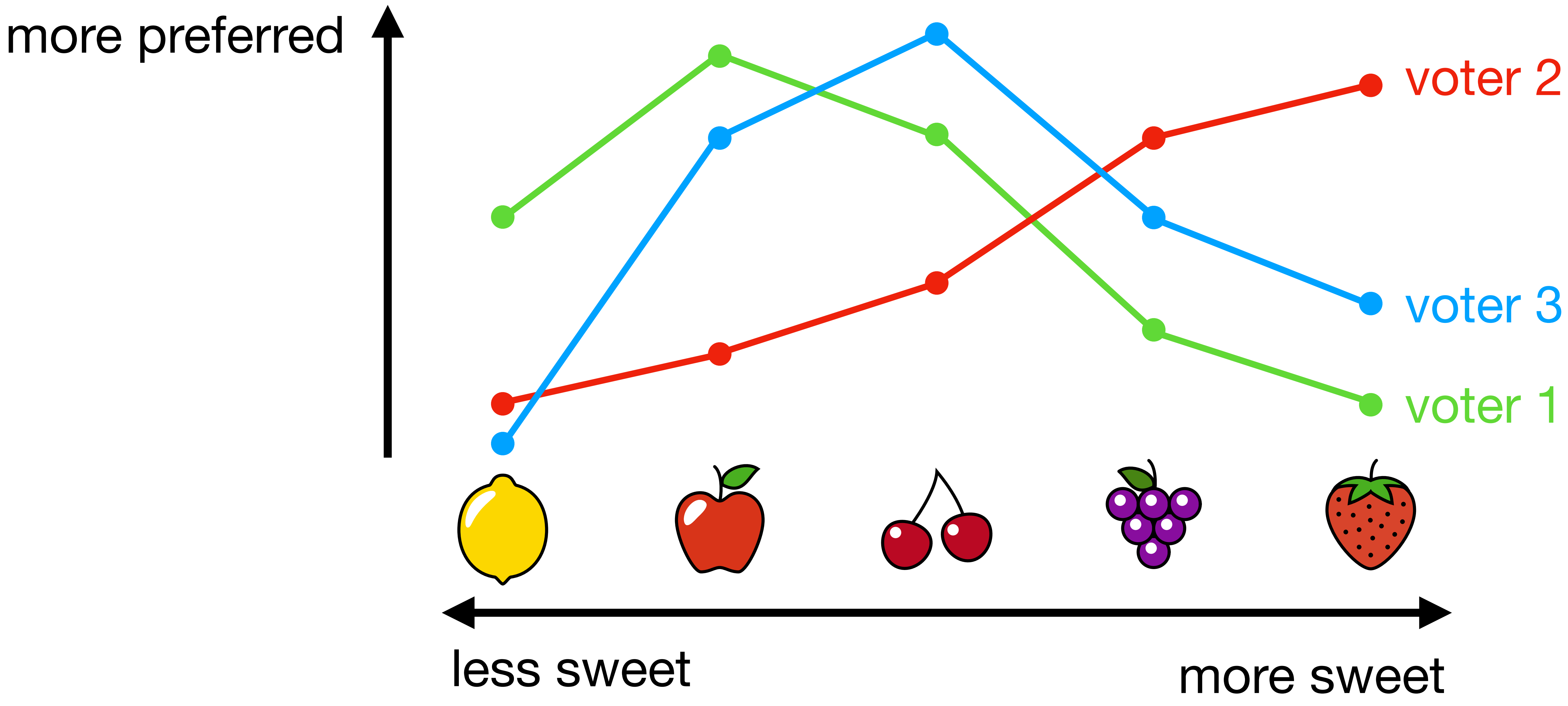

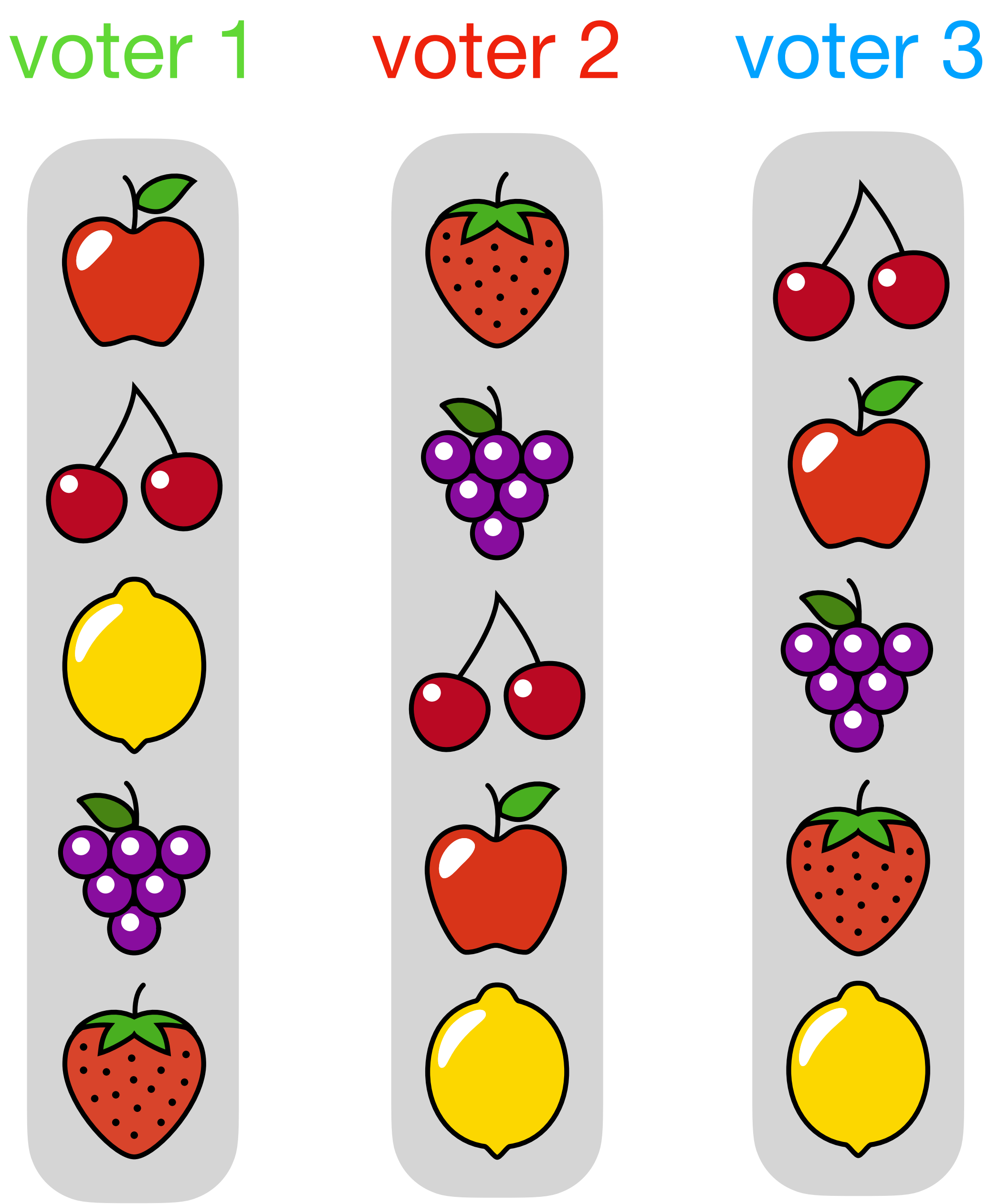

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Open question: more than $\Theta(\sqrt{k})$ truncation winners with single-peaked profiles?

