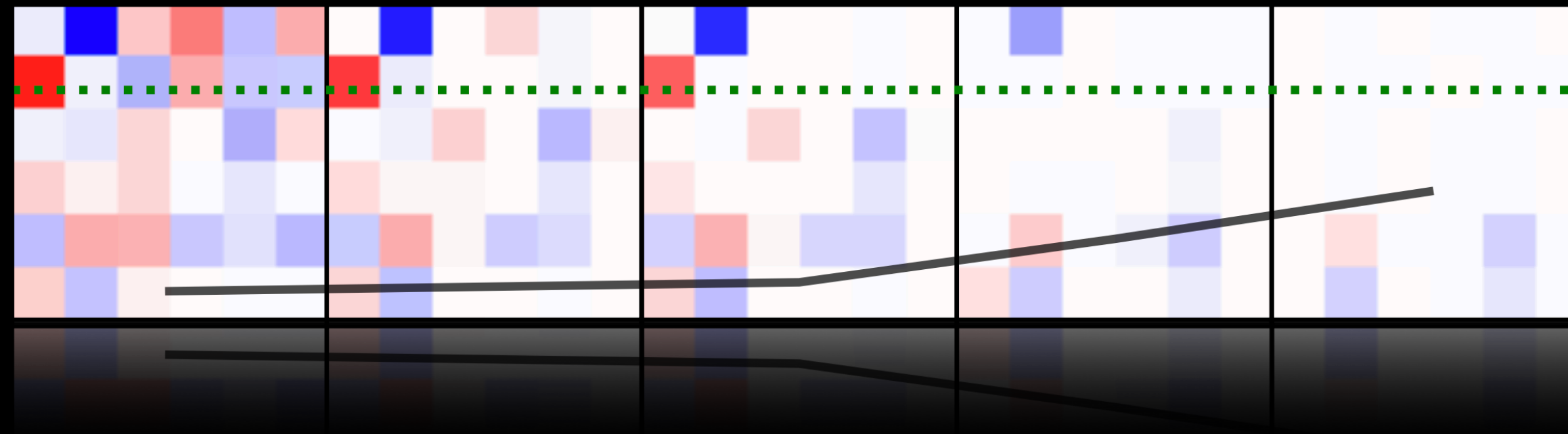
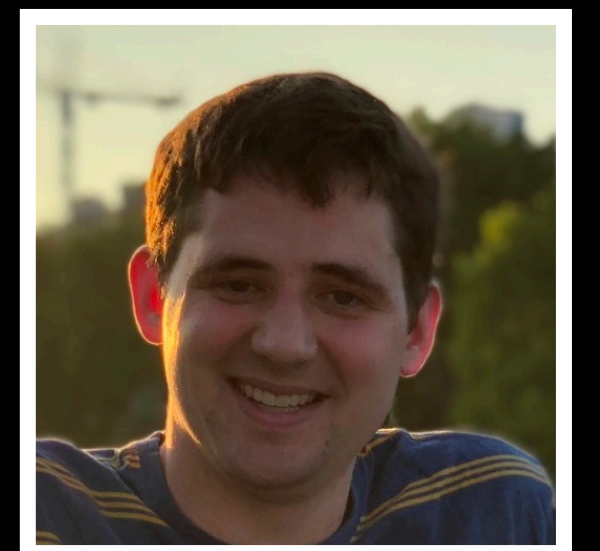


Code: [bit.ly/lcl-code](https://bit.ly/lcl-code)  
Data: [bit.ly/lcl-data](https://bit.ly/lcl-data)  
Slides: [bit.ly/lcl-kdd-slides](https://bit.ly/lcl-kdd-slides)



# Learning Interpretable Feature Context Effects in Discrete Choice

Kiran Tomlinson  
PhD Student, Cornell CS



research with Austin R. Benson

# Choices and context effects



# Discrete choices are everywhere



amazon.com Amazon's Choice

KDD Chocolate Flavored Milk 180ML (18 PACK)  
6 Fl Oz (Pack of 18)  
★★★★☆ ~ 57  
\$27<sup>99</sup> (\$0.26/Fl Oz)  
Save \$2.00 with coupon  
✓prime FREE Delivery Thu, Jun 24

KDD Banana Flavored Milk 180ML (18 PACK)  
6.33 Fl Oz (Pack of 18)  
★★★★☆ ~ 31  
\$27<sup>99</sup> (\$0.26/Fl Oz)  
Save \$2.00 with coupon  
✓prime FREE Delivery Thu, Jun 24

KDD Original Milk 180ML (18 PACK)  
★★★★★ ~ 2  
\$27<sup>99</sup> (\$4.60/Ounce)  
✓prime FREE Delivery Thu, Jun 24

Ad

**Best Western University Inn**  
Ithaca

**Black Friday / Cyber Monday Deals Now**  
Free Shuttle Transportation, Grab & Go Breakfast, WiFi & Parking. Pet friendly, Outdoor Pool, Fitness Center. Sanitizing Daily

Breakfast included

3.9/5 Good (999 reviews)

**\$63**  
per night  
**\$71 total**  
Includes taxes & fees

Ad

**Quality Inn Ithaca - University Area**  
Ithaca

**Black Friday / Cyber Monday Deals Now**  
Complimentary Breakfast. Free Airport Shuttle, WiFi & parking. Close to Ithaca College & Cornell University. Pets welcome.

Breakfast included

3.6/5 Good (694 reviews)

**Member Price available**

**\$59**  
per night  
**\$66 total**  
Includes taxes & fees

Ad

**Hotel Ithaca**  
Ithaca

4.0/5 Very Good (842 reviews)

**Member Price available**

**\$94**  
per night  
**\$106 total**  
Includes taxes & fees

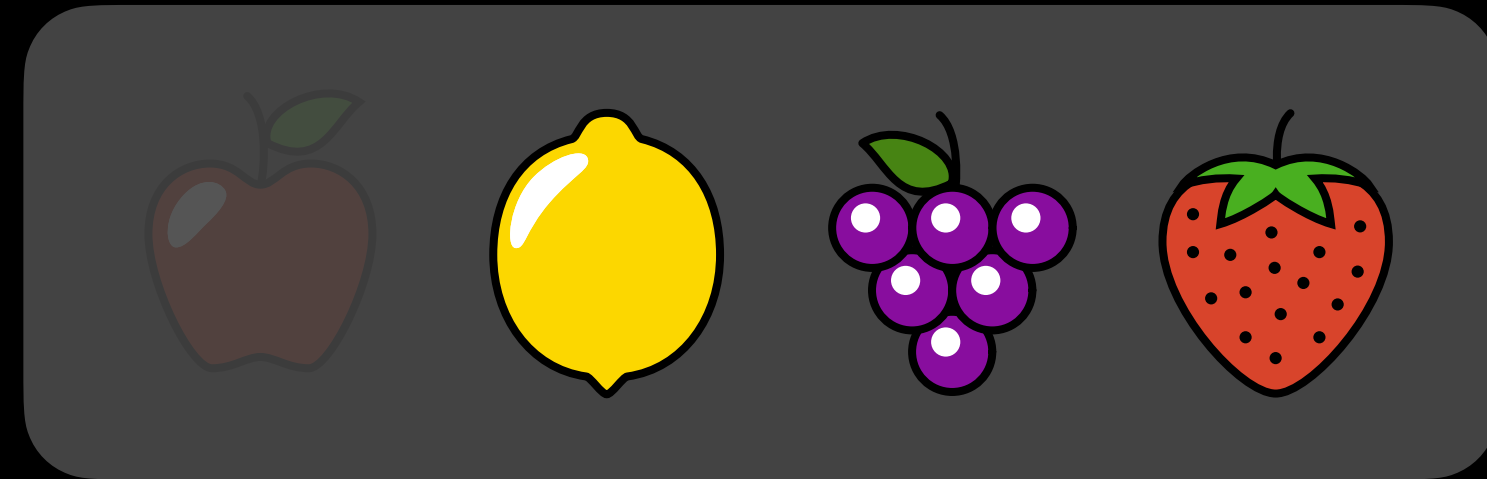


**“The fundamental problem of discrete choice”**



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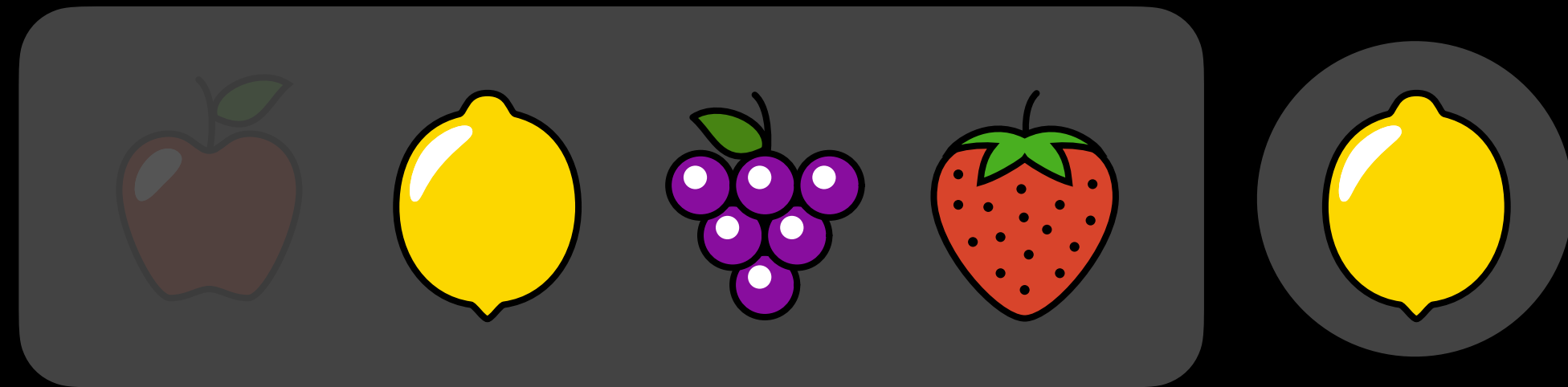
*choice set*



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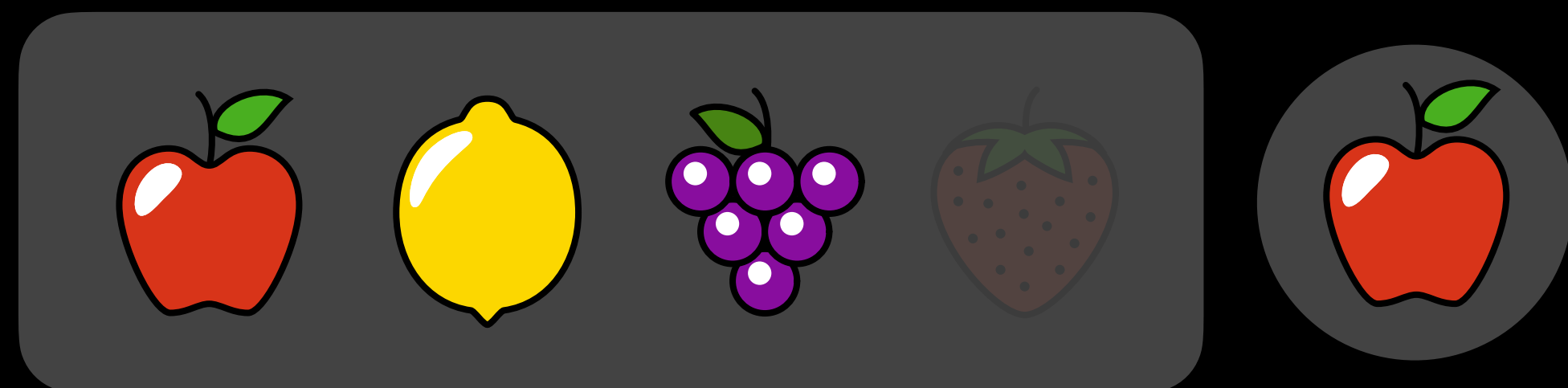
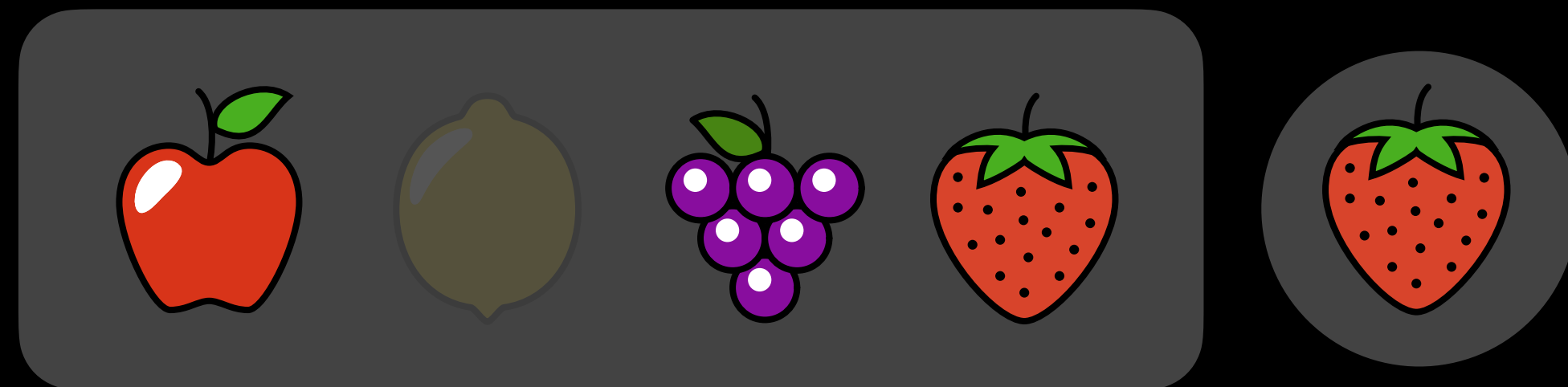
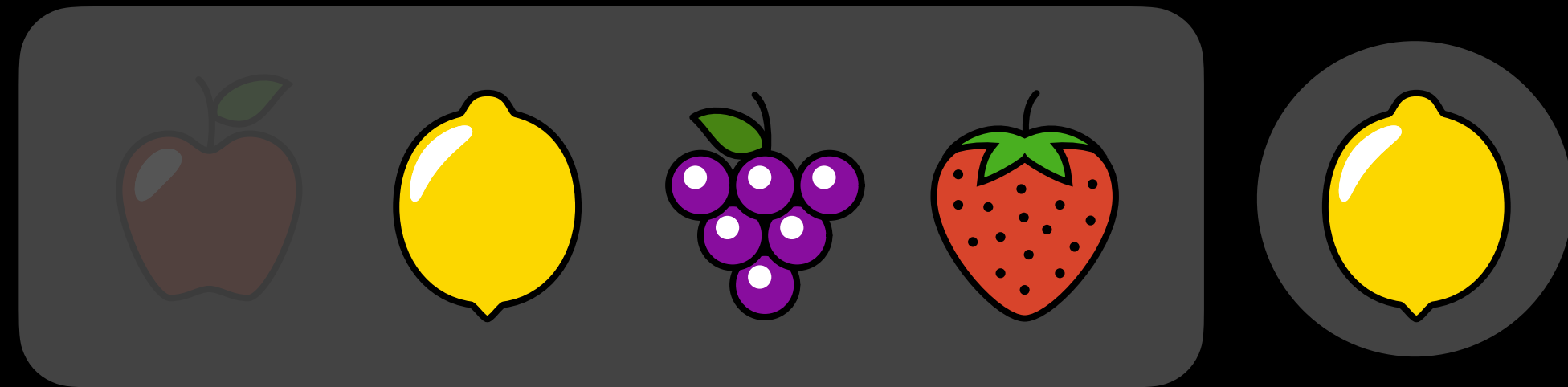




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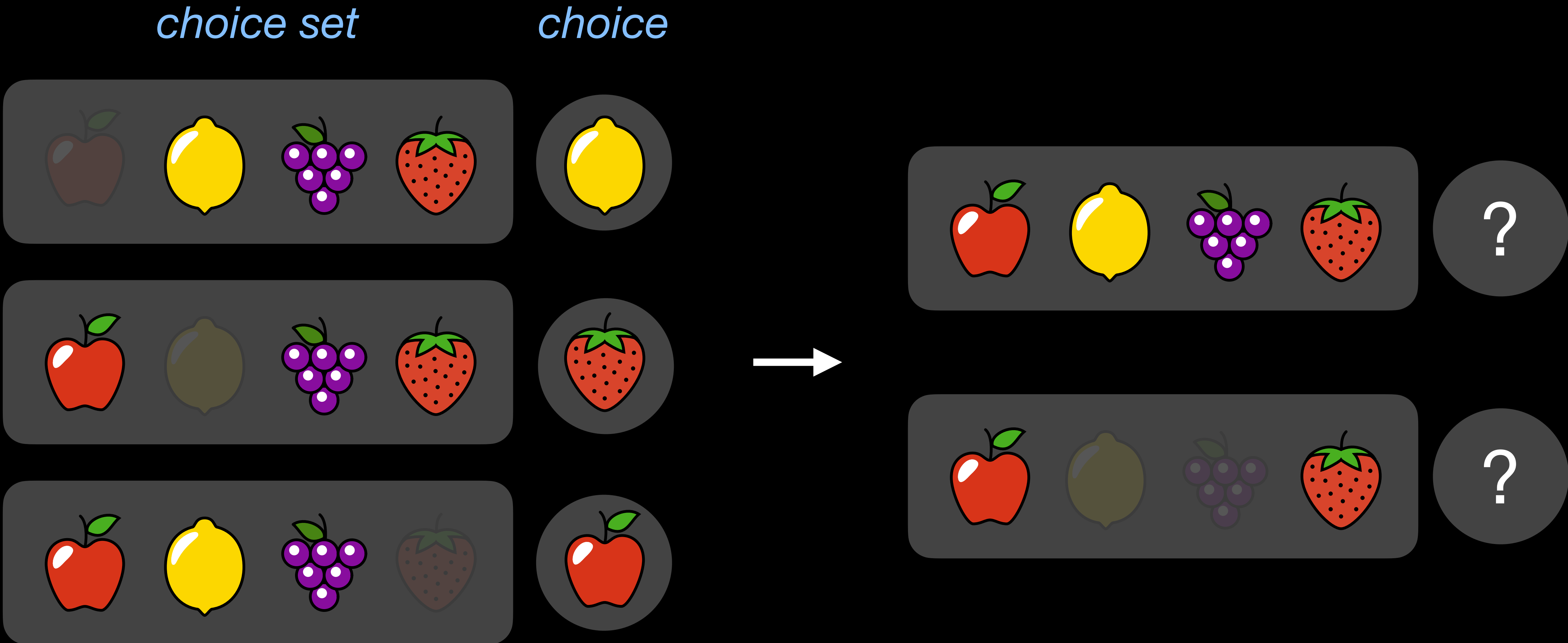
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(McFadden, *Frontiers in Econometrics* 1973)

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$u_i$	1	-1	0	2
$\Pr(i   C)$	.24	.03	.09	.64

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Unique choice model satisfying  
*independence of irrelevant alternatives (IIA)*:

(Luce, *Individual Choice Behavior* 1959)

$$\frac{\Pr(i | C)}{\Pr(j | C)} = \frac{\Pr(i | C')}{\Pr(j | C')}$$



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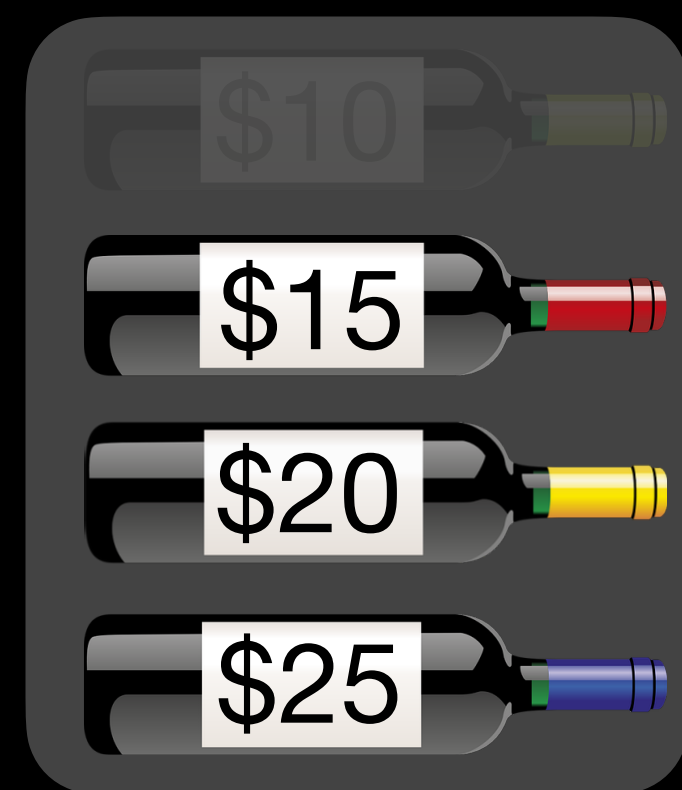
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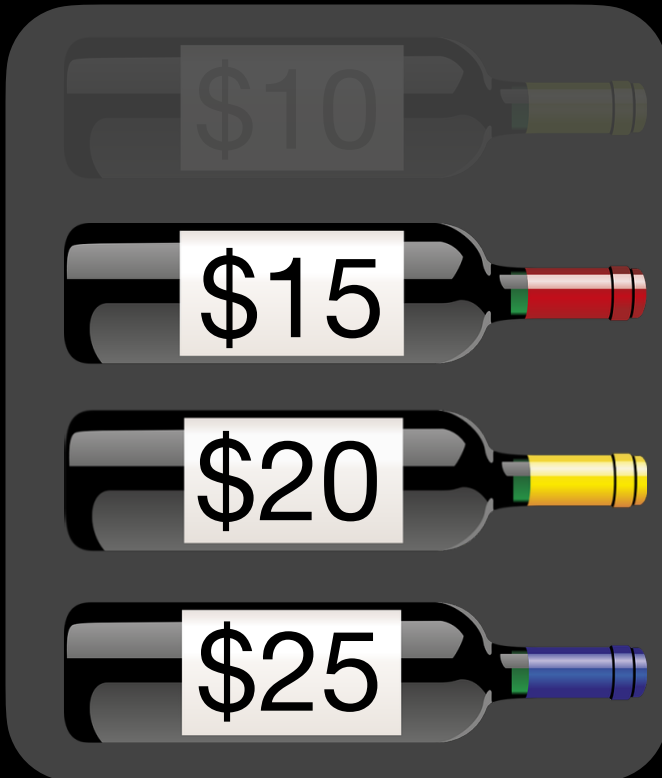
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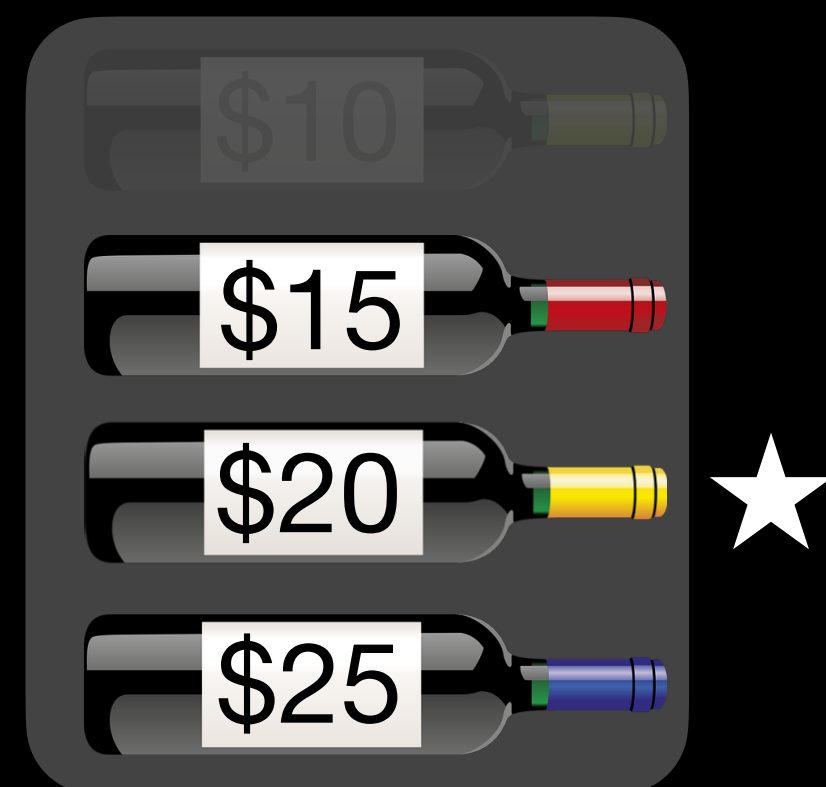