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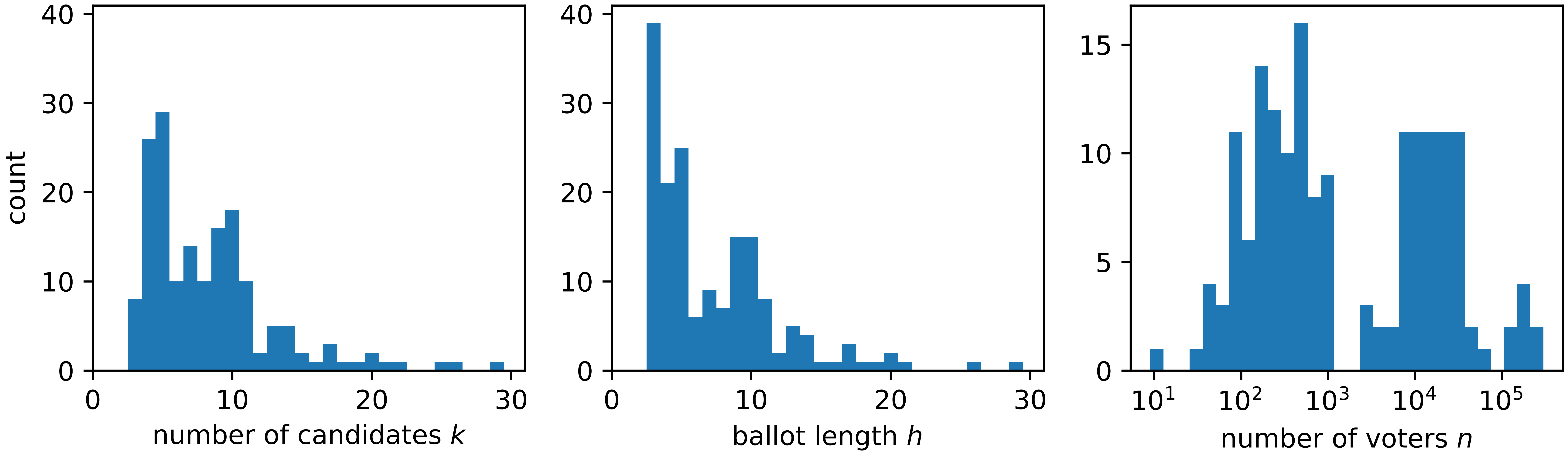

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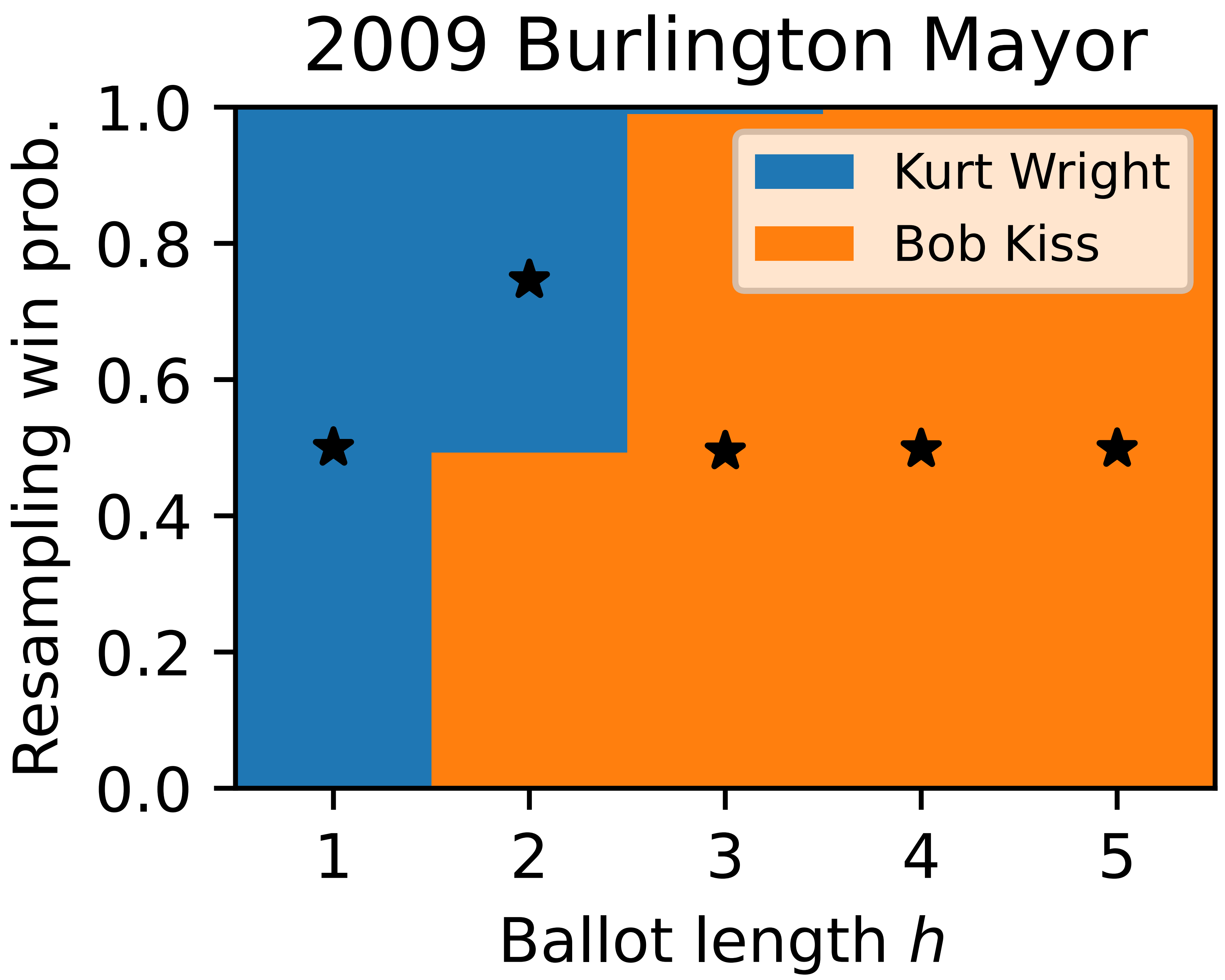

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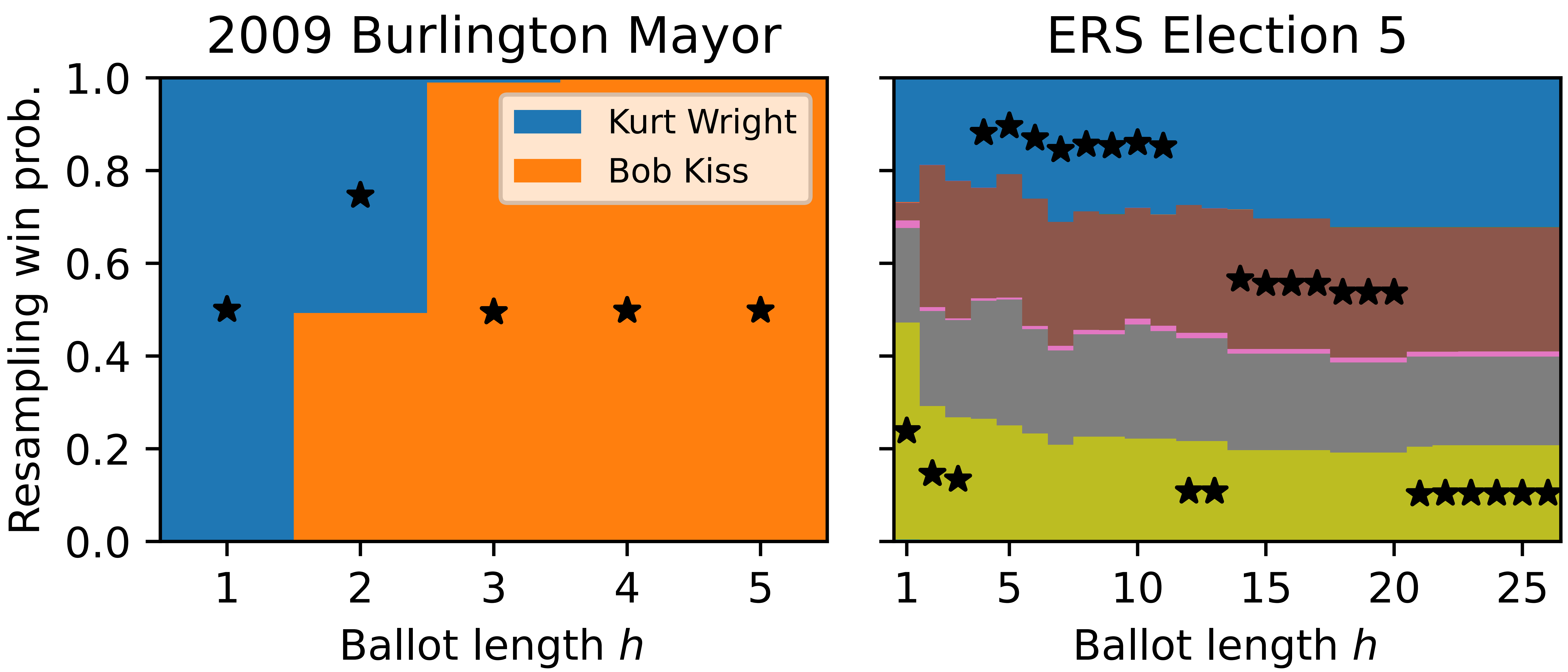
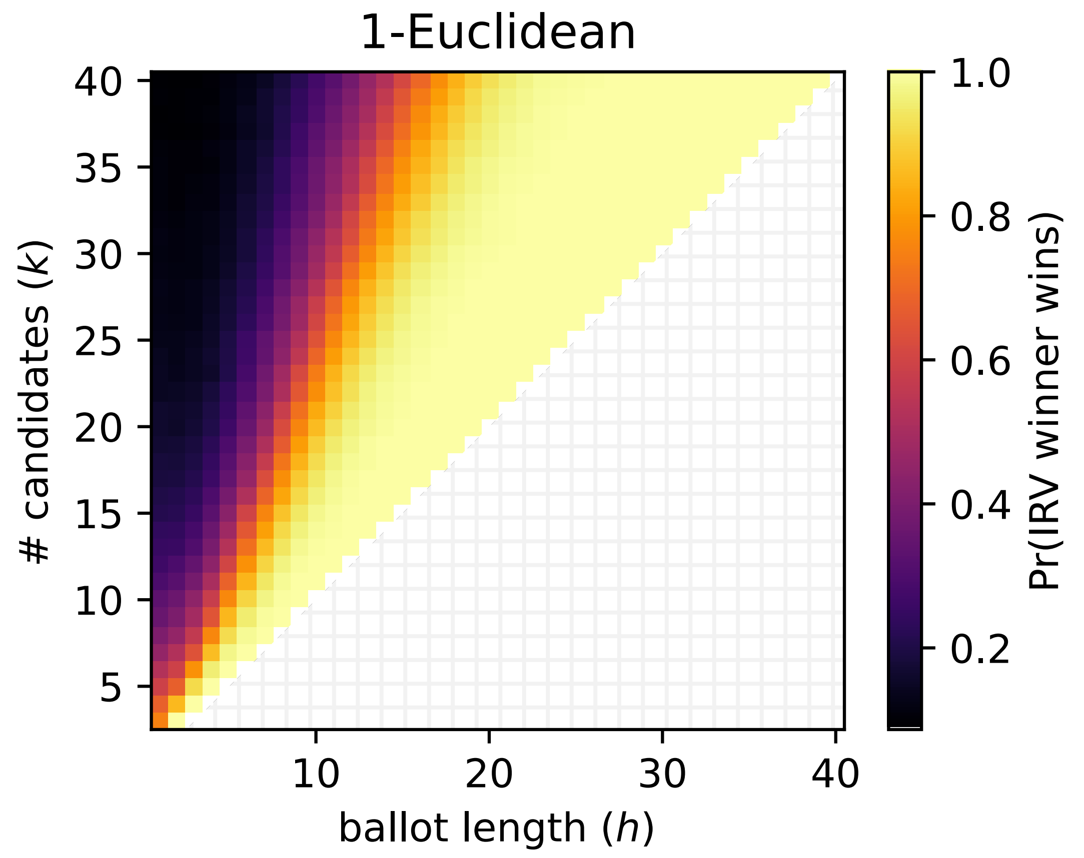
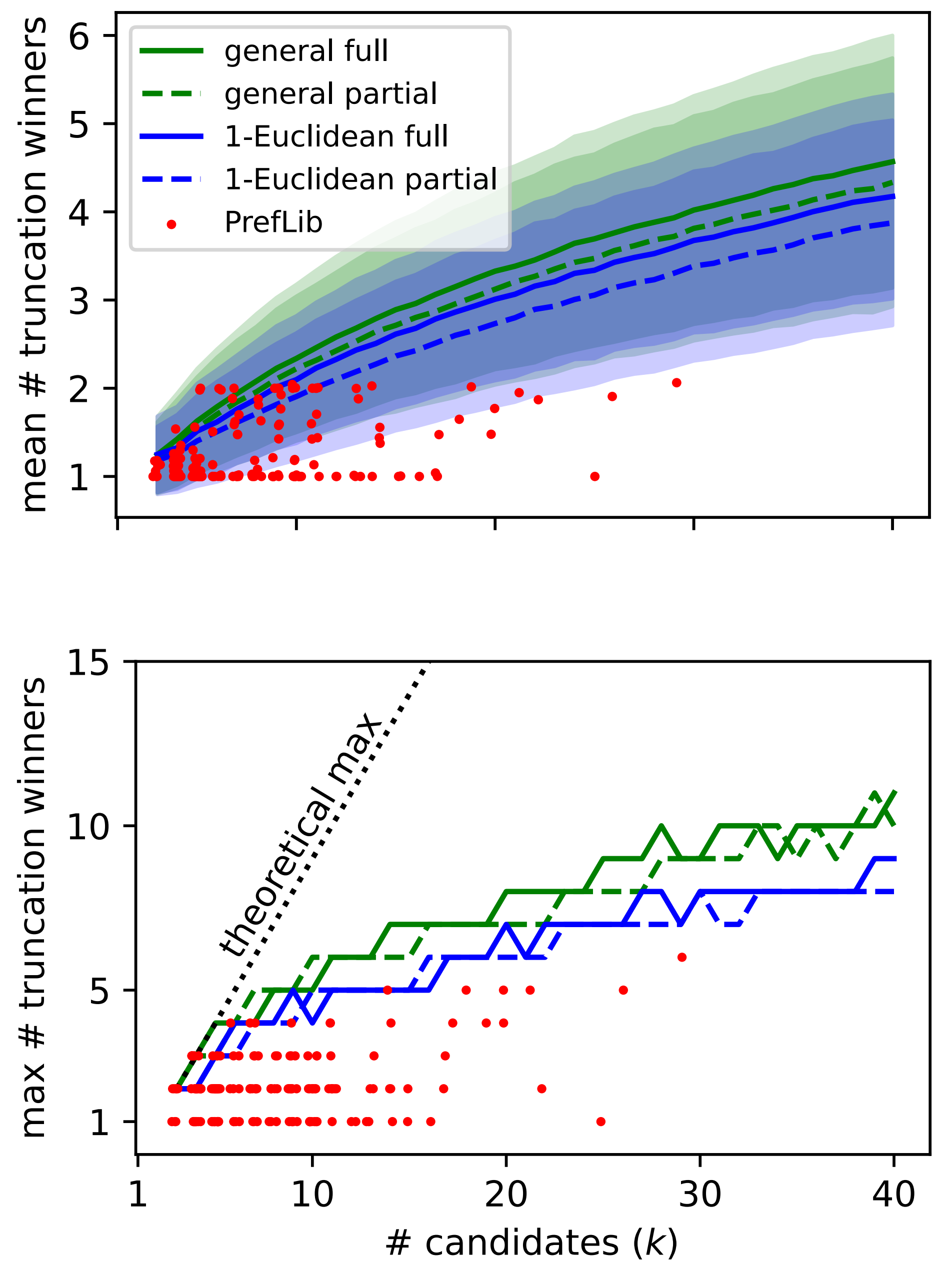