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*IIA violations:*

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# Item features and the LCL

# Choice models with *item features*



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Use item features:



*genre*: drama,  
*in\_top\_10*: True,  
*has\_new\_episodes*: True,  
*producer*: Netflix



*genre*: comedy,  
*in\_top\_10*: False,  
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*producer*: NBC



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*producer*: Banijay

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Preference coefficient  $\theta_k$  is easy to interpret: importance of the  $k^{\text{th}}$  feature

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$\rightarrow u_{i,C} = (\theta + Ax_C)^T x_i$  ( $x_C = \frac{1}{|C|} \sum_{j \in C} x_j$  is the *mean feature vector*)

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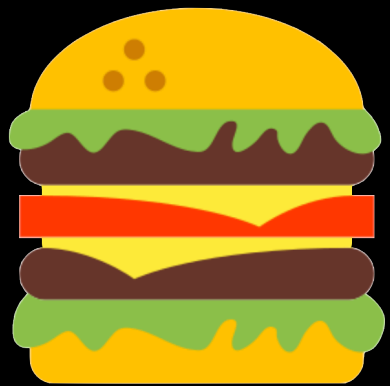
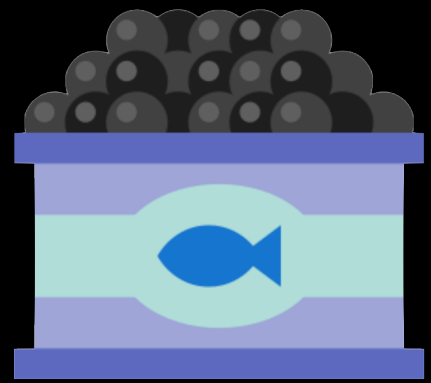
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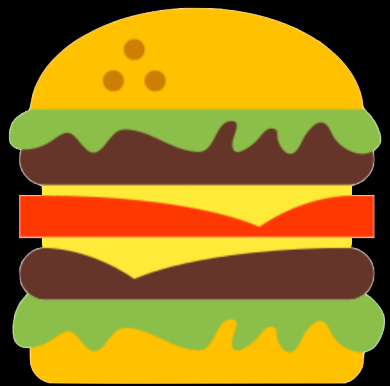
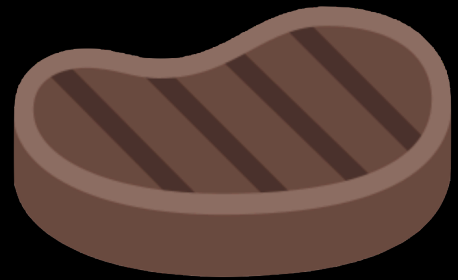
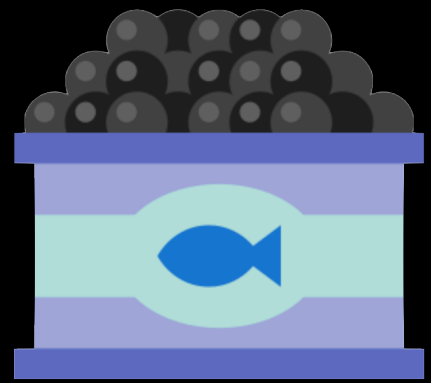
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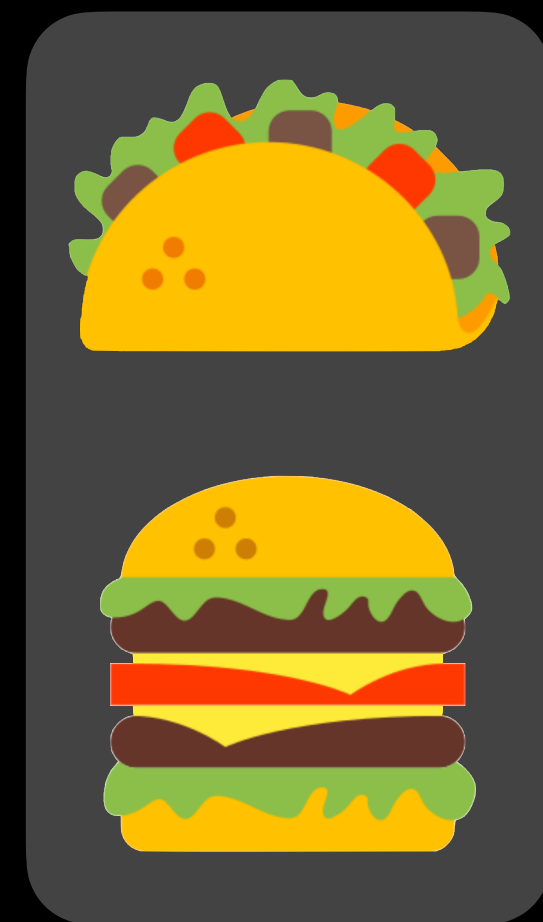
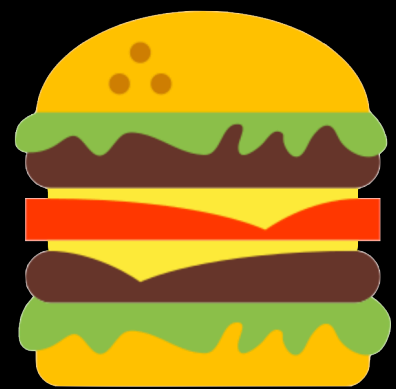
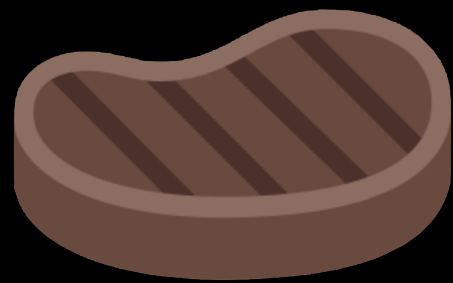
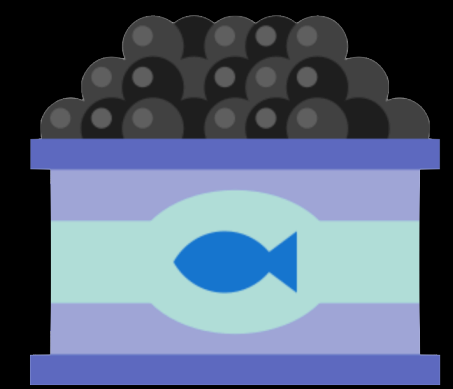
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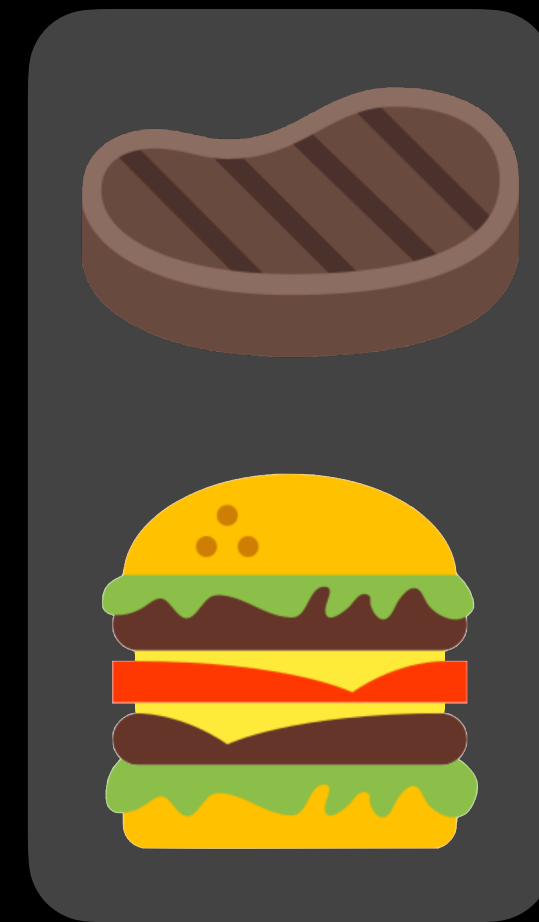
item features:

- price
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$C_1$



$C_2$

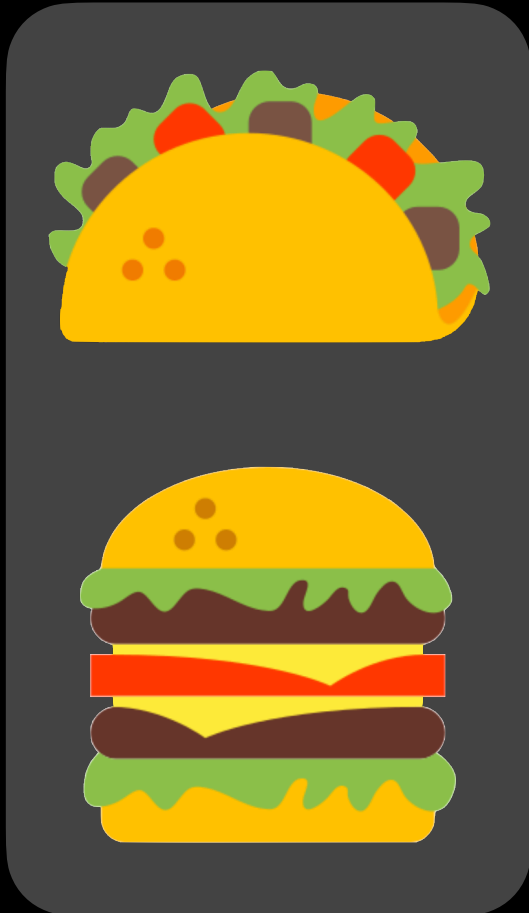
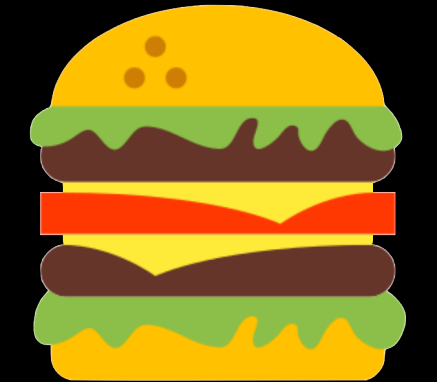
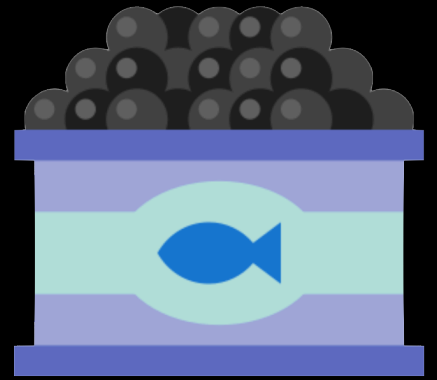


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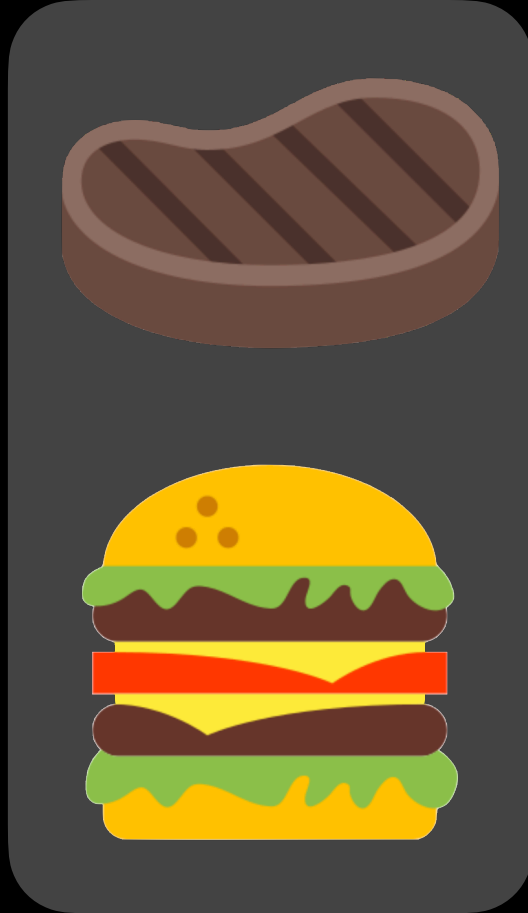
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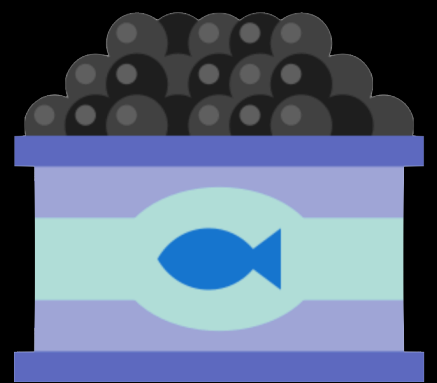


mean choice set price

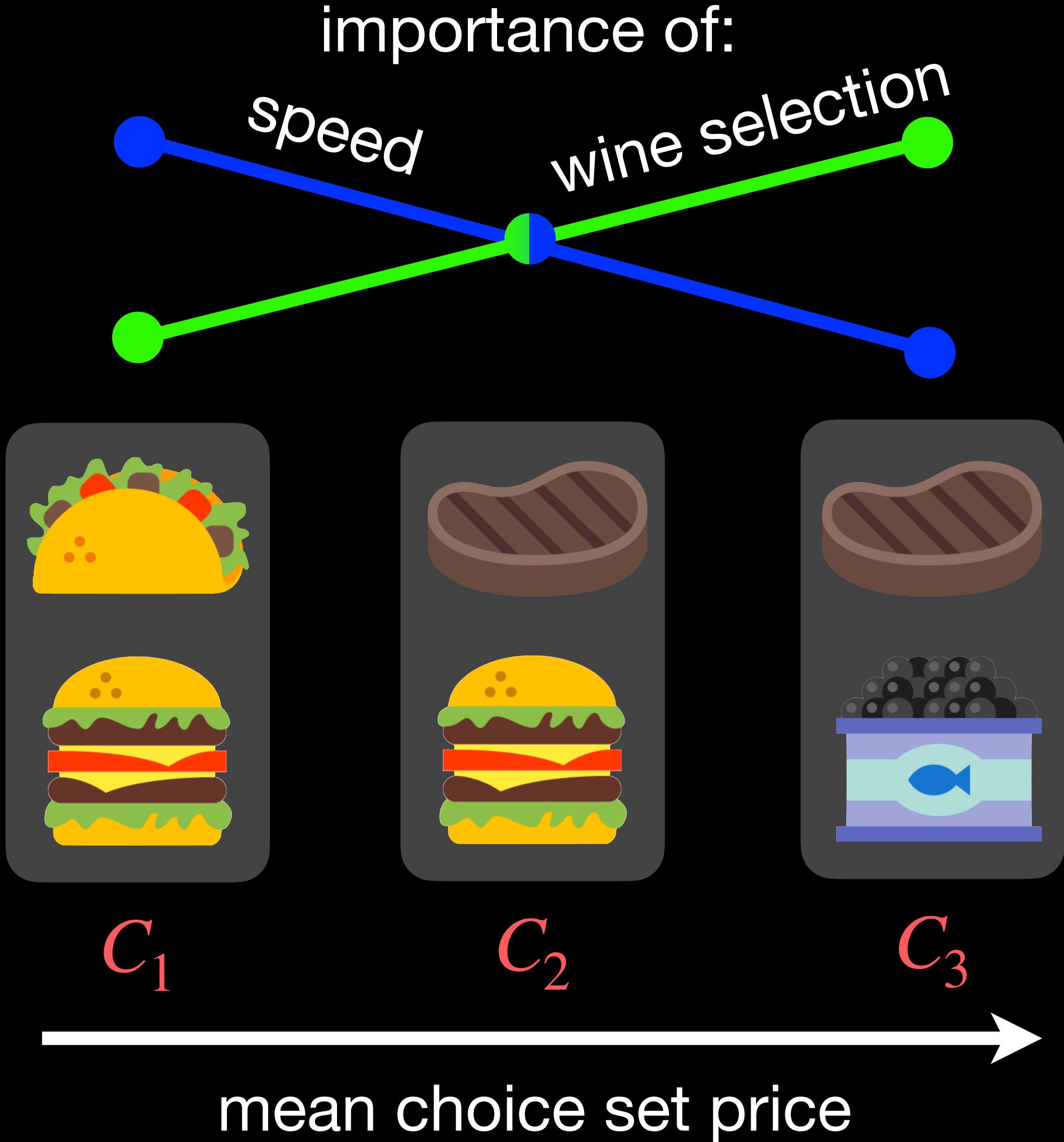
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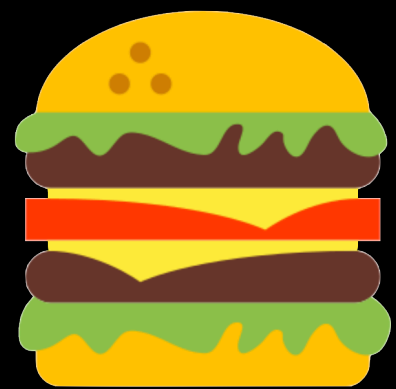
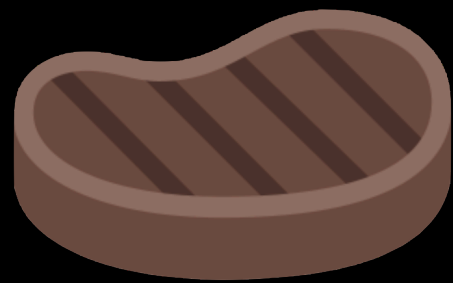
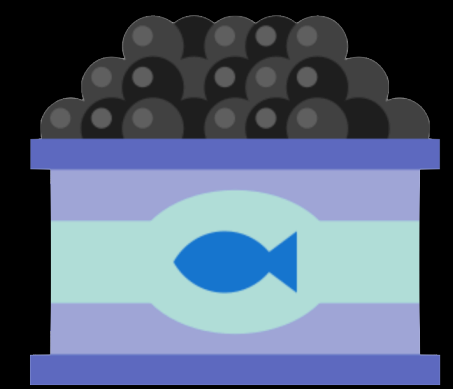
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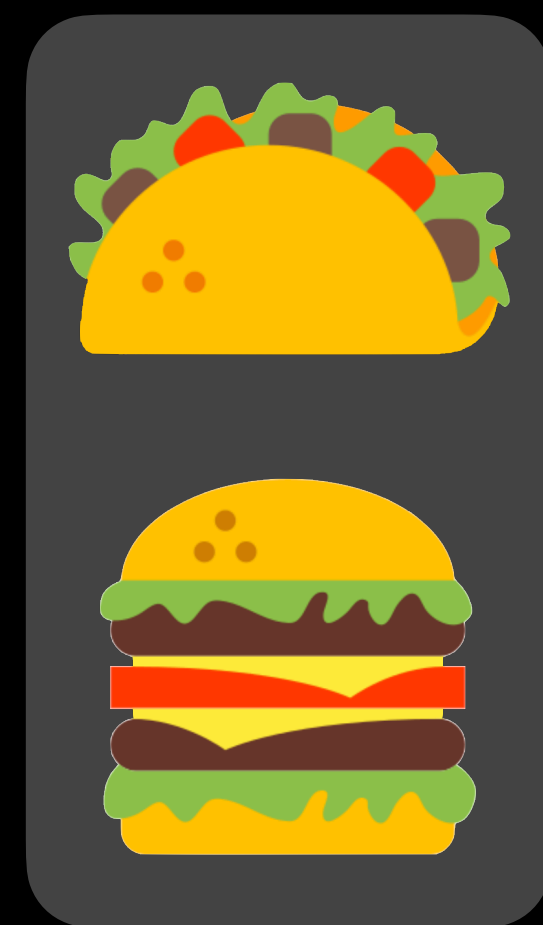
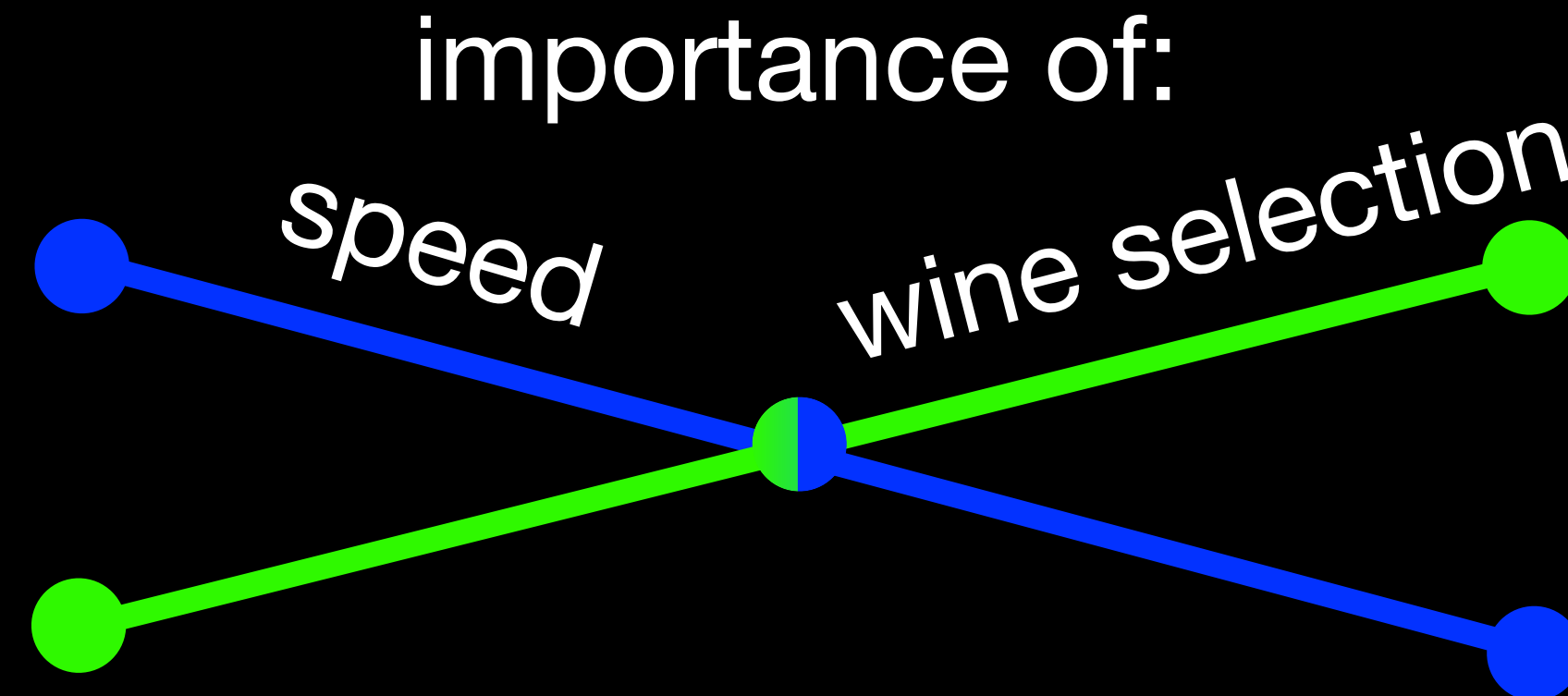