More things in the paper





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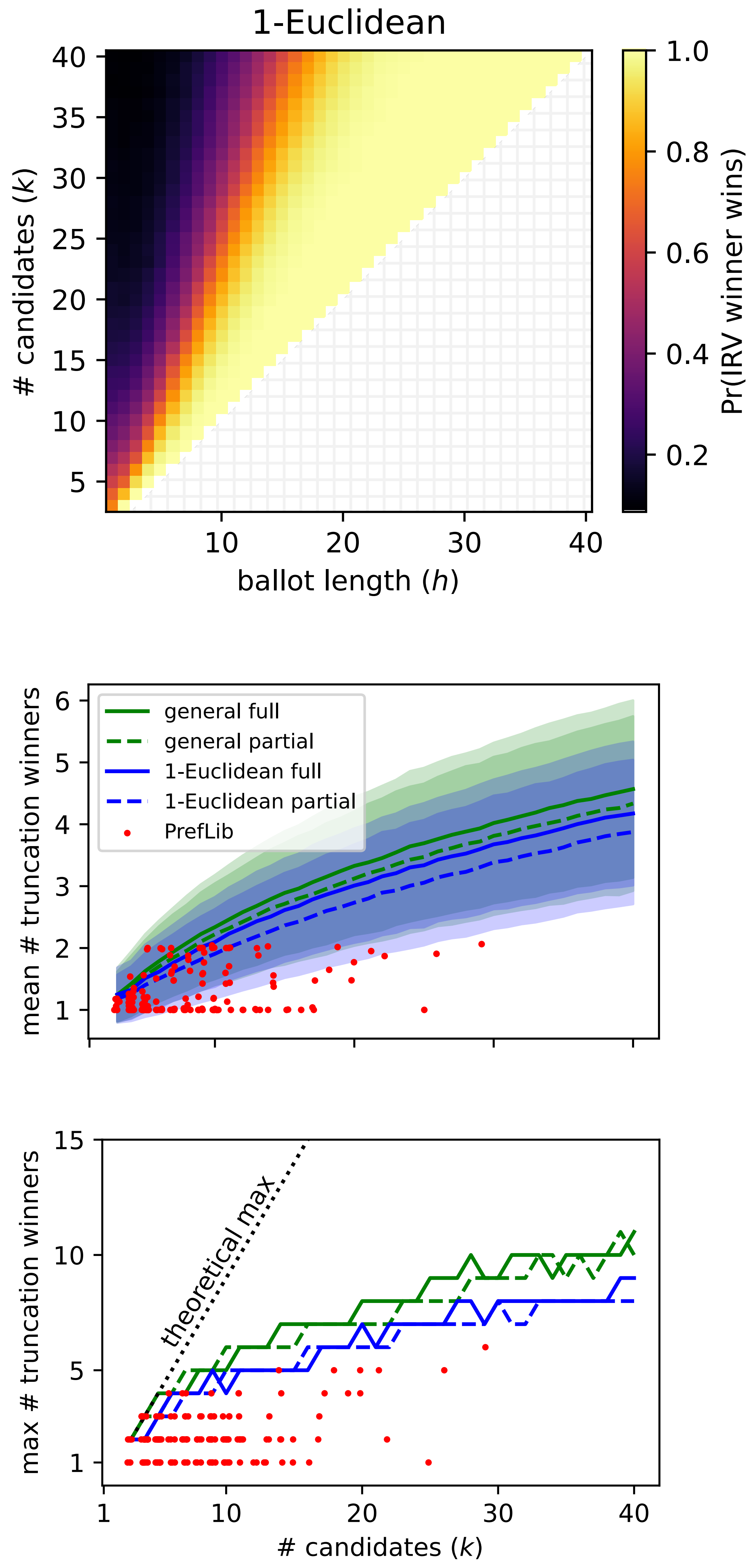

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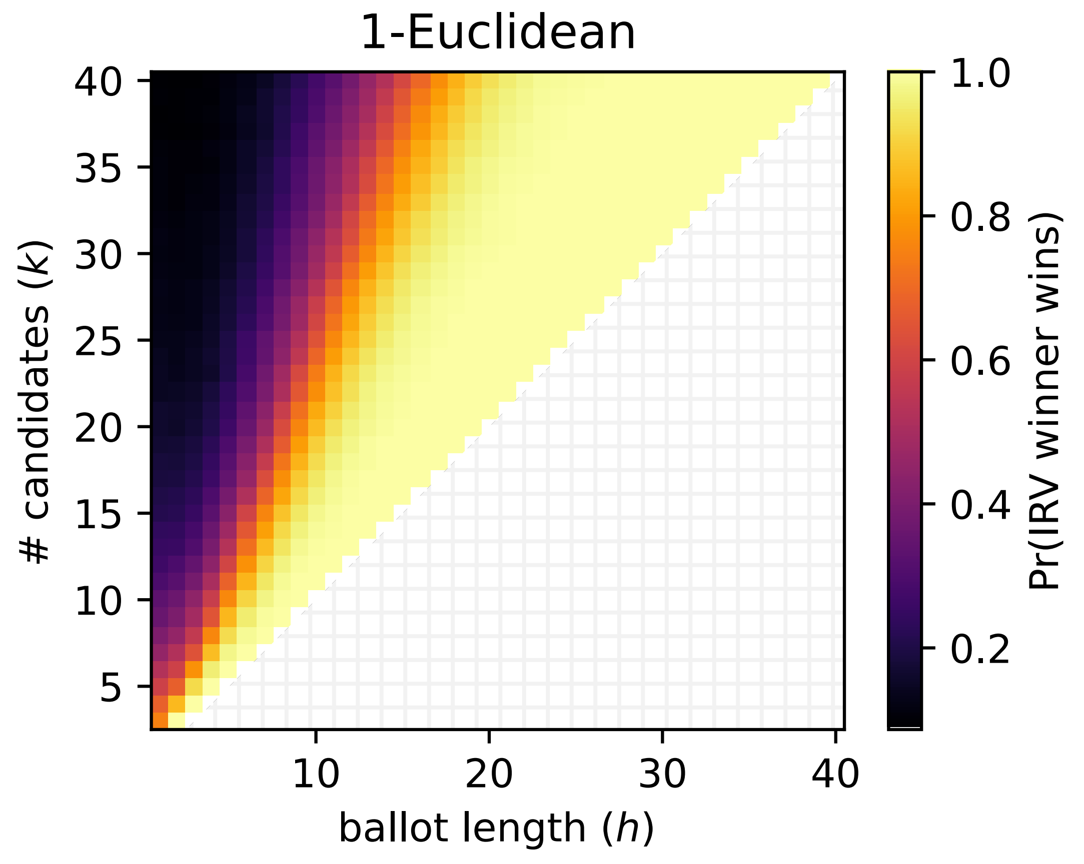