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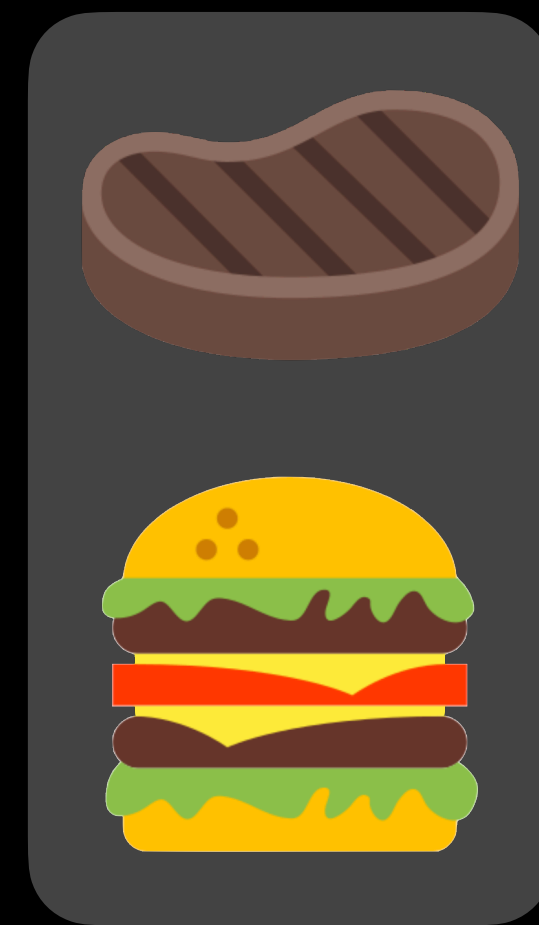


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$C_1$



$C_2$



$C_3$

mean choice set price

$A$

$$\begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

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*Theorem 1.* A  $d$ -feature linear context logit is identifiable from a dataset  $\mathcal{D}$  if and only if

$$\text{span} \left\{ \begin{bmatrix} x_C \\ 1 \end{bmatrix} \otimes (x_i - x_C) \mid C \in \mathcal{C}_{\mathcal{D}}, i \in C \right\} = \mathbb{R}^{d^2+d}. \quad (6)$$

( $\mathcal{C}_{\mathcal{D}}$ : unique choice sets in  $\mathcal{D}$ ,  $\otimes$ : Kronecker product)

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→ important for inference and interpretation

*Theorem 1.* A  $d$ -feature linear context logit is identifiable from a dataset  $\mathcal{D}$  if and only if

$$\text{span} \left\{ \begin{bmatrix} x_C \\ 1 \end{bmatrix} \otimes (x_i - x_C) \mid C \in \mathcal{C}_{\mathcal{D}}, i \in C \right\} = \mathbb{R}^{d^2+d}. \quad (6)$$

( $\mathcal{C}_{\mathcal{D}}$ : unique choice sets in  $\mathcal{D}$ ,  $\otimes$ : Kronecker product)

*intuition:* need varied choice sets containing varied items



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- see paper for details

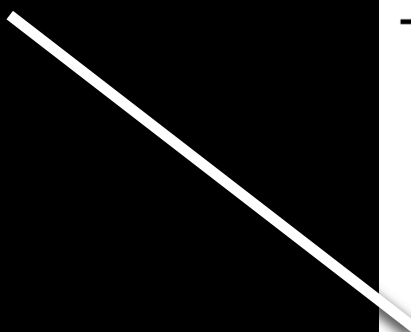
# Results on choice data

# Choice datasets

<b>Dataset</b>	<b>Choices</b>	<b>Features</b>	<b>Largest Choice Set</b>
DISTRICT	5376	27	2
DISTRICT-SMART	5376	6	2
SUSHI	5000	6	10
EXPEDIA	276593	5	38
CAR-A	2675	4	2
CAR-B	2206	5	2
CAR-ALT	4654	21	6

# Choice datasets

favorite sushi types



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hotel bookings

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# LCL improves model fit

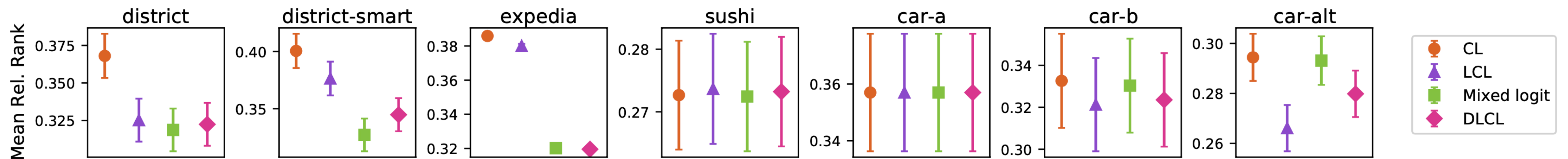
whole-dataset negative log-likelihood (lower = better)

	CL	LCL	Mixed logit	DLCL
DISTRICT	3313	<b>3130</b>	3258	3206
DISTRICT-SMART	3426	<b>3278*</b>	3351	3303 <sup>†</sup>
EXPEDIA	839505	837649*	839055	<b>837569<sup>†</sup></b>
SUSHI	9821	9773*	9793	<b>9764</b>
CAR-A	1702	1694	1696	<b>1692</b>
CAR-B	1305	1295	1297	<b>1284</b>
CAR-ALT	7393	<b>6733*</b>	7301	7011 <sup>†</sup>

\*significant likelihood ratio test vs MNL ( $p < 0.001$ )

<sup>†</sup>significant likelihood ratio test vs mixed logit ( $p < 0.001$ )

# LCL can improve out-of-sample prediction performance



**Figure 2: Mean relative rank of predictions on held-out test data (lower is better). Error bars show standard error of the mean.**

**LCL can test individual effects for significance**

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Compute std. errs. (and z-scores) for each parameter estimate using MLE *asymptotic normality*

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**Table 4: Five largest context effects in SUSHI.**

Effect ( $q$ on $p$ )	$A_{pq}$ (std. err.)	$p$ -value
<i>popularity</i> on <i>popularity</i>	-0.28 (0.15)	0.066
<i>availability</i> on <i>is maki</i>	0.24 (0.14)	0.087
<i>oiliness</i> on <i>oiliness</i>	-0.20 (0.08)	0.0089
<i>popularity</i> on <i>availability</i>	0.19 (0.14)	0.16
<i>availability</i> on <i>oiliness</i>	-0.18 (0.10)	0.064

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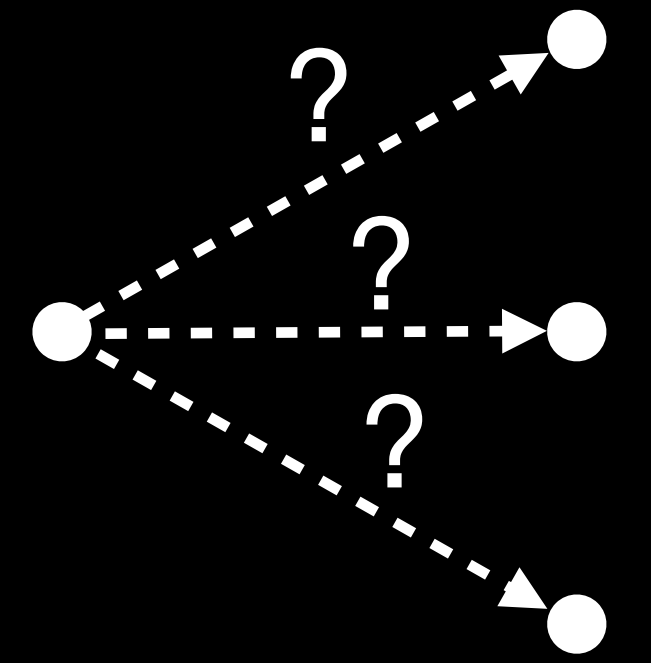
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**Table 5: Five largest context effects in EXPEDIA.**

Effect ( $q$ on $p$ )	$A_{pq}$ (std. err.)	$p$ -value
<i>location score on price</i>	-0.47 (0.05)	$< 10^{-16}$
<i>on promotion on price</i>	0.27 (0.03)	$< 10^{-16}$
<i>review score on price</i>	-0.19 (0.03)	$1.4 \times 10^{-9}$
<i>star rating on price</i>	0.15 (0.04)	$6.7 \times 10^{-5}$
<i>price on star rating</i>	0.10 (0.00)	$< 10^{-16}$

# Social network application

# What factors drive edge formation?

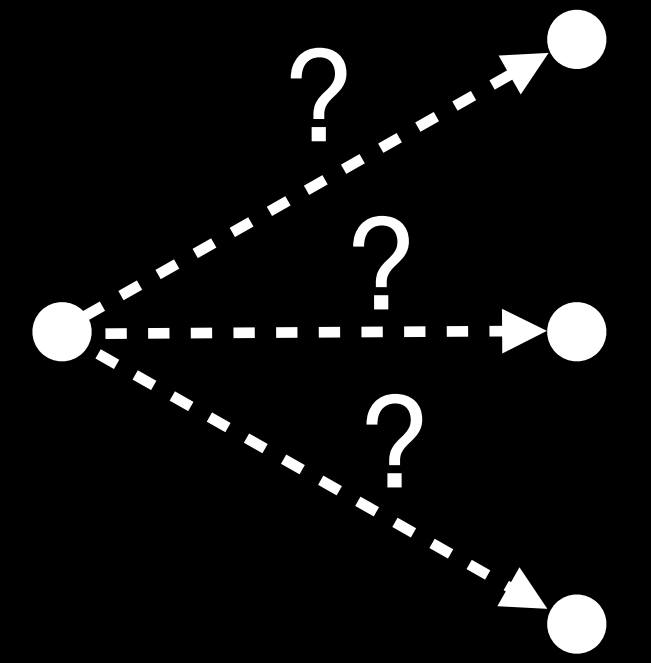
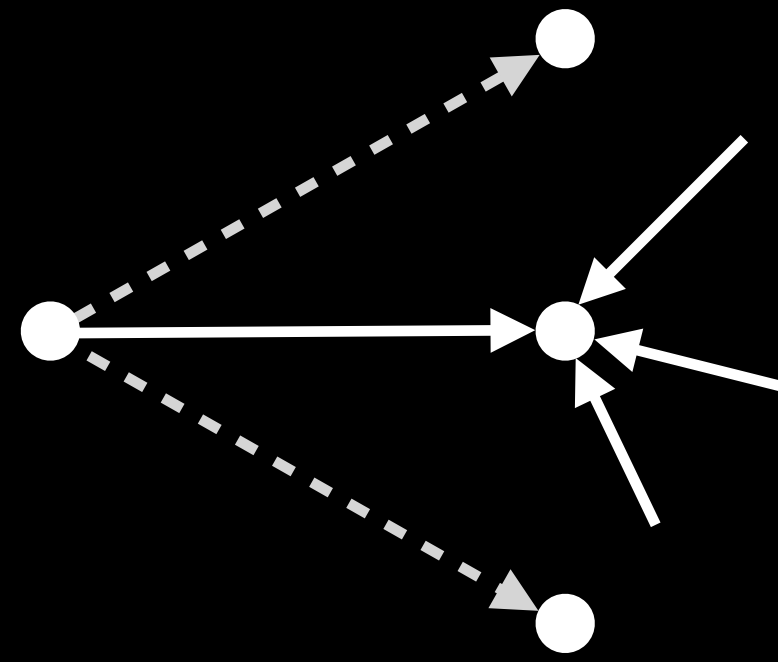




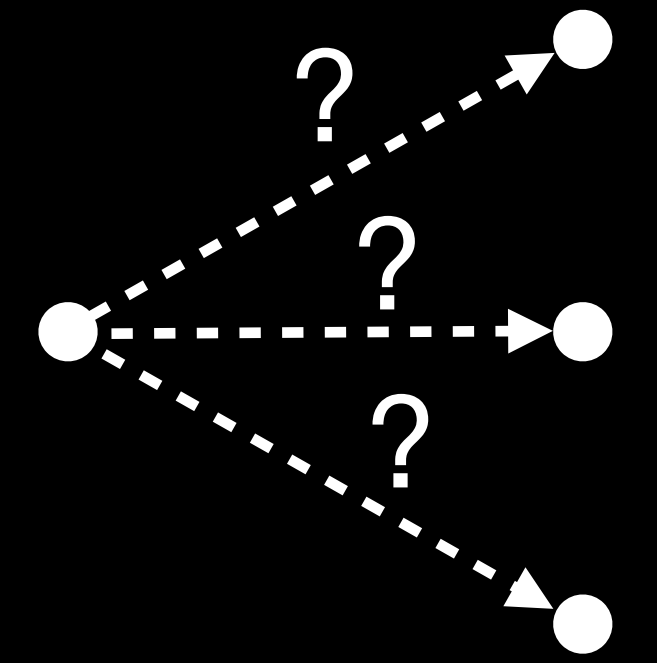
# What factors drive edge formation?

## *Preferential attachment*

(Barabási & Albert, *Science* 1999)

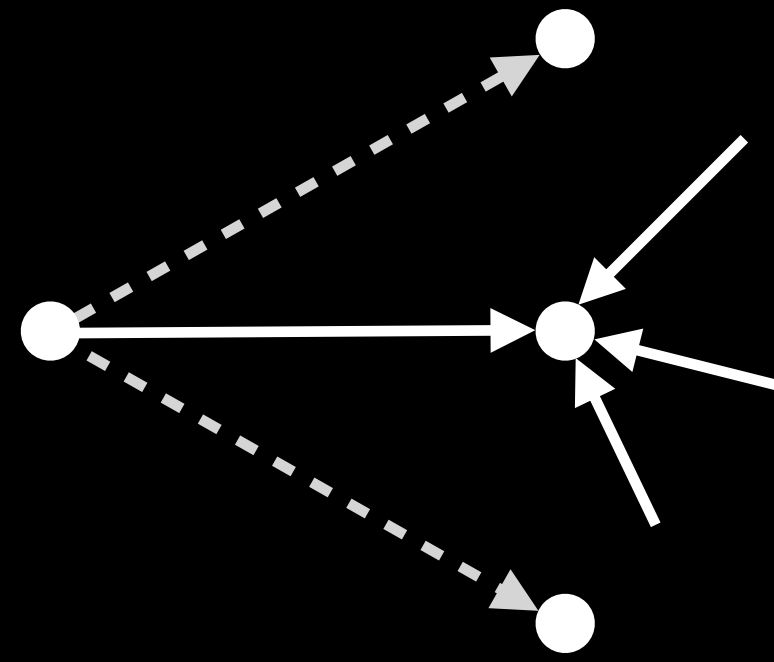


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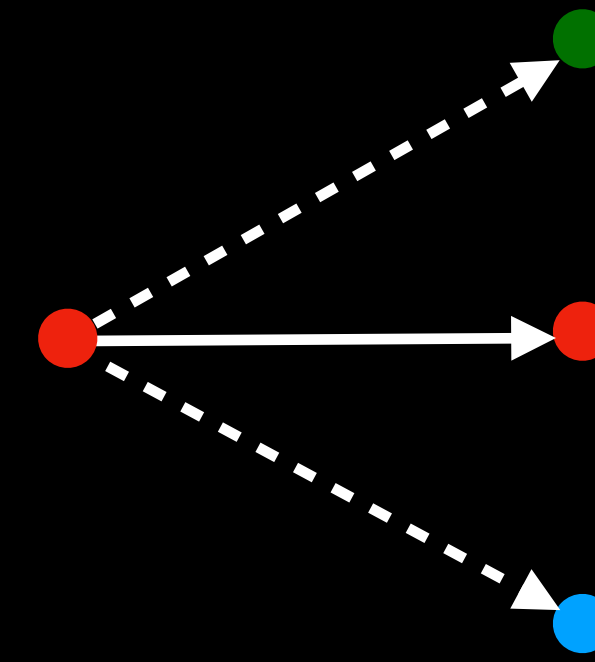
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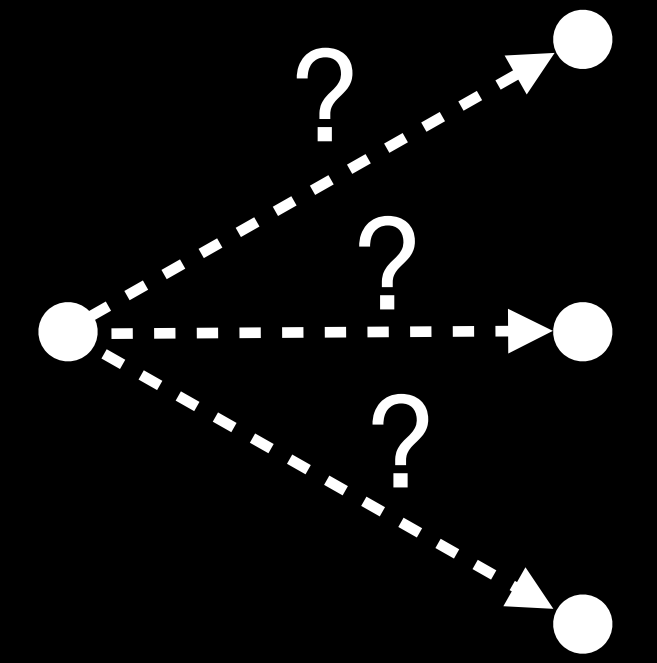
## *Homophily*

(McPherson et al., *Annual Review of Sociology* 2001)

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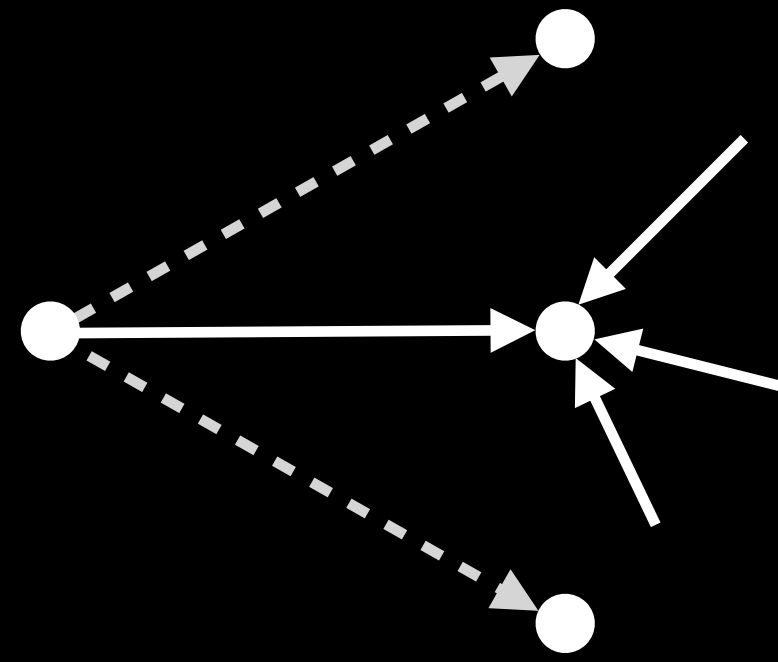


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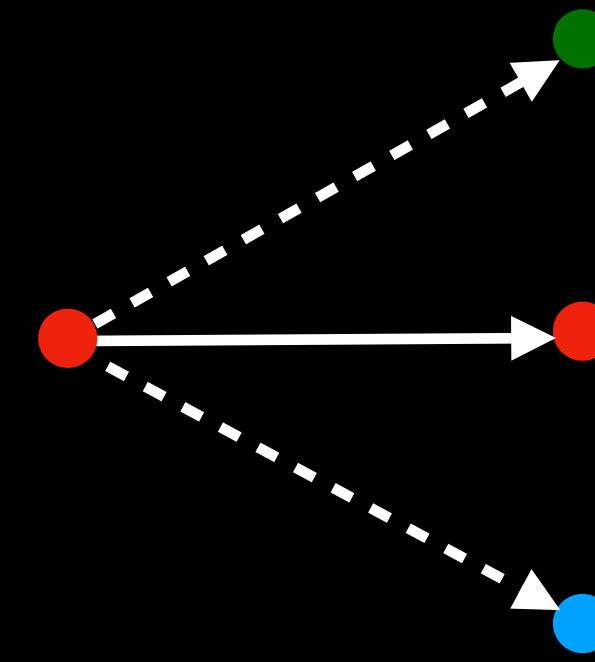
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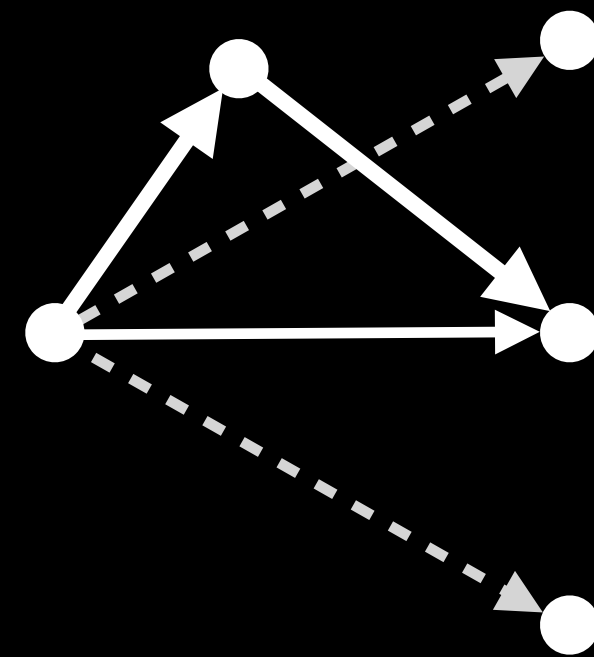
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## *Triadic closure*

(Rapoport, *Bulletin of Mathematical Biophysics* 1953)  
(Jin et al., *Physical Review E* 2001)



# “Choosing to grow a graph”

(Overgoor et al., *SINM* '19 & *WWW* '19)

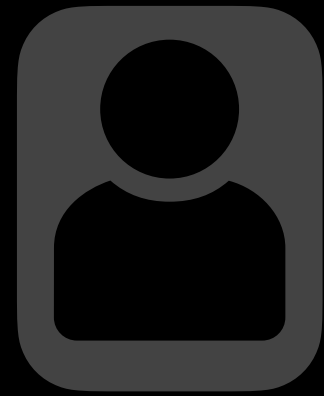
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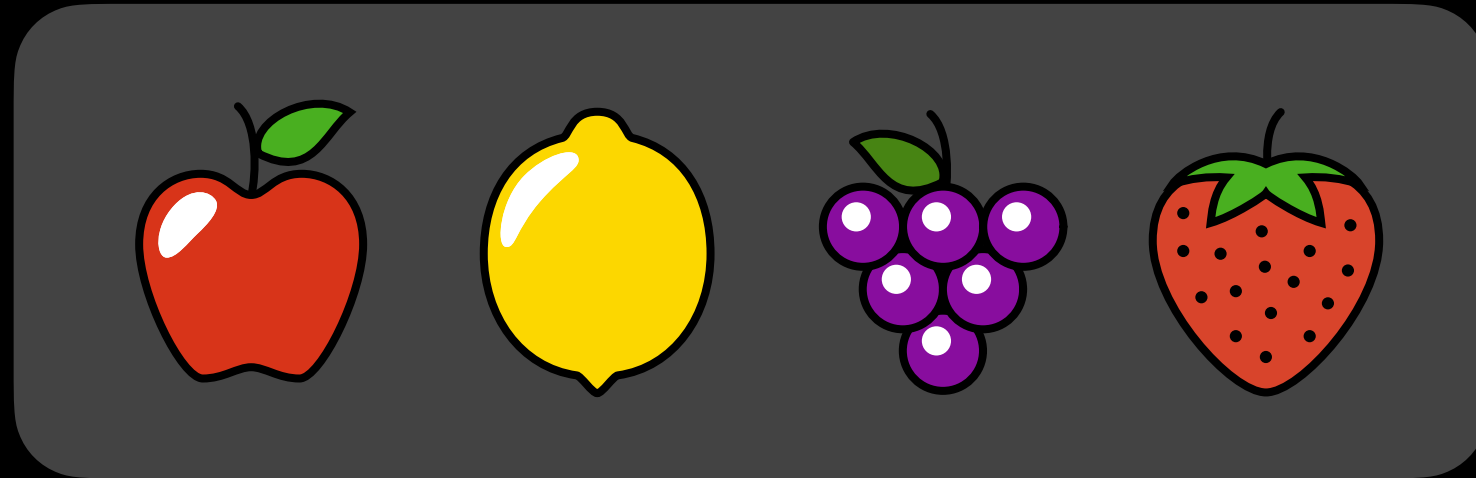
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so far:



*chooser*



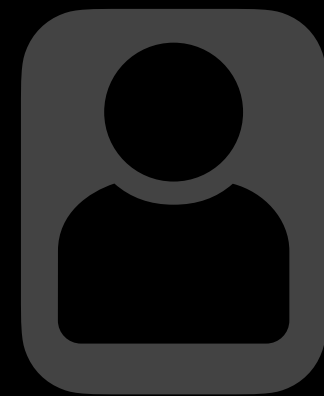
*choice set*

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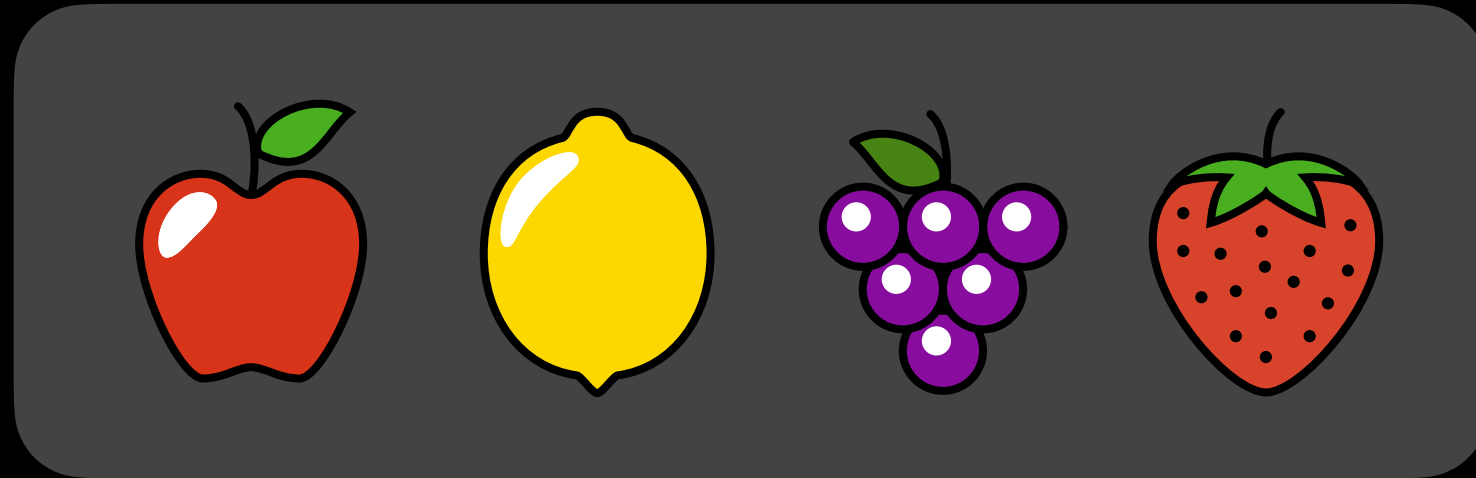
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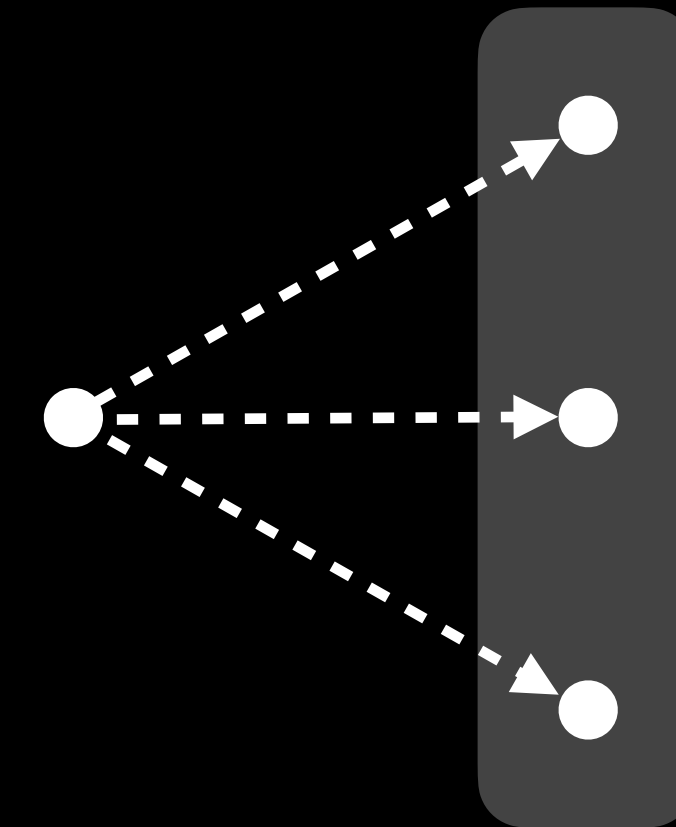
*chooser*



*choice set*

in network growth:

*chooser*



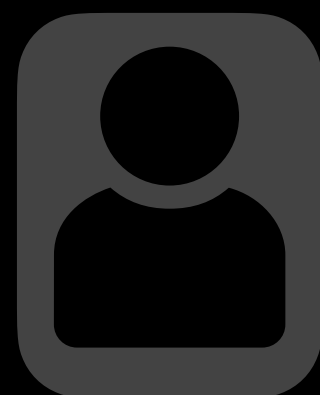
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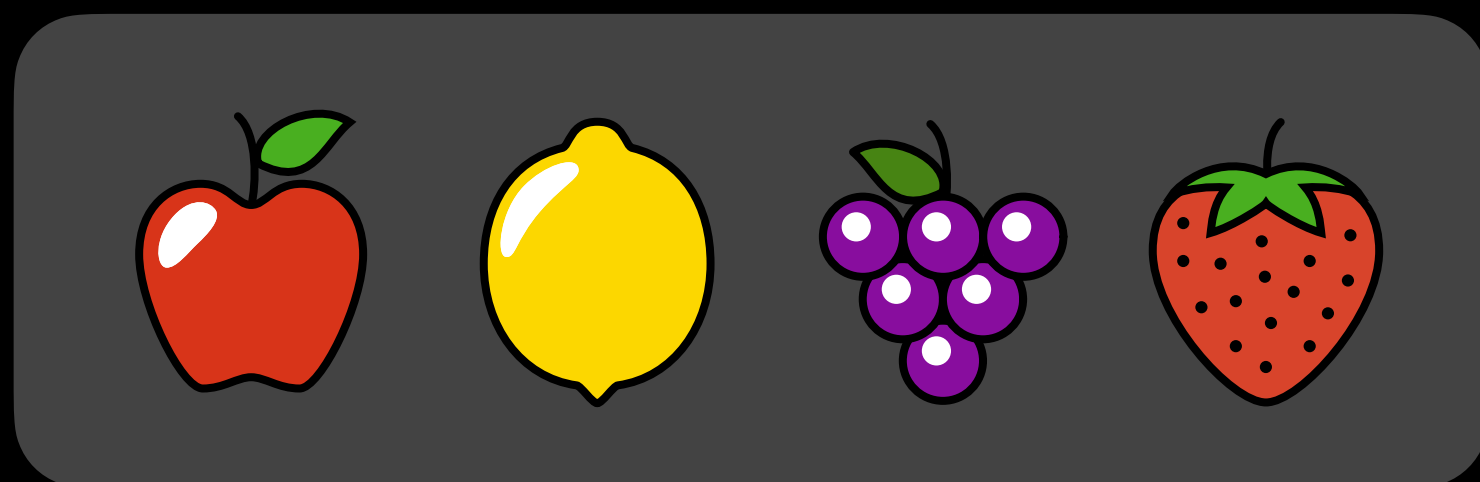
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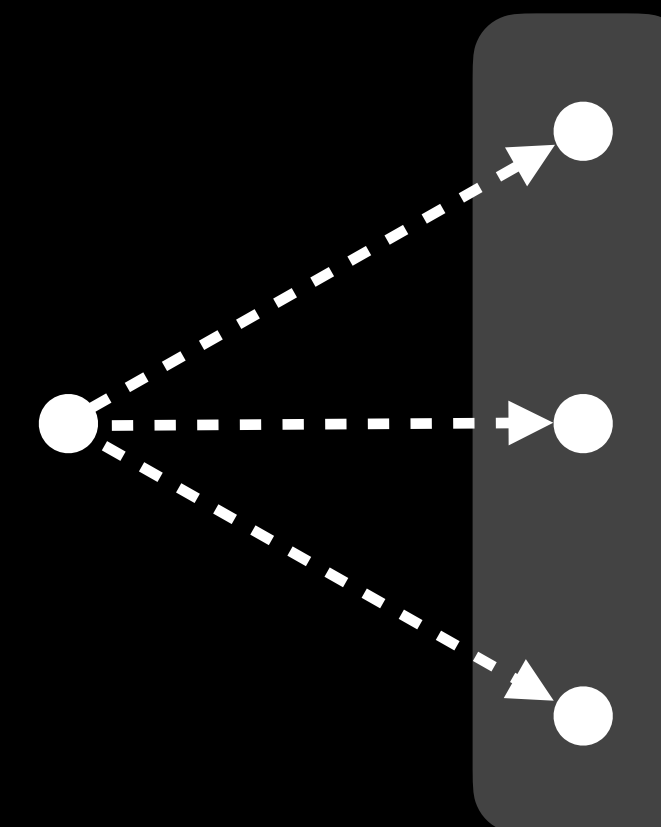
*chooser*



*choice set*

in network growth:

*chooser*



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## Key usage

Timestamped edges

→ meaningful choice sets

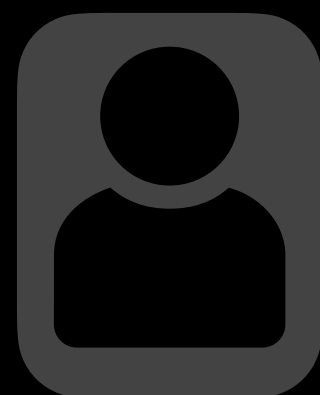
Infer relative importance of edge formation mechanisms from data

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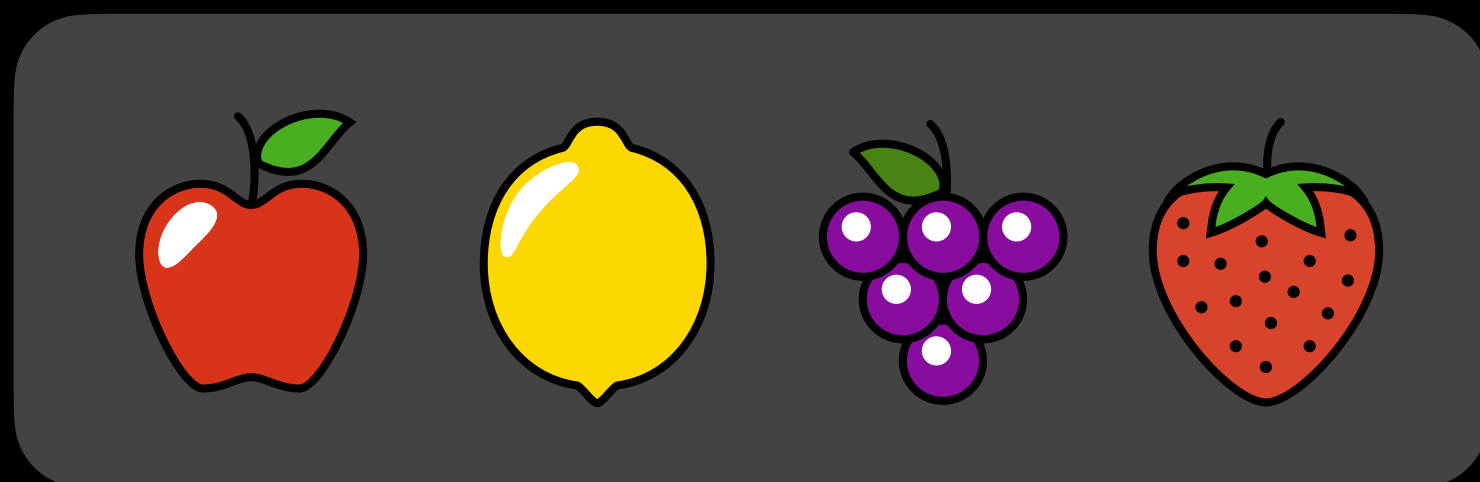
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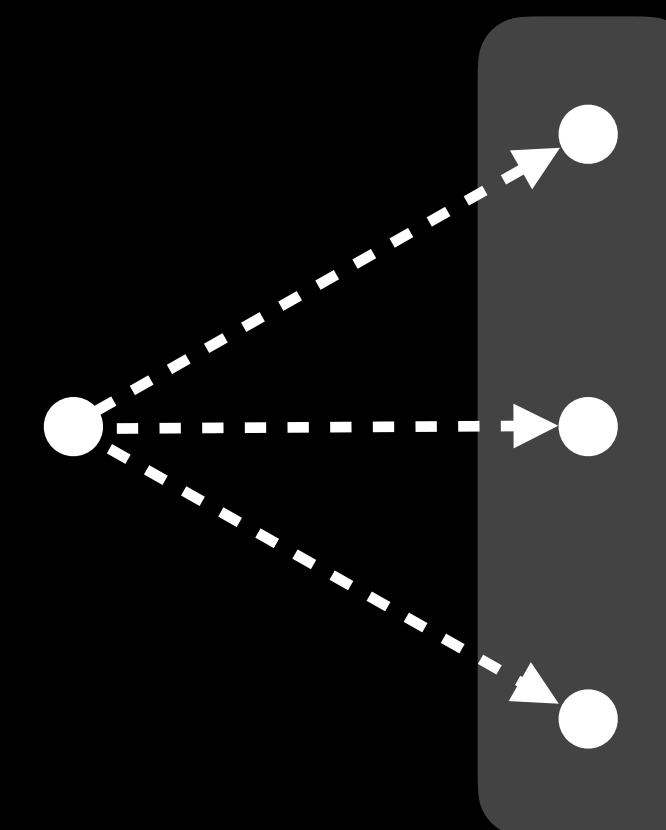
*chooser*



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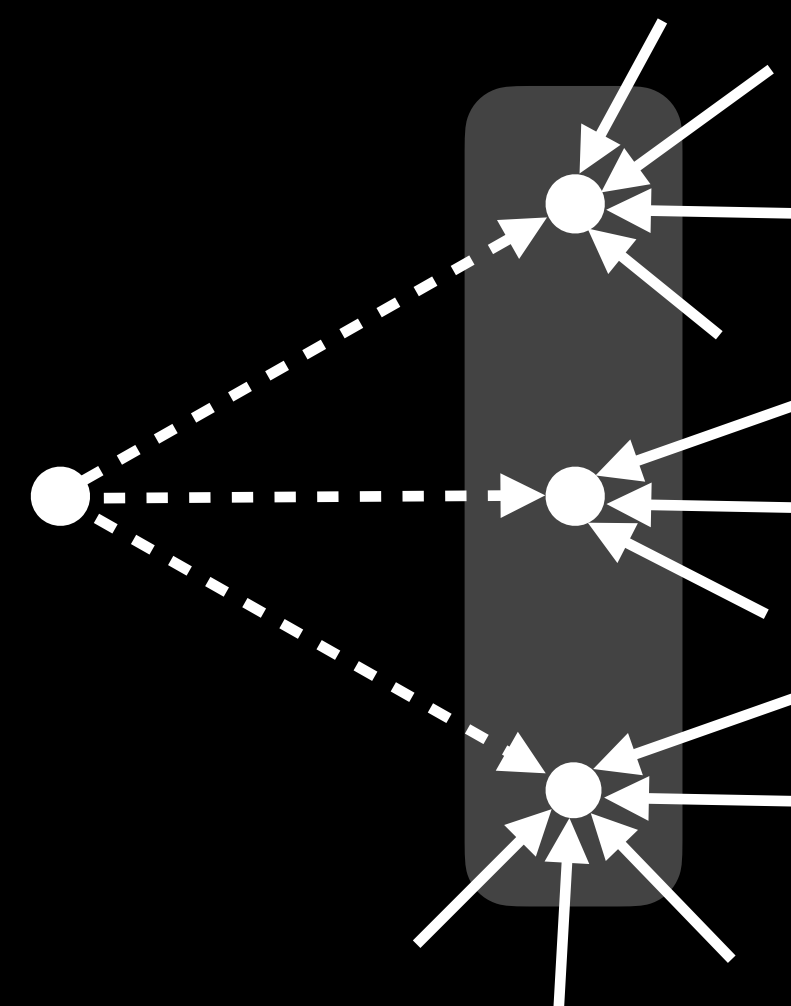
in network growth:

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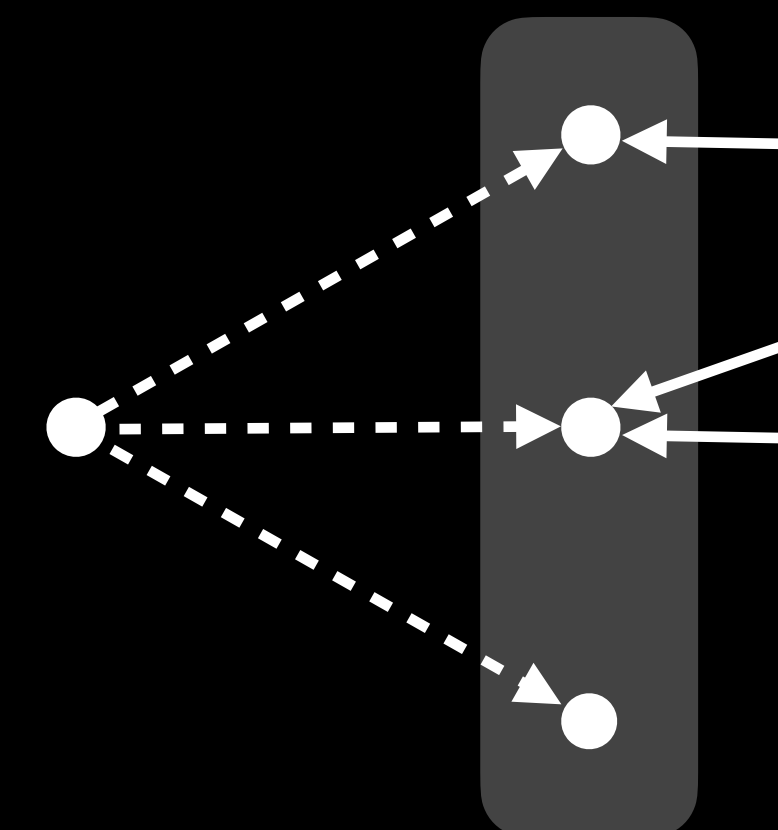


*choice set*

feature context effects:



vs.