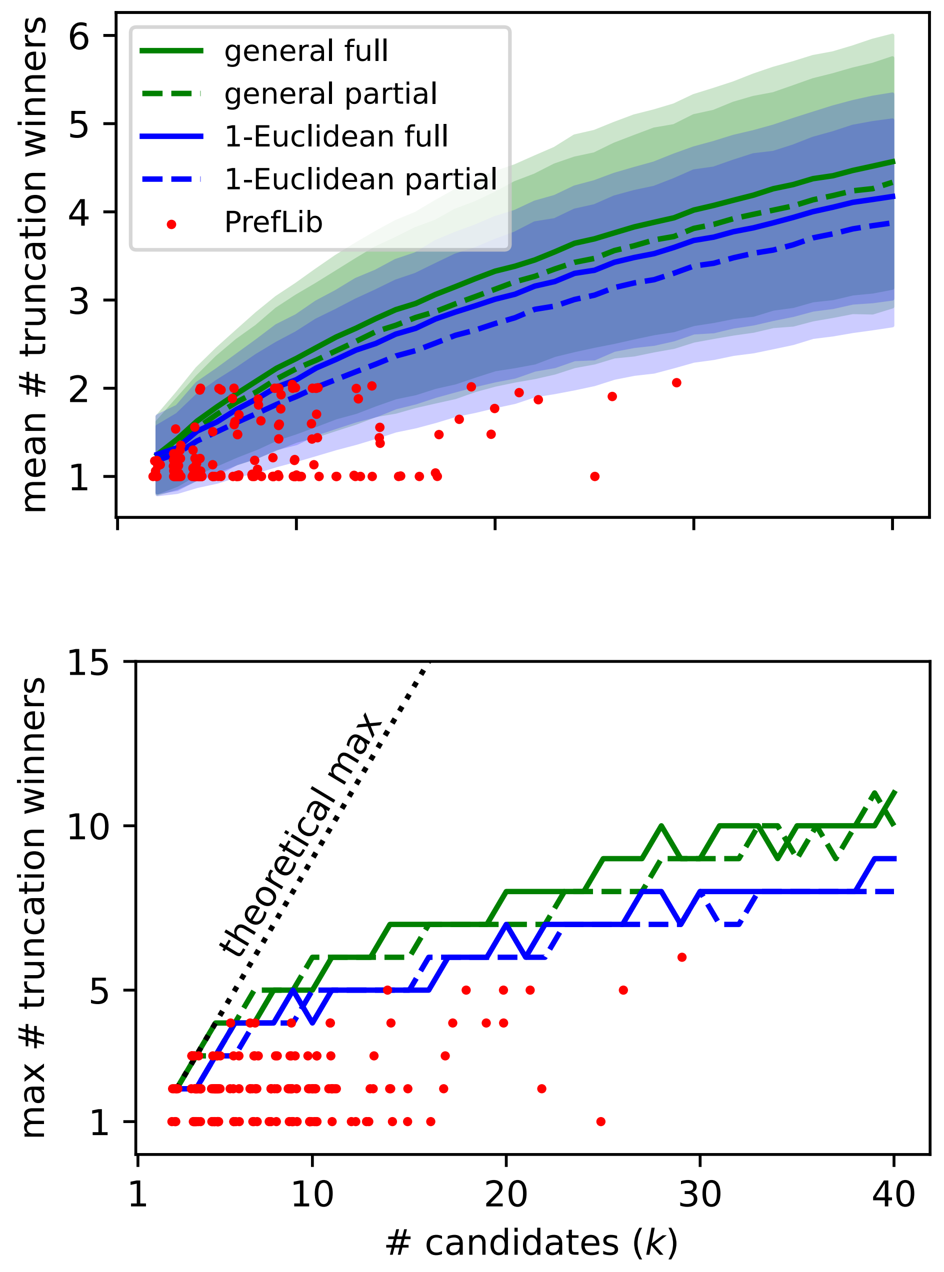

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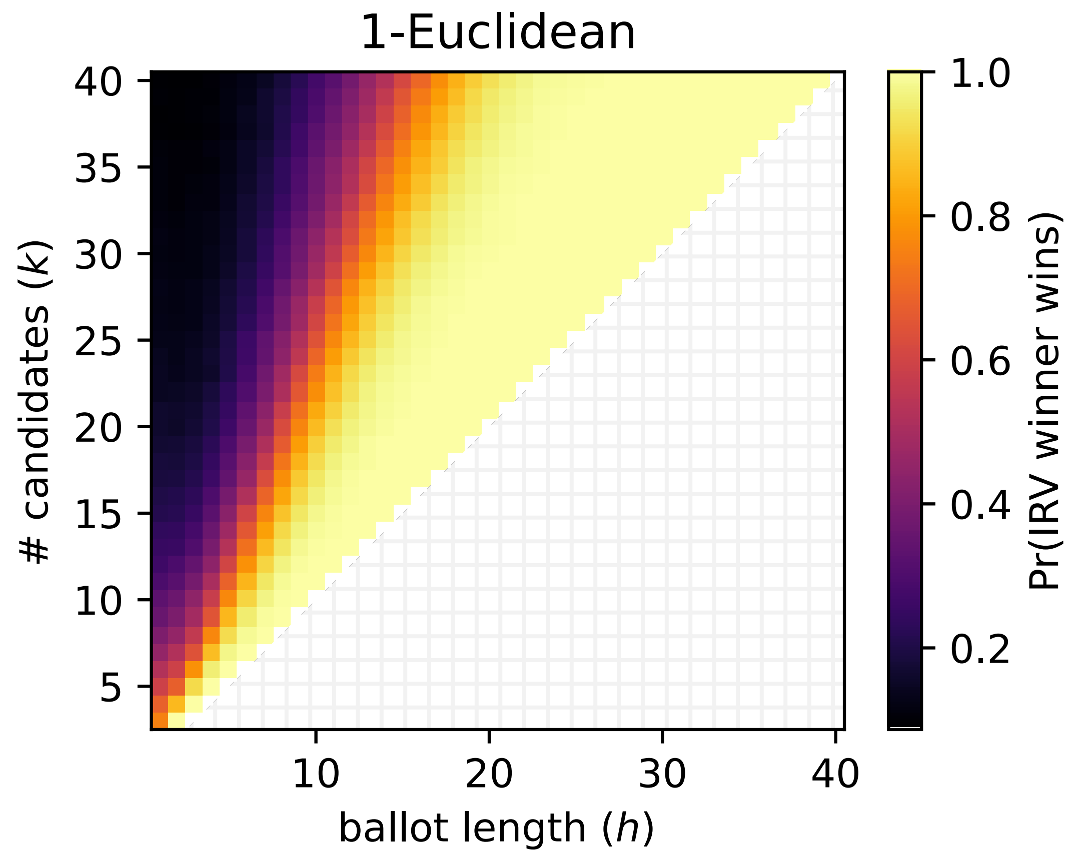