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Infer relative importance of edge formation mechanisms from data



# Choosing to close triangles

- Triadic closure* offers small choice sets
- tractable inference
- varied choice sets

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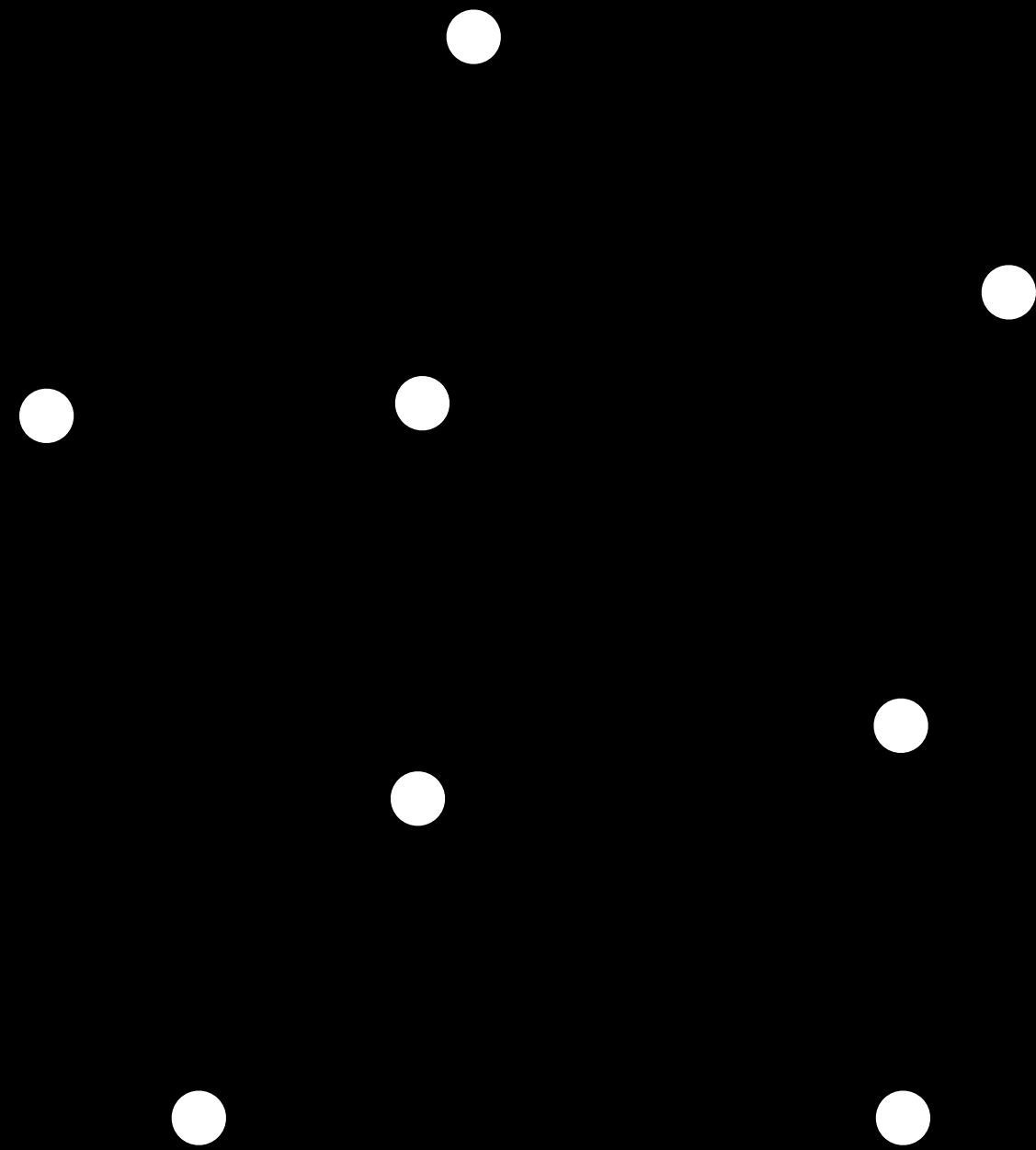
**Our data**  
Timestamped edges  
(including repeats)

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## Our data

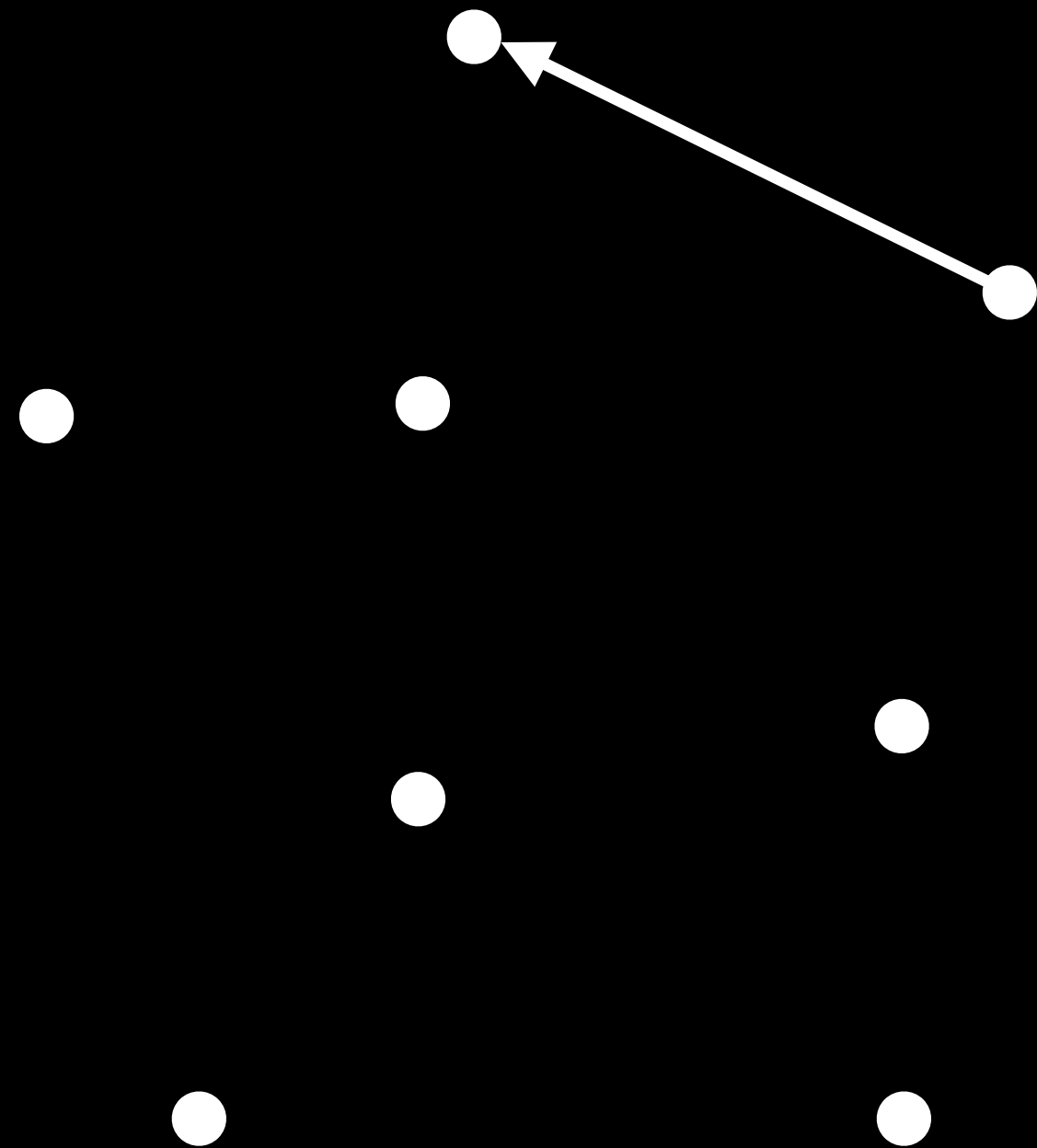
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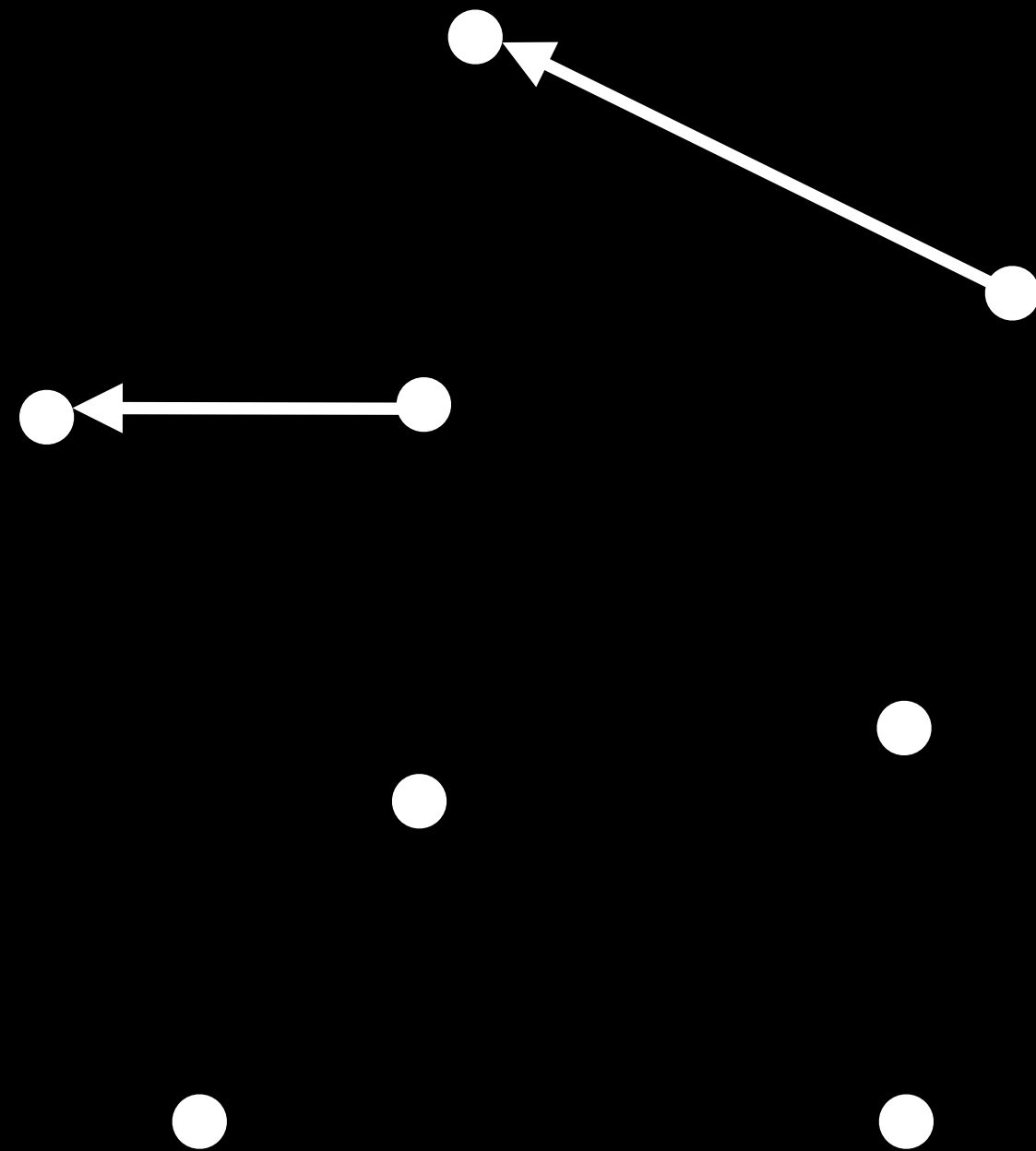


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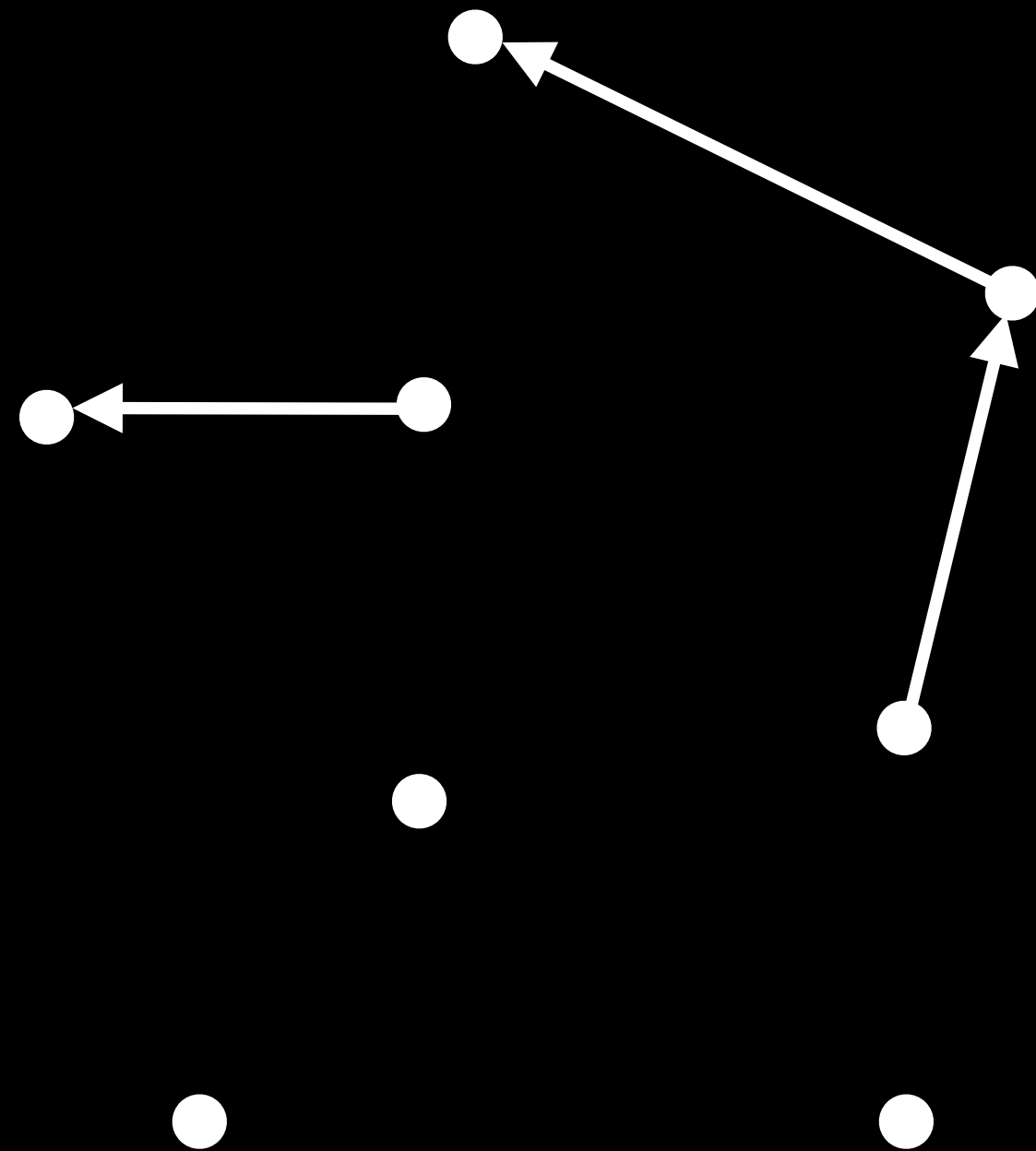


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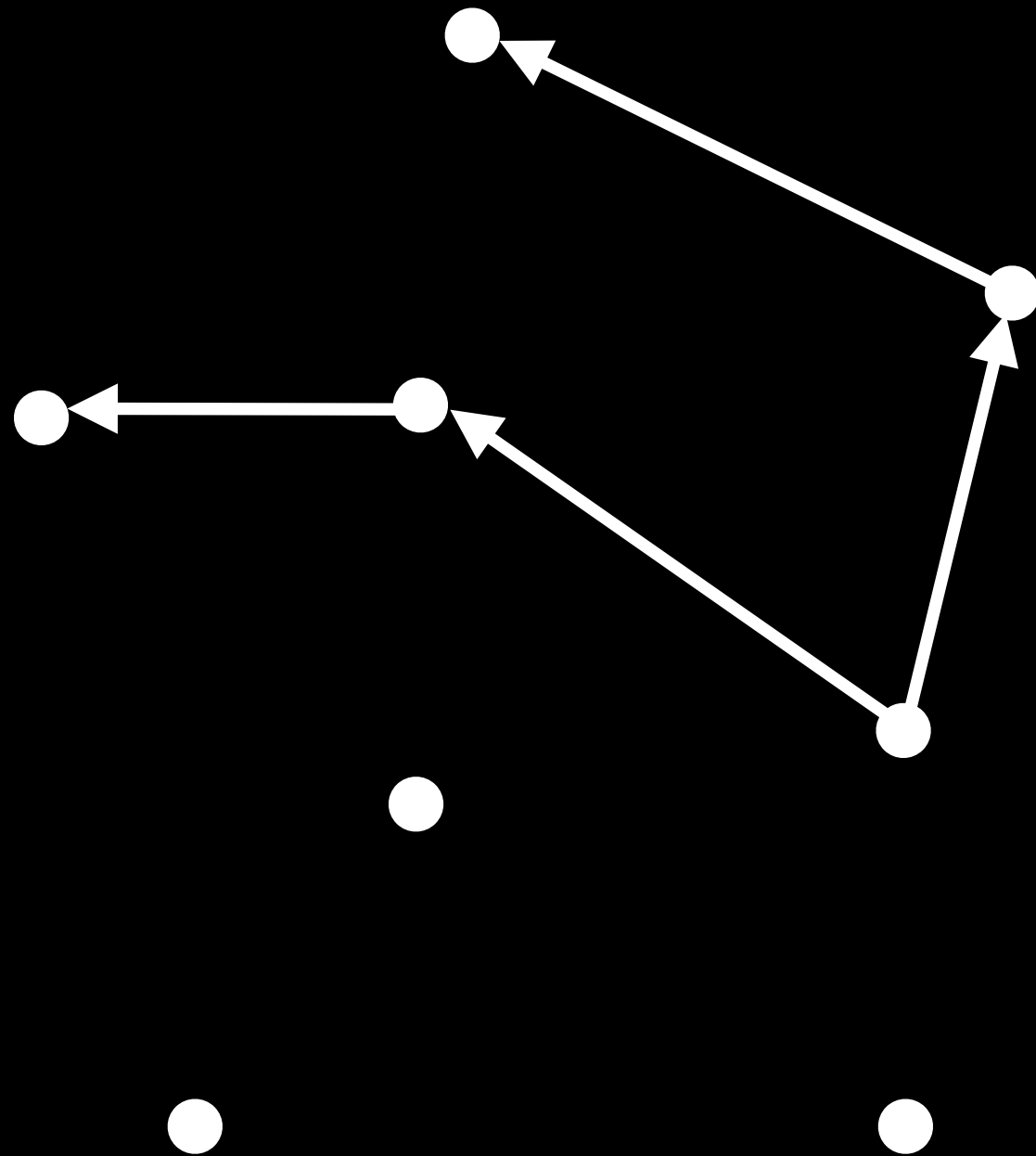


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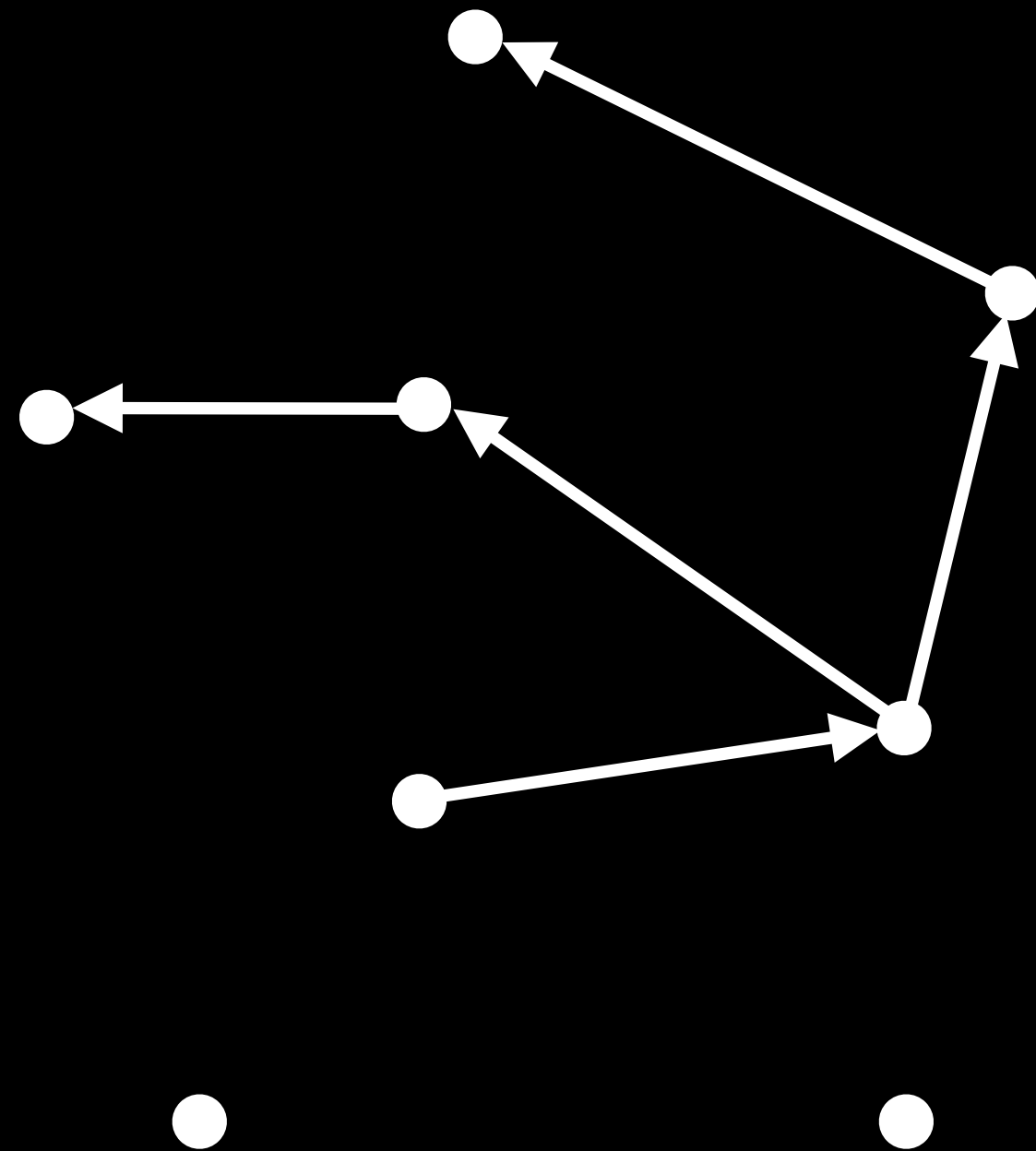


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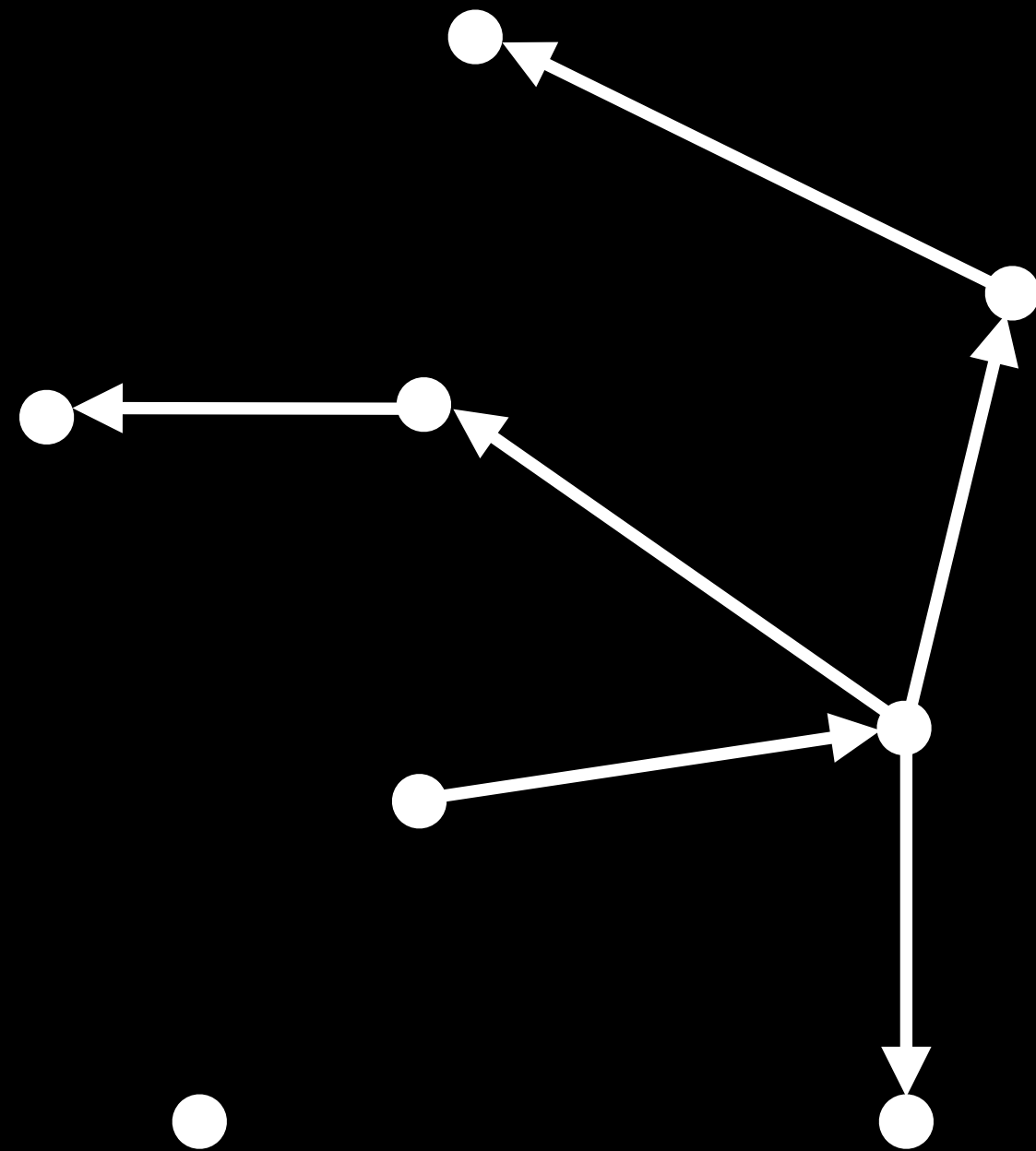


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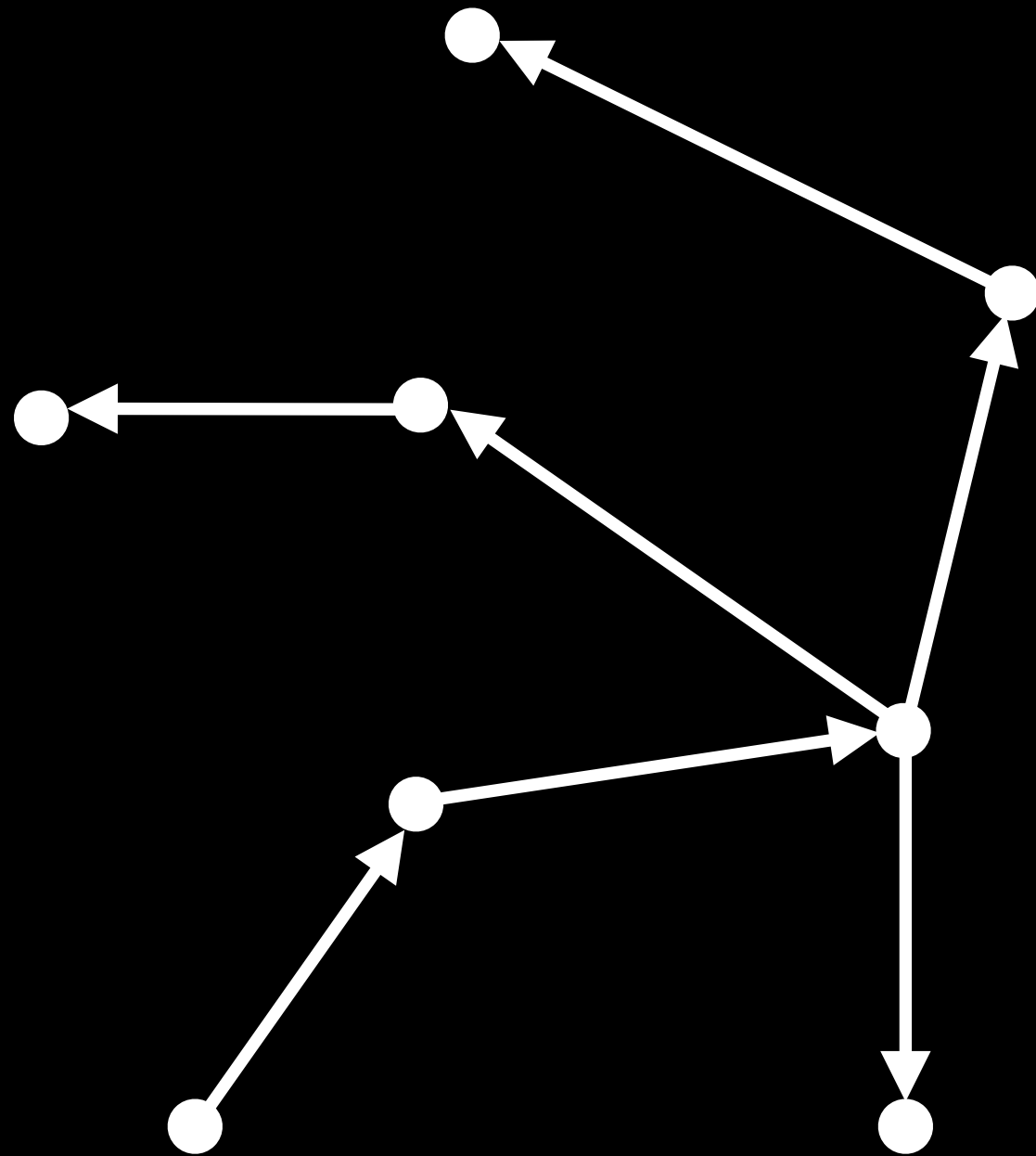


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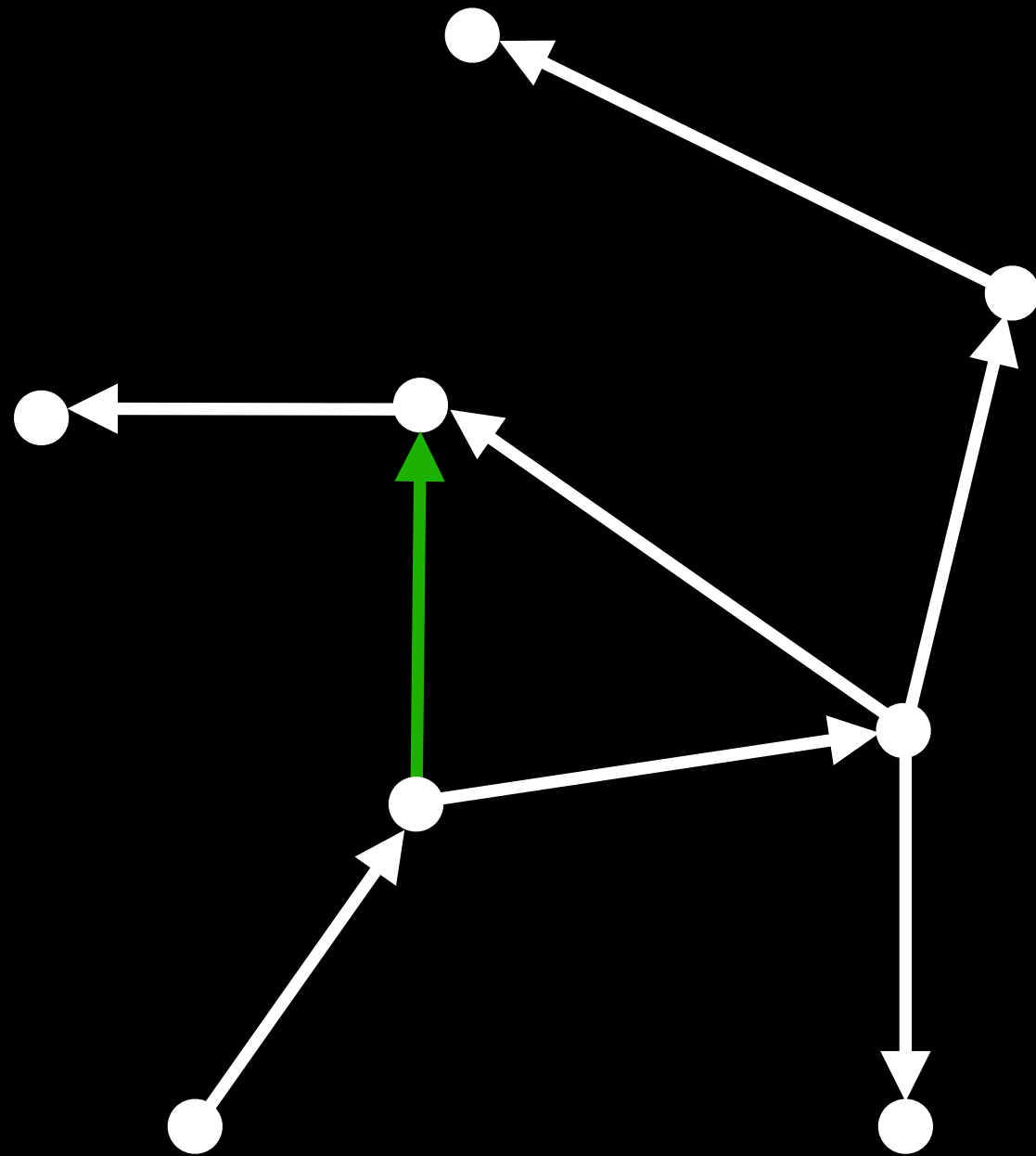
Timestamped edges  
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# Choosing to close triangles

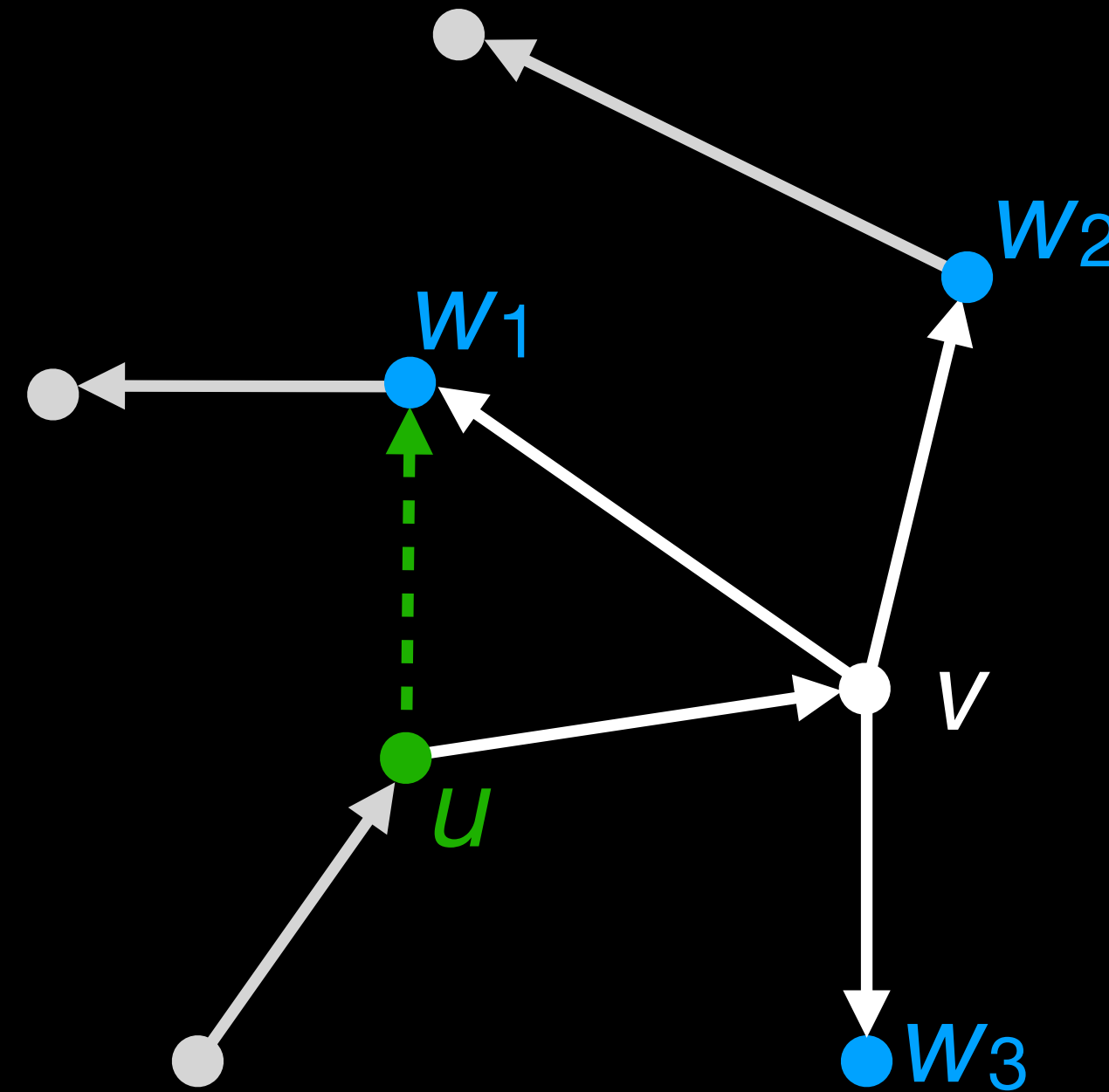
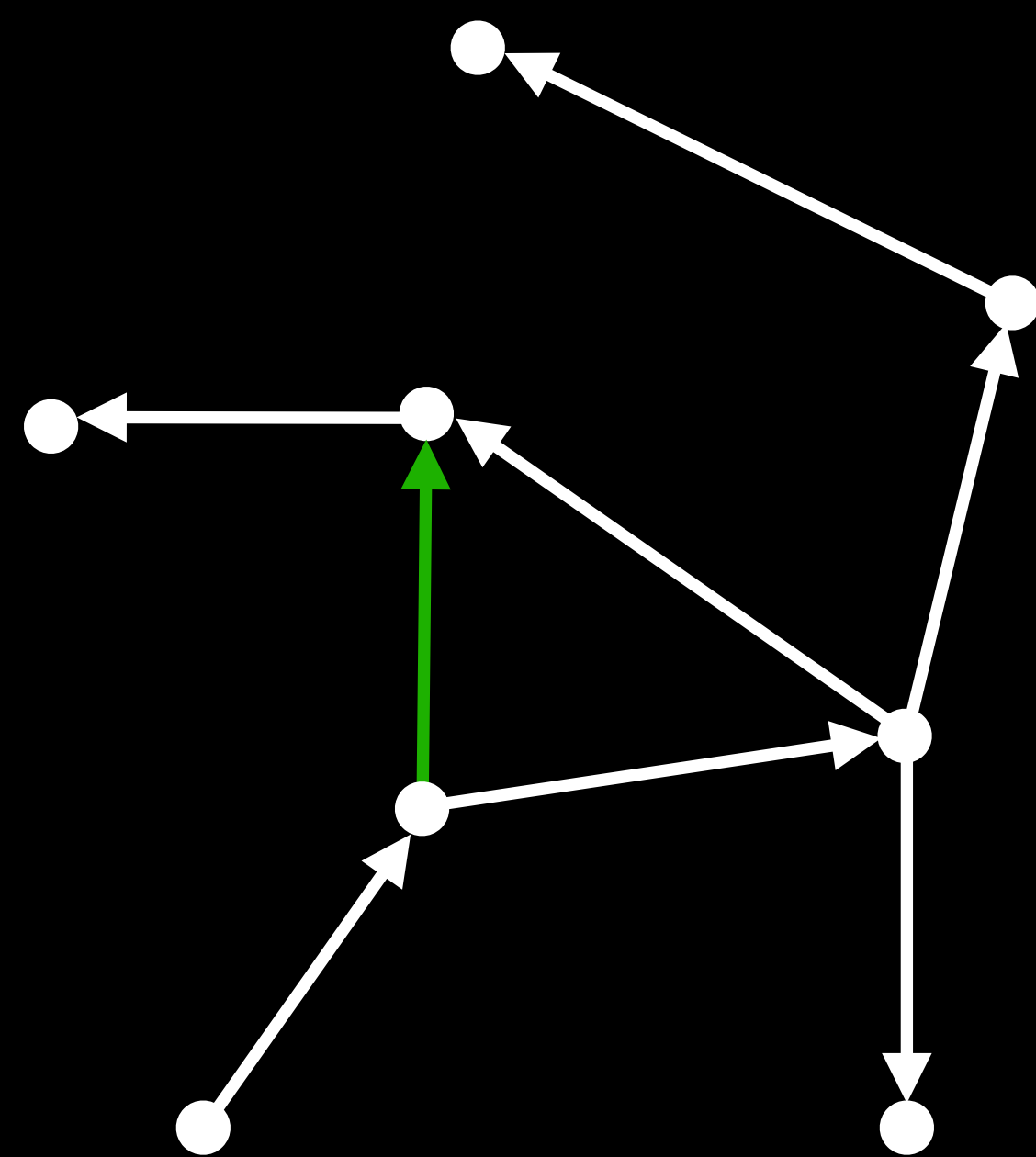
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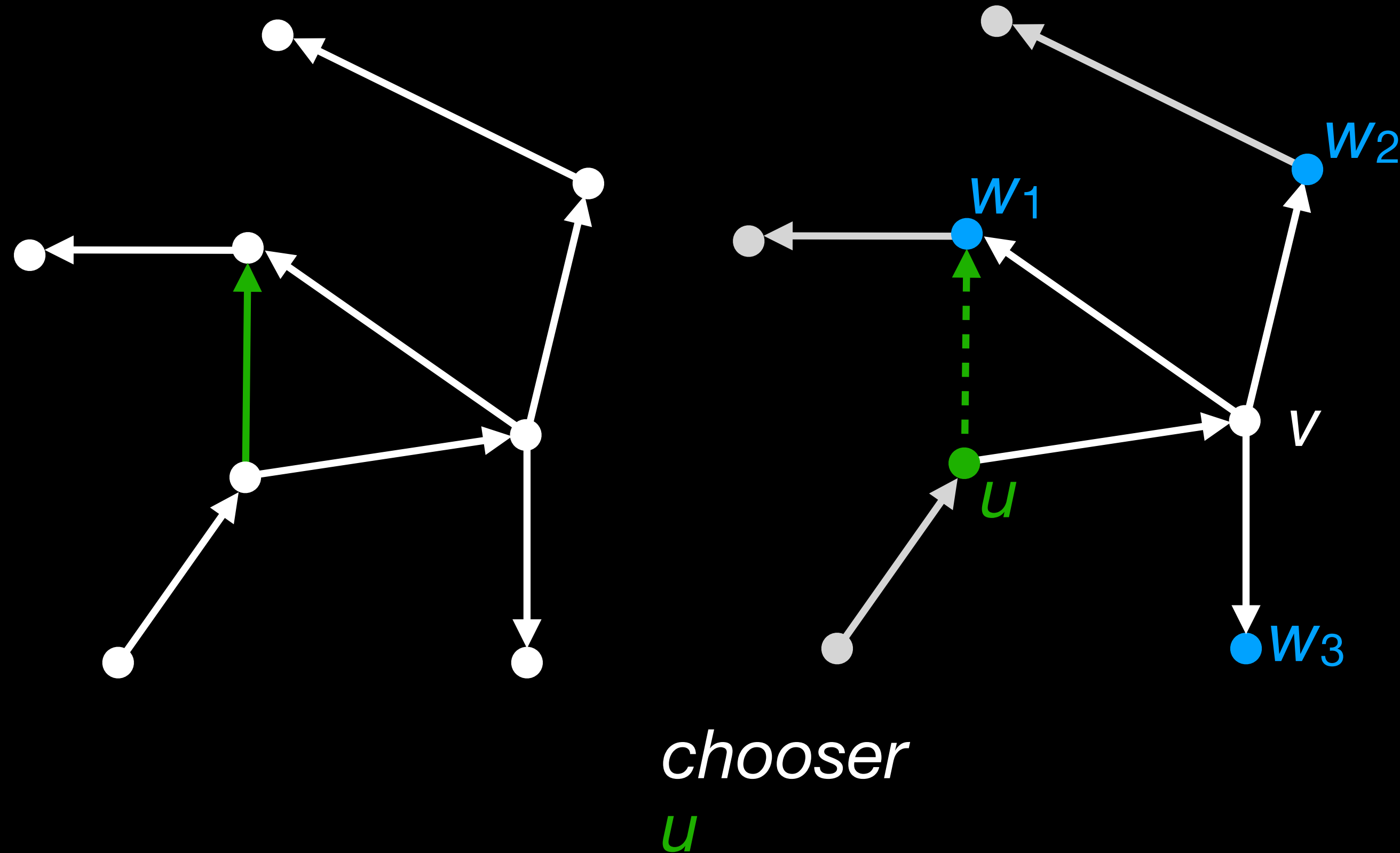
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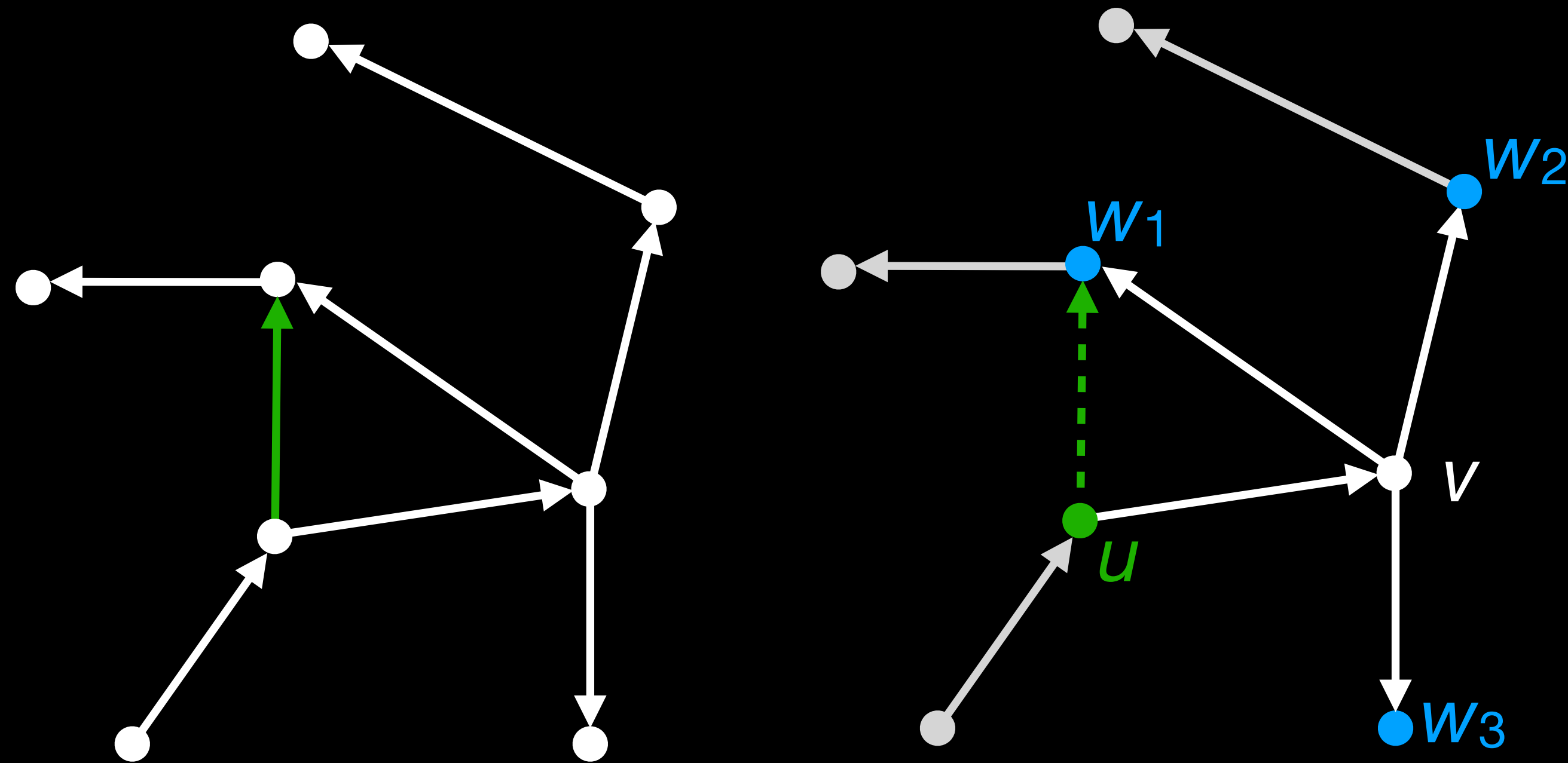
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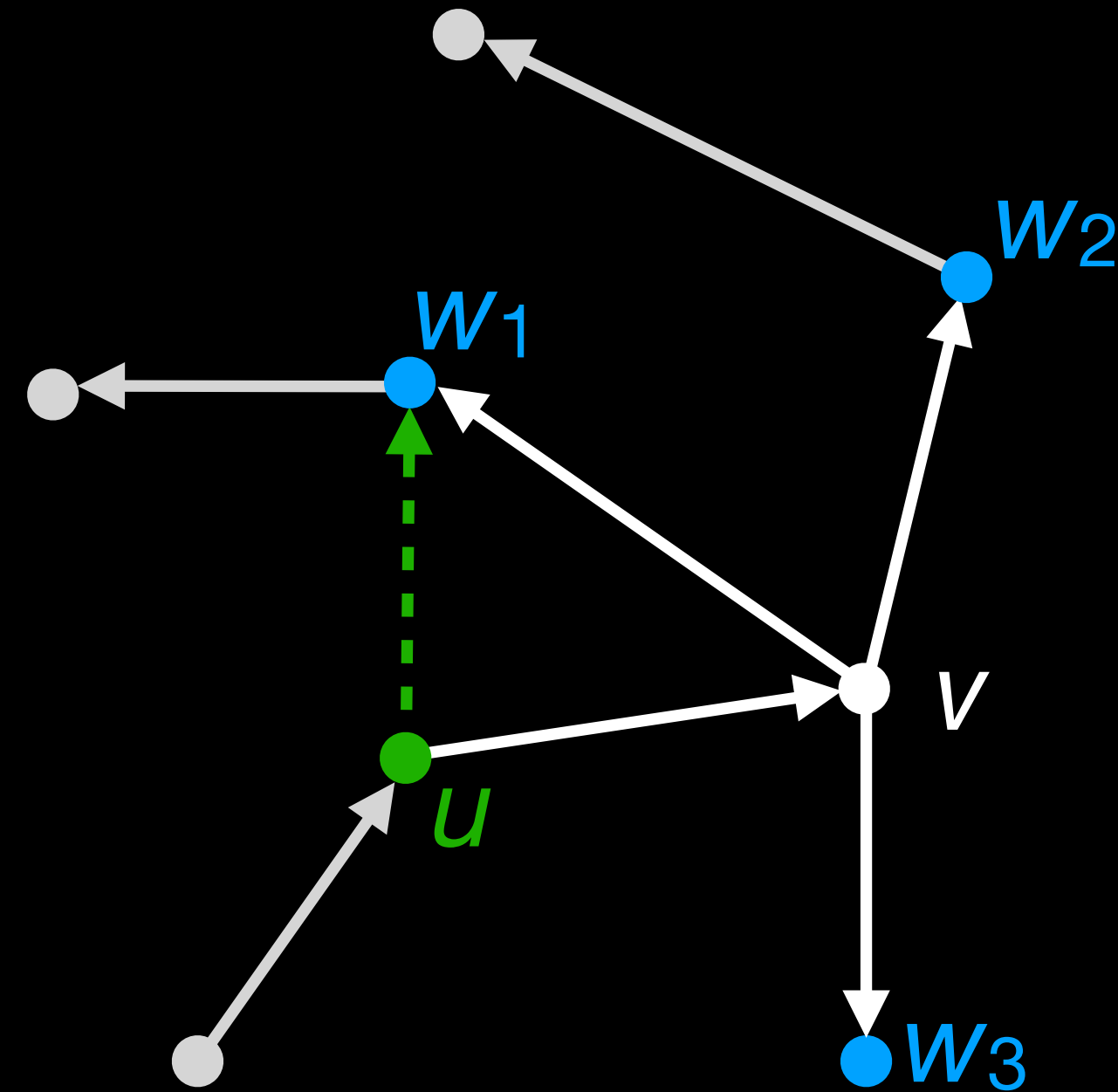
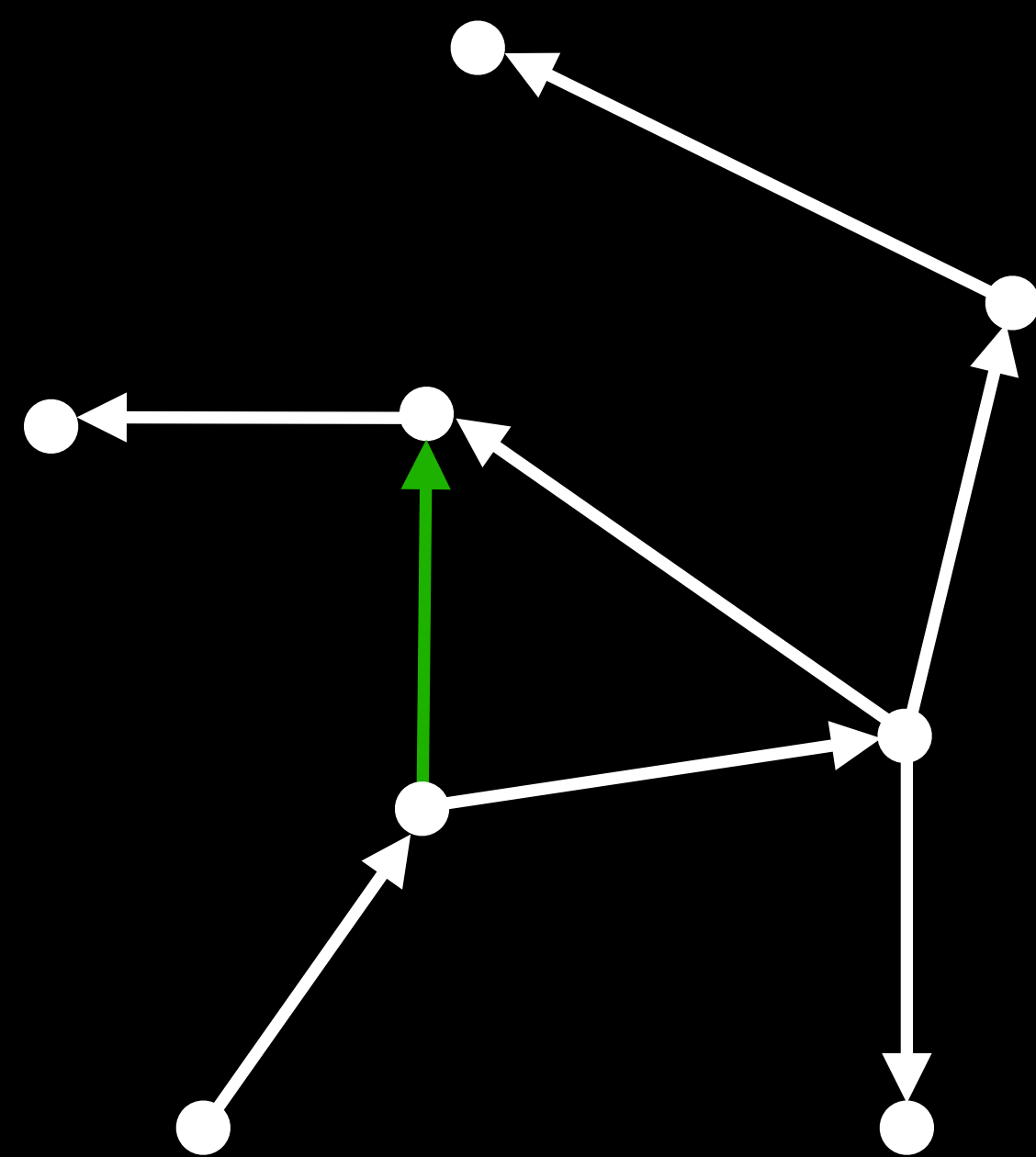
*chooser*  
 $u$