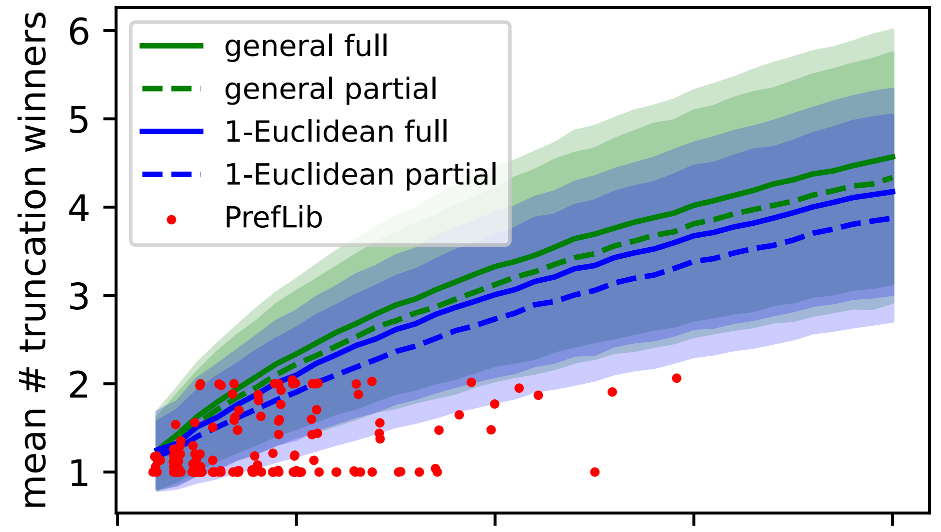

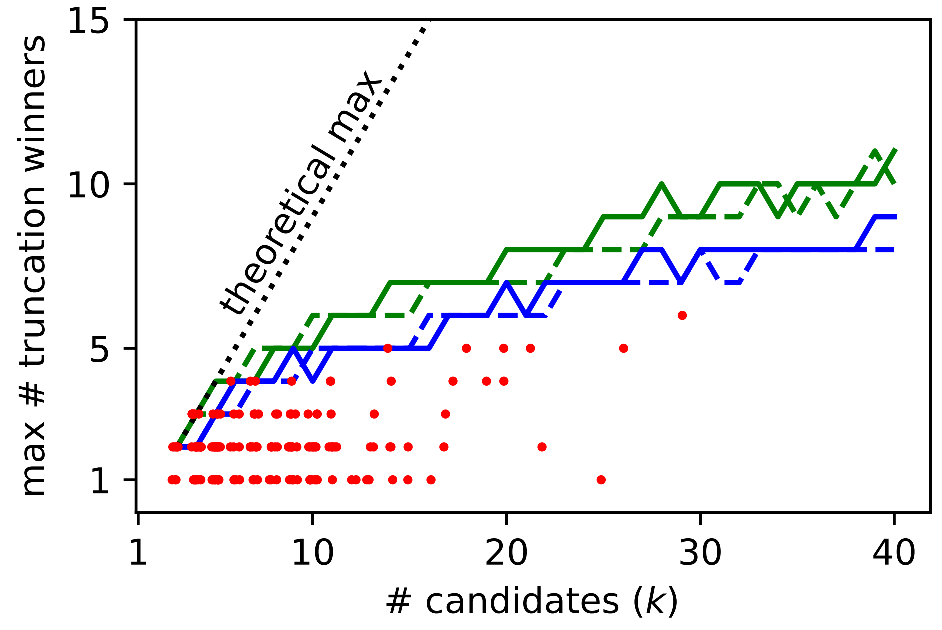

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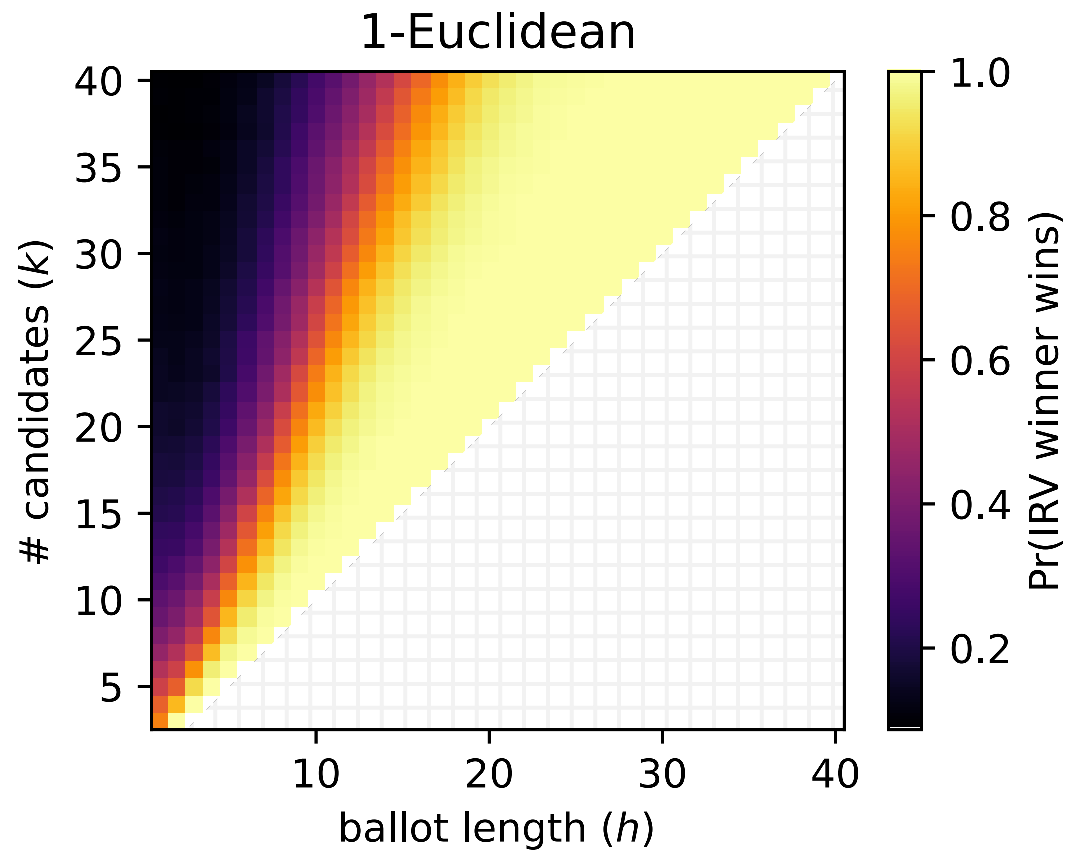