*choice set*  
 $\{W_1, W_2, W_3\}$

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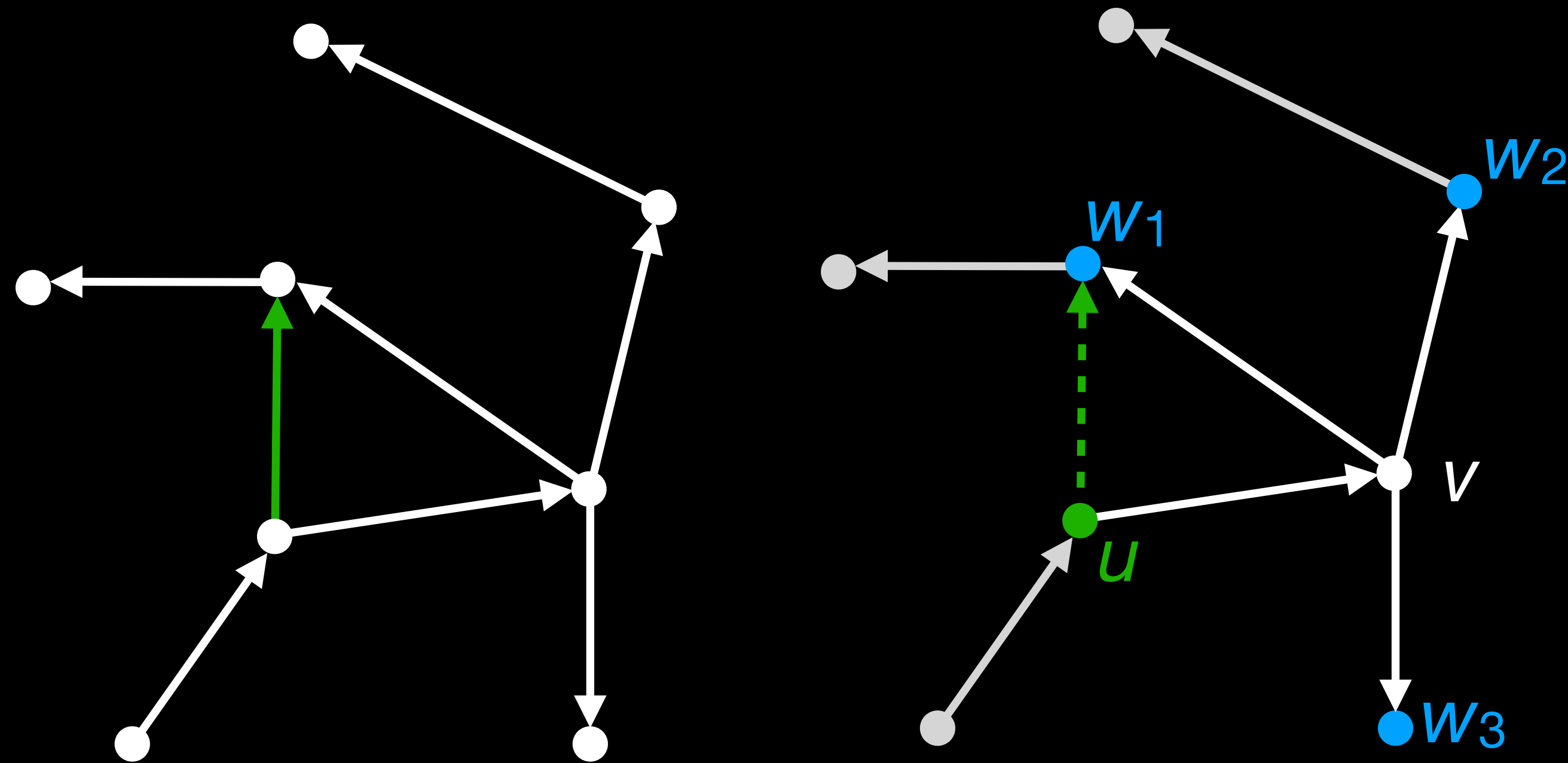
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 $u$

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## Our data

Timestamped edges  
(including repeats)

## Node features

1. in-degree of  $w$
2. # shared neighbors of  $u, w$
3. weight of edge  $w \rightarrow u$
4. time since last edge into  $w$
5. time since last edge out of  $w$
6. time since last  $w \rightarrow u$  edge



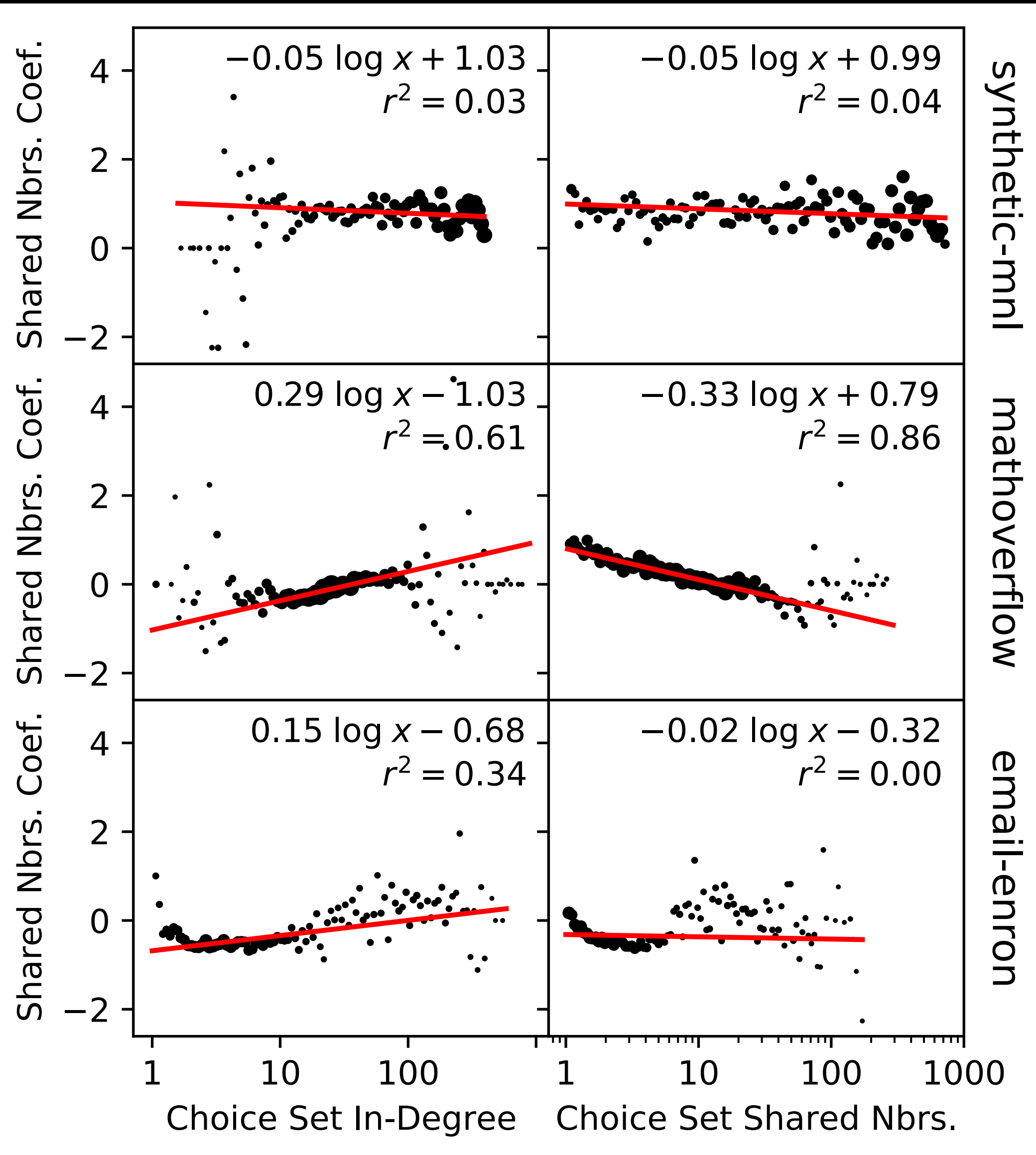
# Context matters in triadic closure

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## Datasets

email-enron  
email-eu  
email-w3c  
wiki-talk  
reddit-hyperlink  
bitcoin-alpha  
bitcoin-otc  
mathoverflow  
college-msg  
facebook-wall  
sms-a  
sms-b  
sms-c

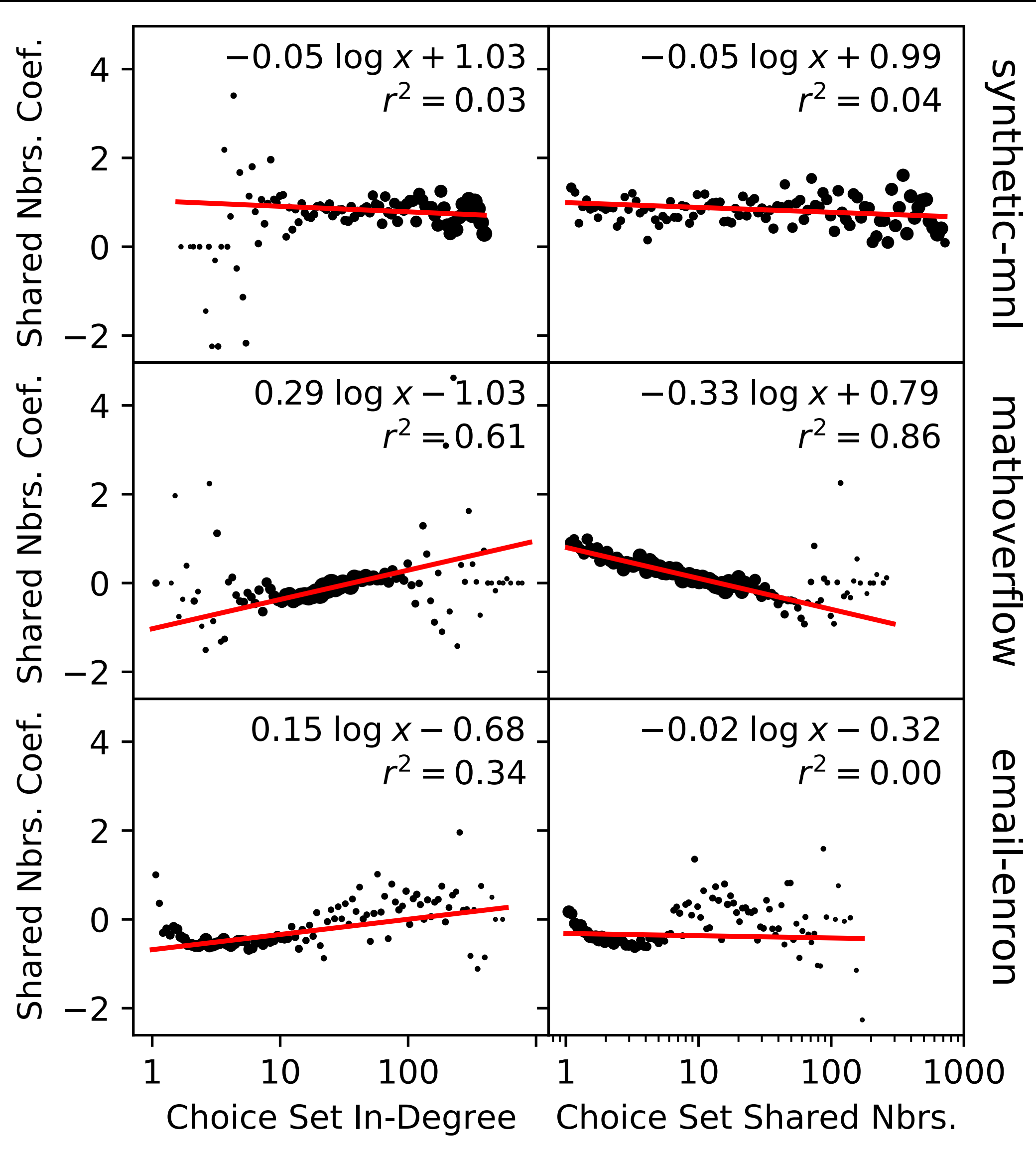
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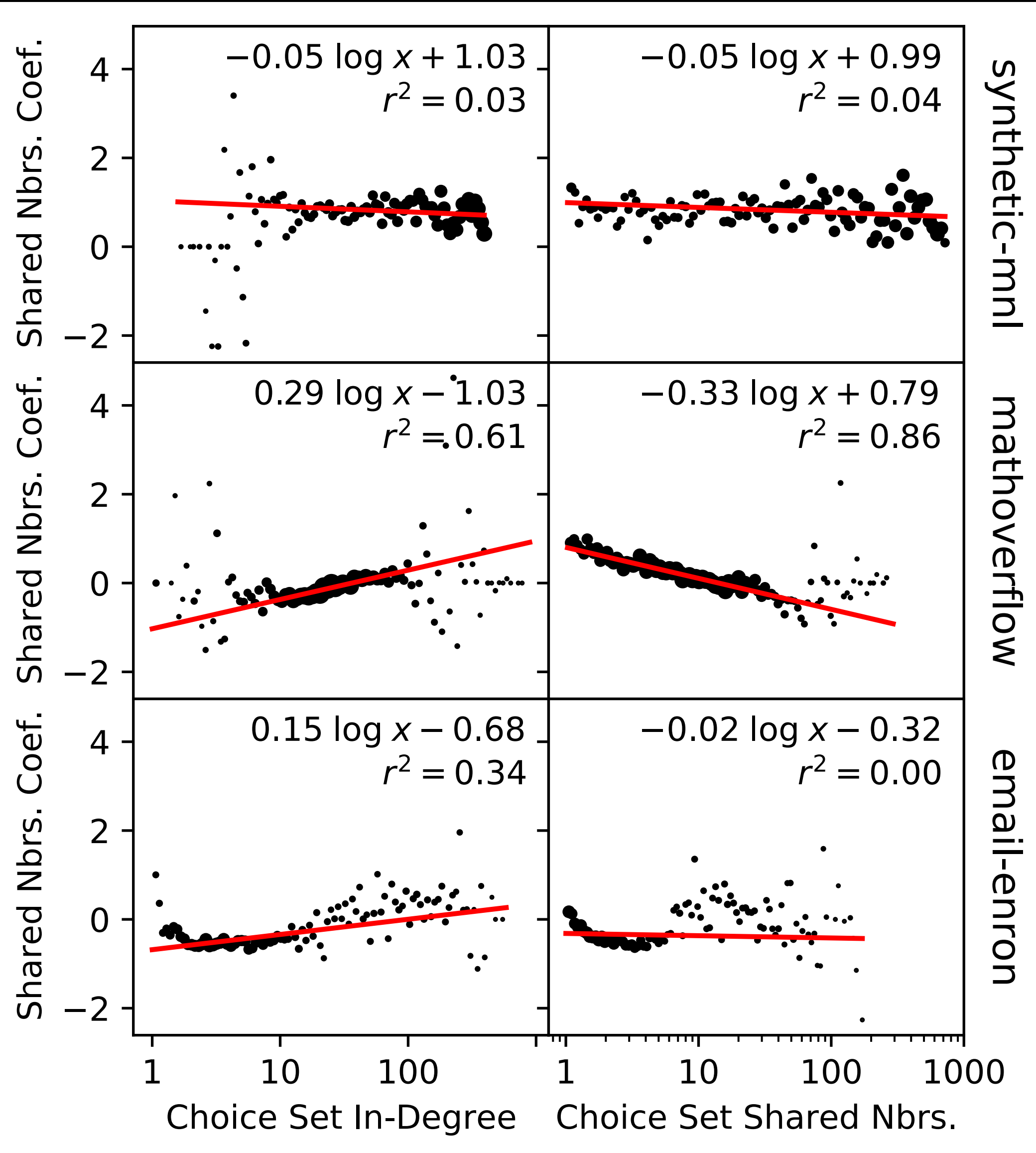
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Synthetic data,  
no context effects