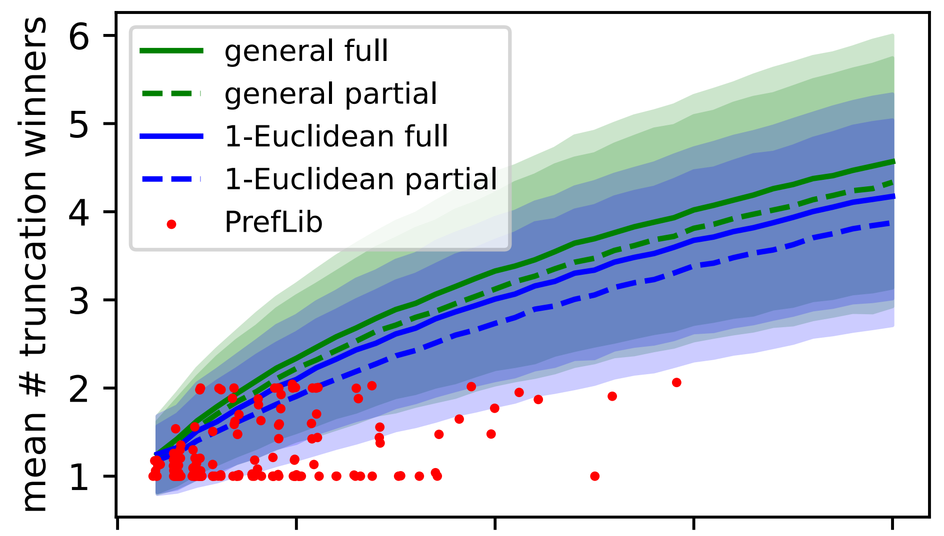

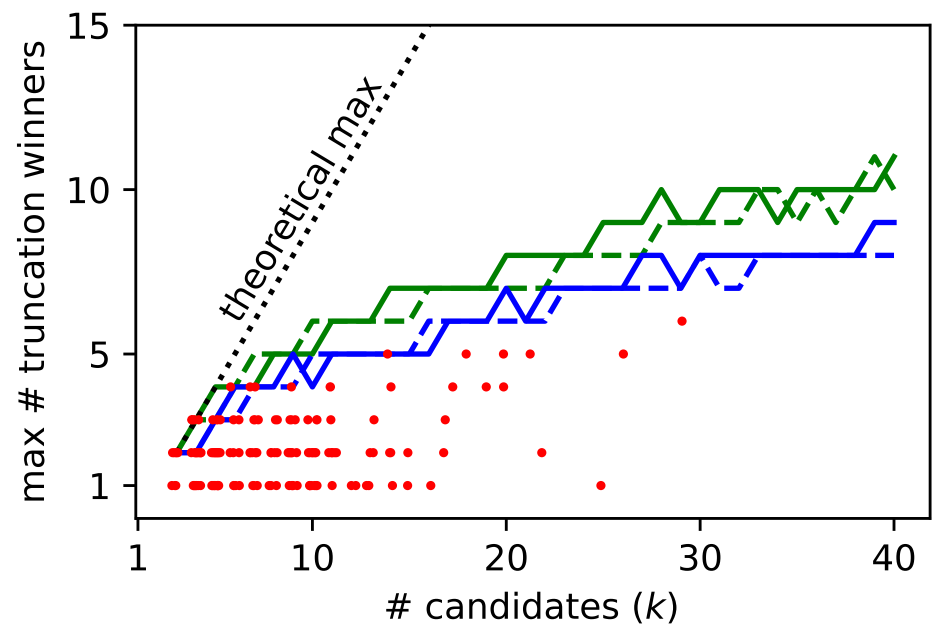

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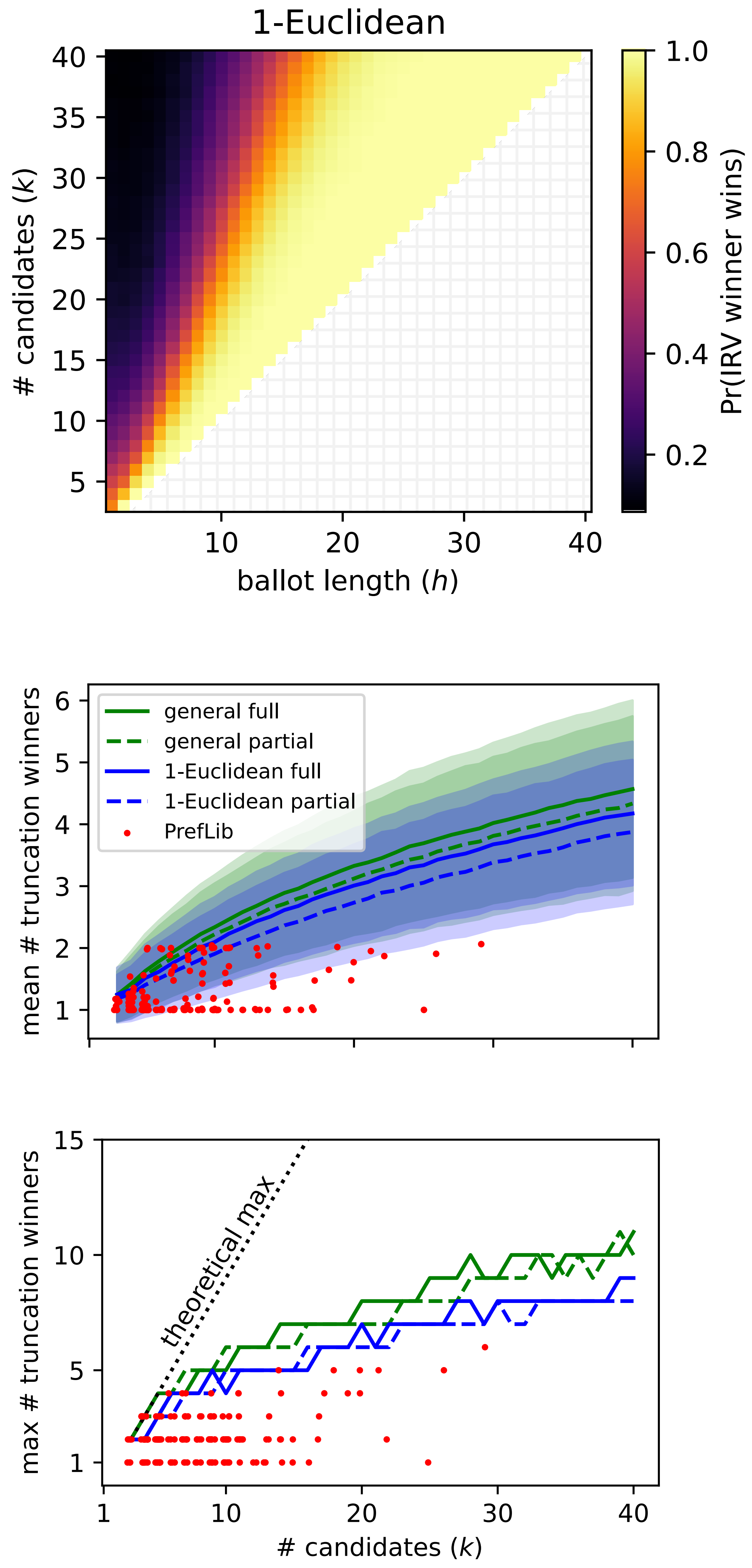

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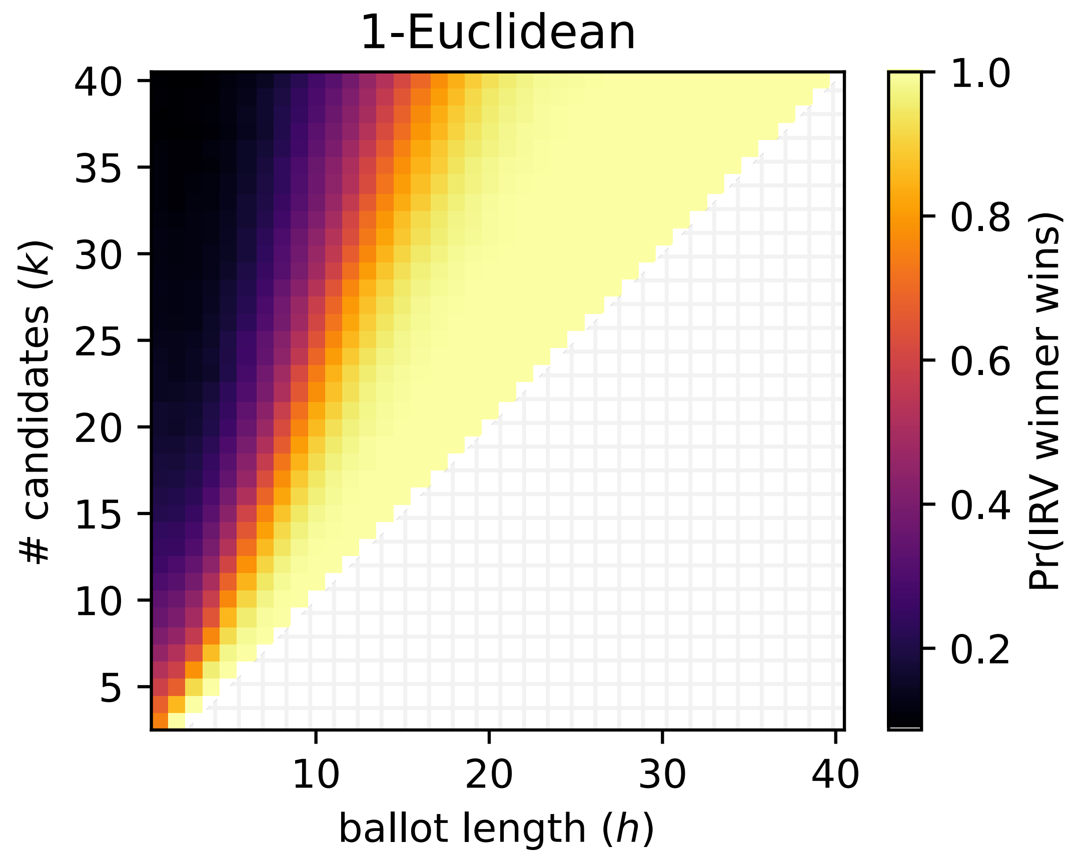

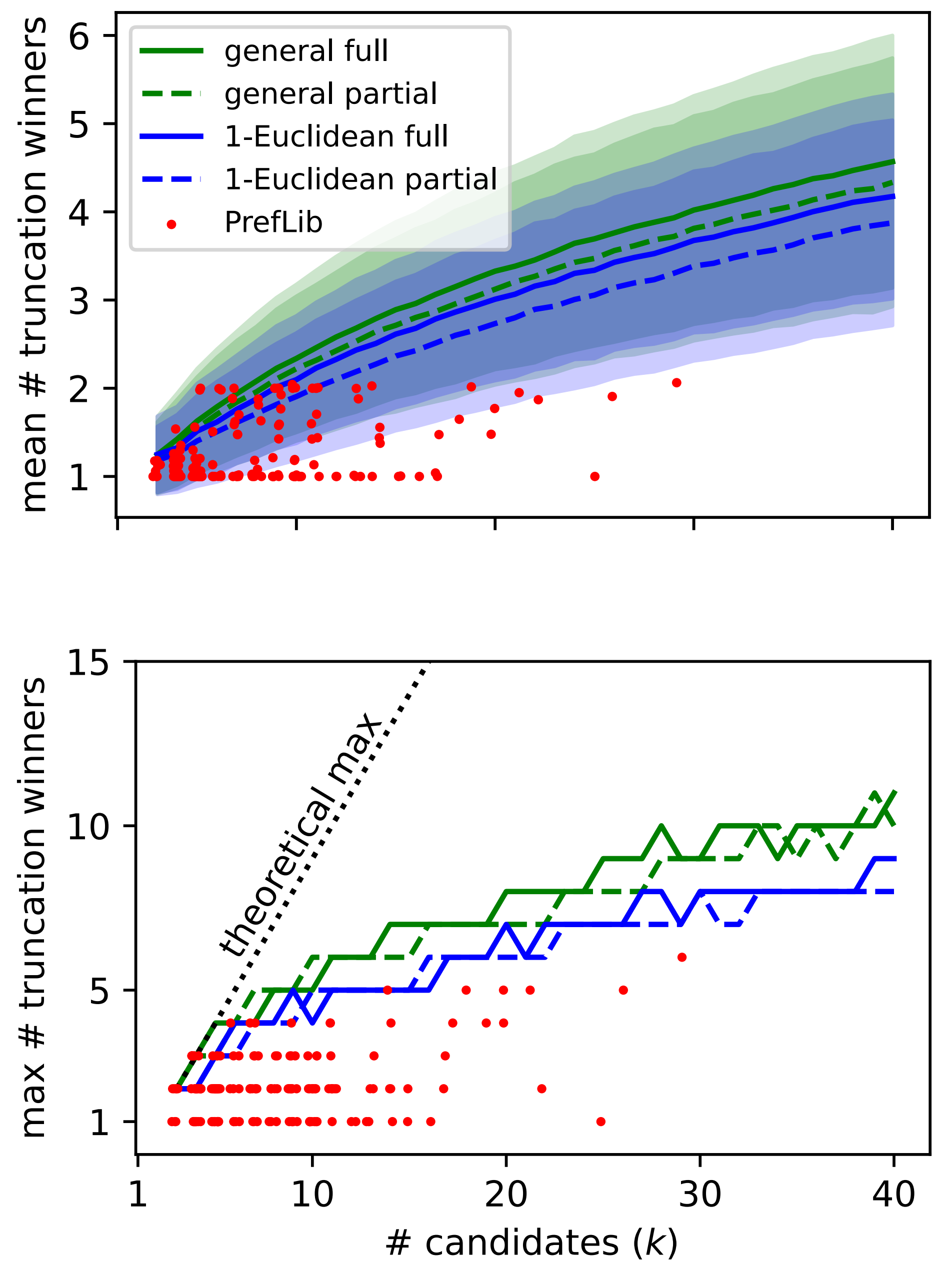

Thank you!

Code and data:
github.com/tomlinsonk/irv-ballot-length

Extended version:
arxiv.org/abs/2207.08958

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