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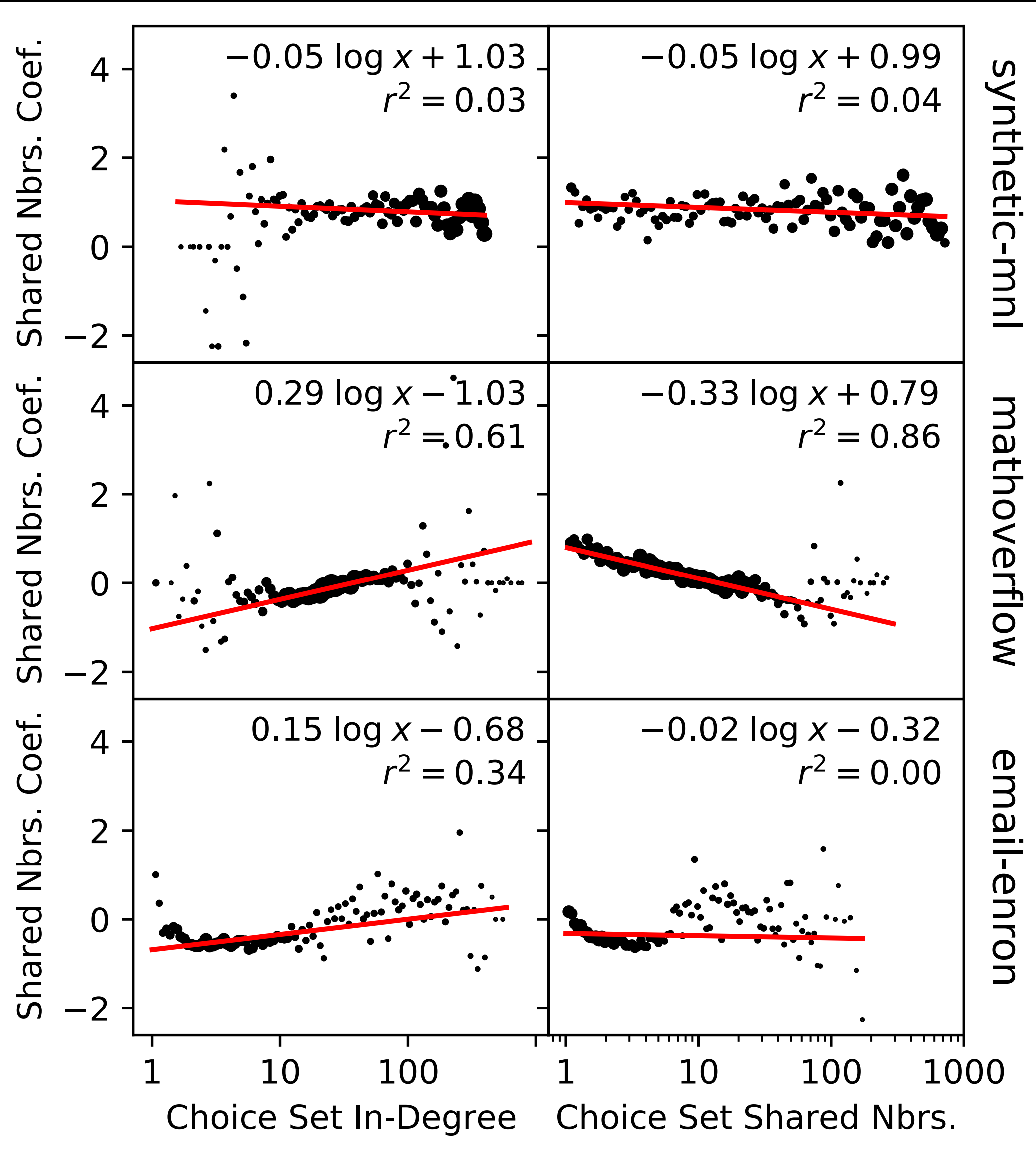


Synthetic data,  
no context effects

Commenting network,  
linear context effects

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# Context matters in triadic closure



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no context effects

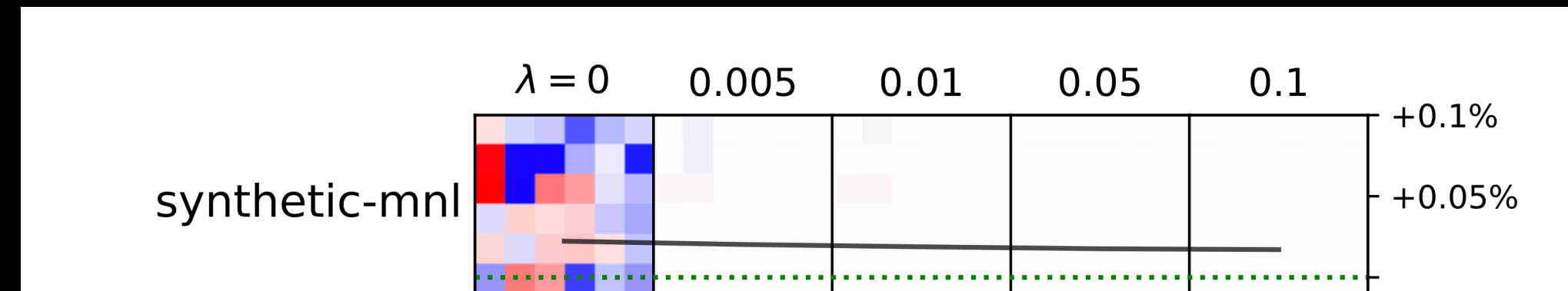
Commenting network,  
linear context effects

Email network,  
nonlinear context effects?

- Datasets**
- email-enron
  - email-eu
  - email-w3c
  - wiki-talk
  - reddit-hyperlink
  - bitcoin-alpha
  - bitcoin-otc
  - mathoverflow
  - college-msg
  - facebook-wall
  - sms-a
  - sms-b
  - sms-c

# LCL reveals interpretable feature context effects

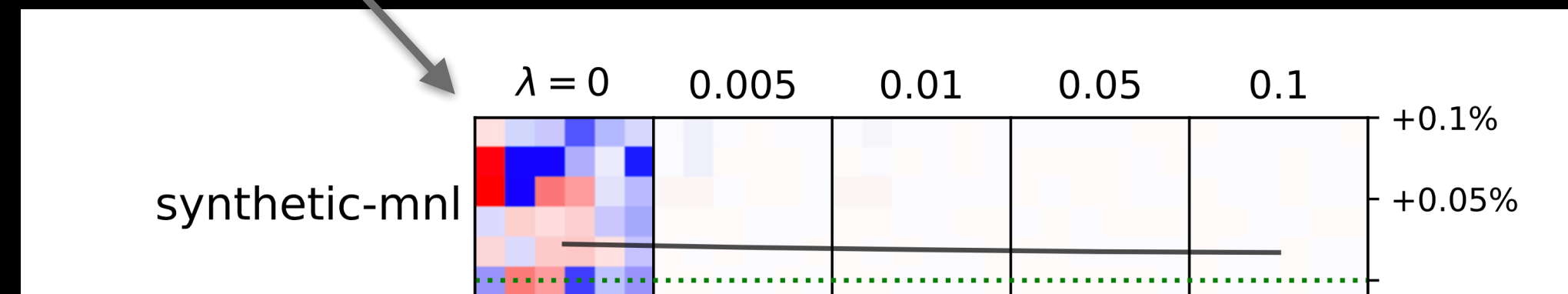
# LCL reveals interpretable feature context effects





# LCL reveals interpretable feature context effects

context effect matrix  $A$   
red: +, blue: -, white: 0  
(column acts on row)



## Node features

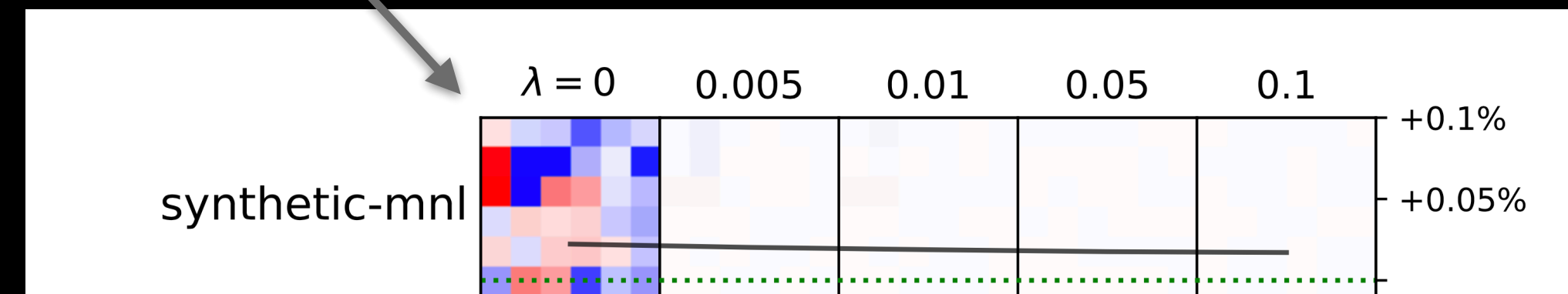
(left-right, top-bottom)

1. in-degree
2. shared neighbors
3. reciprocal weight
4. send recency
5. receive recency
6. reciprocal recency

# LCL reveals interpretable feature context effects

context effect matrix  $A$   
red: +, blue: -, white: 0  
(column acts on row)

increasing  $L_1$  regularization on  $A$



## Node features

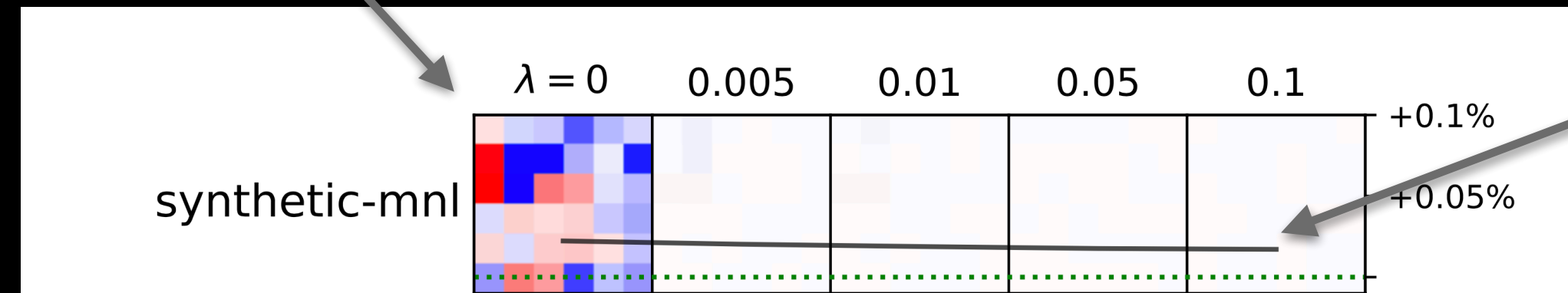
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NLL (lower = better fit)

## Node features

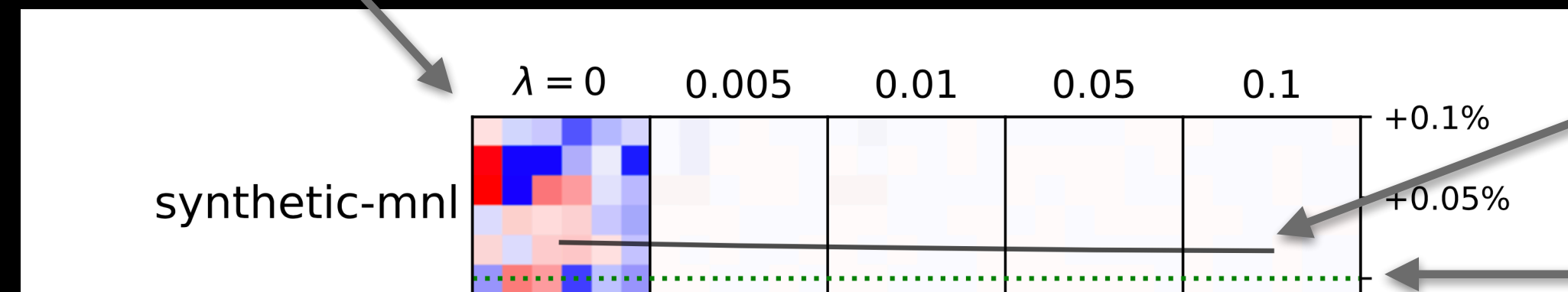
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$p = 0.001$  LRT threshold

## Node features

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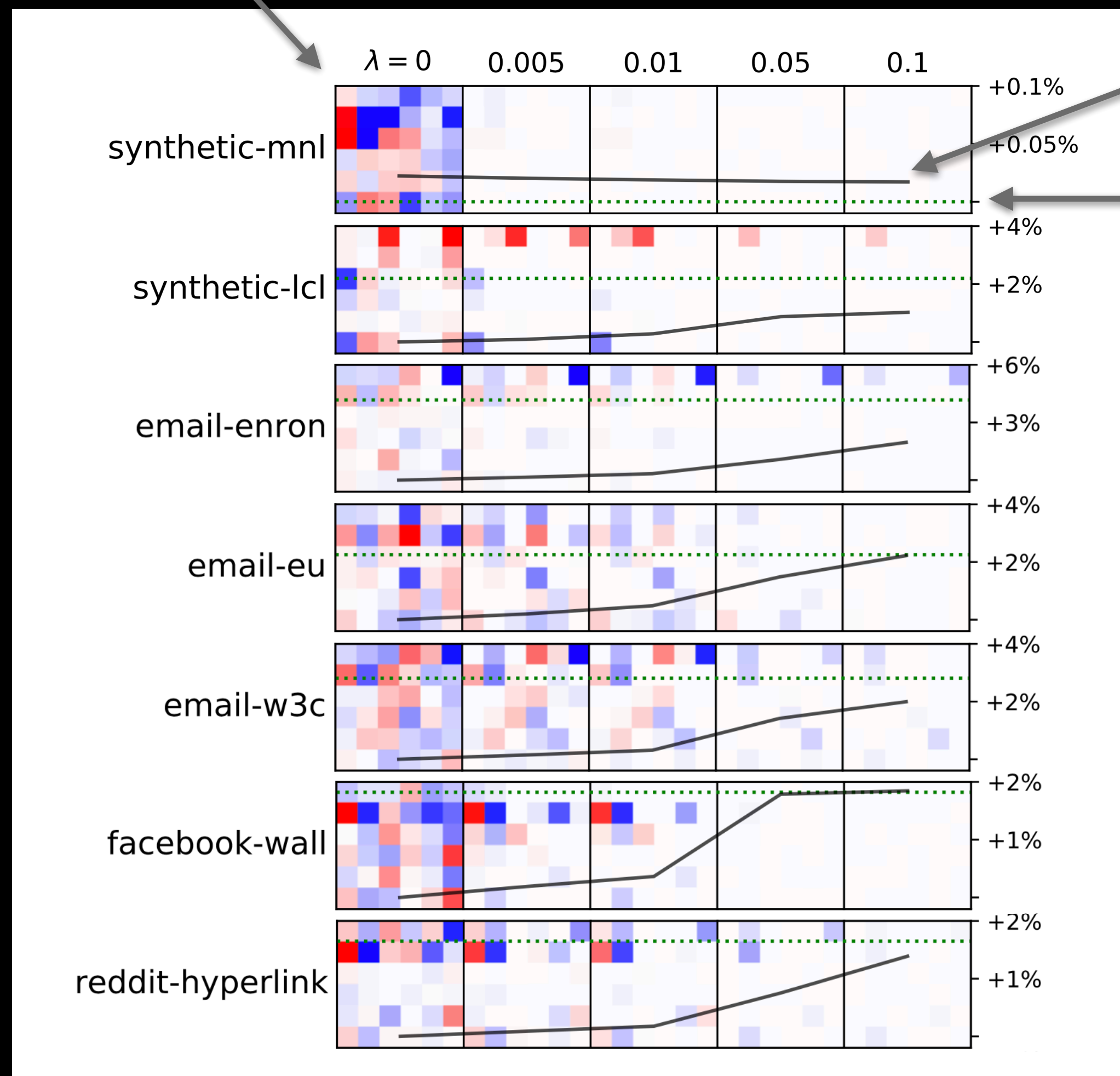
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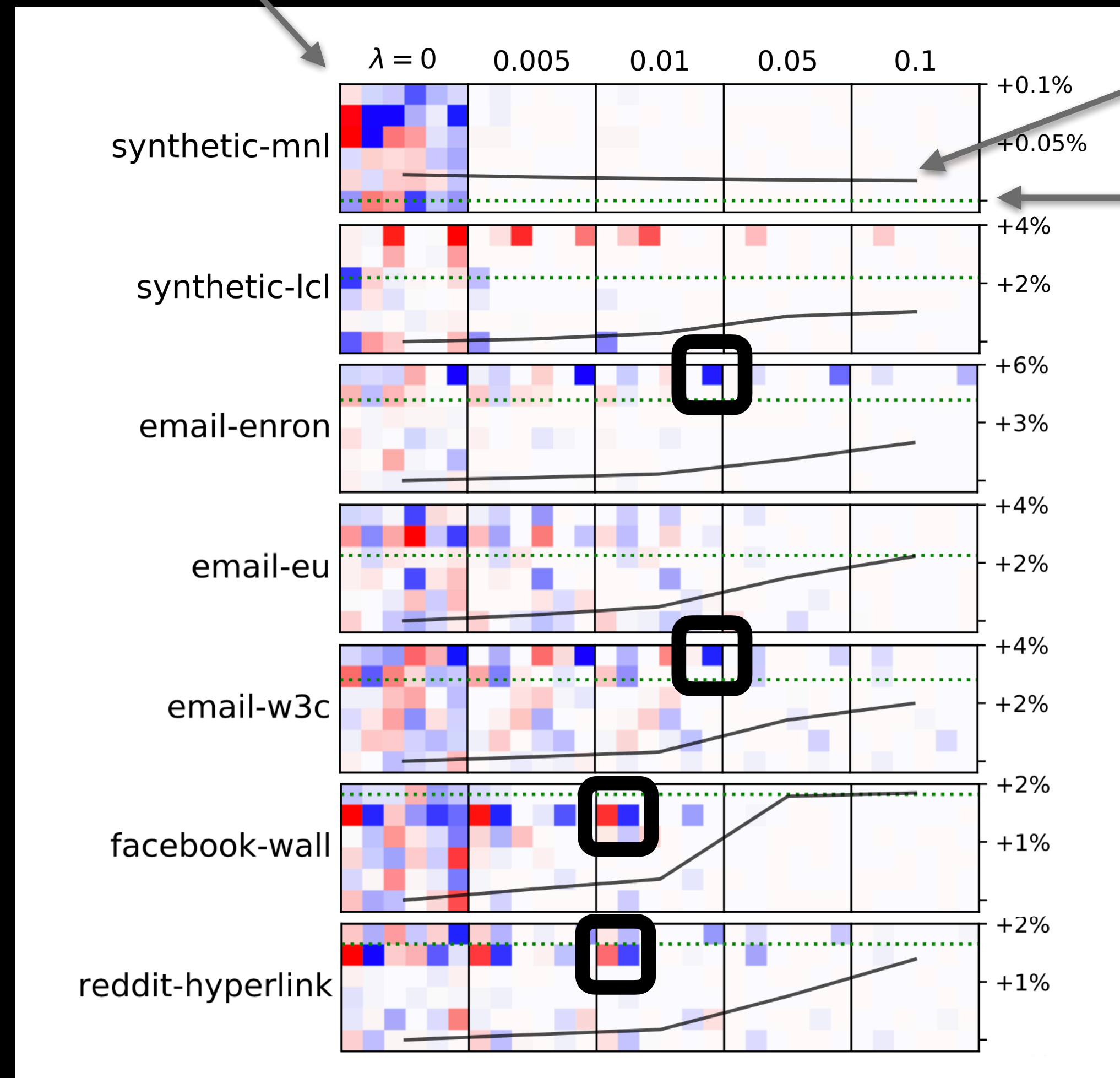
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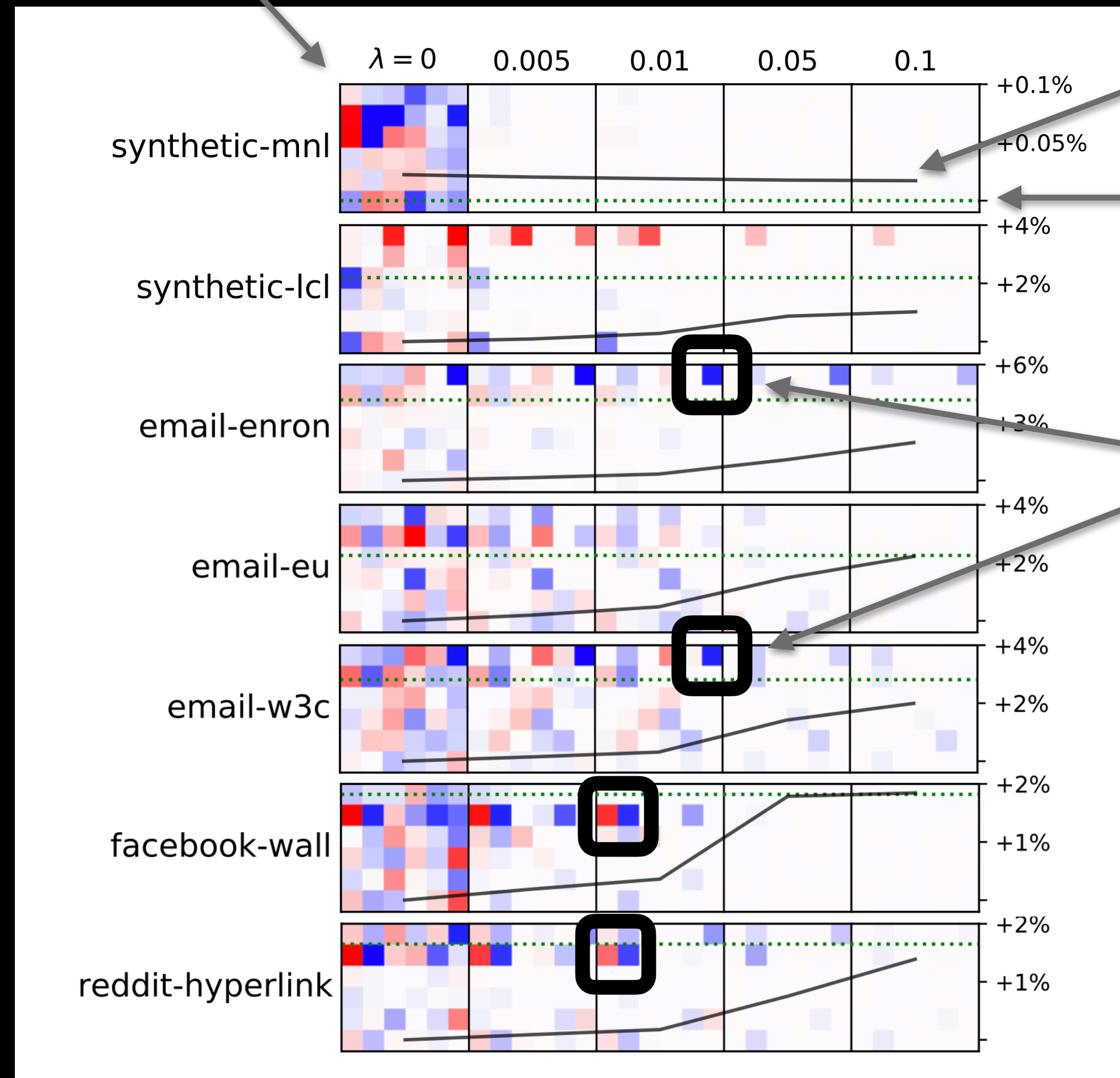
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## Node features (left-right, top-bottom)

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2. shared neighbors
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4. send recency
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NLL (lower = better fit)

$p = 0.001$  LRT threshold

“cluttered inbox”  
 high choice set reciprocal recency  
 → in-degree less important



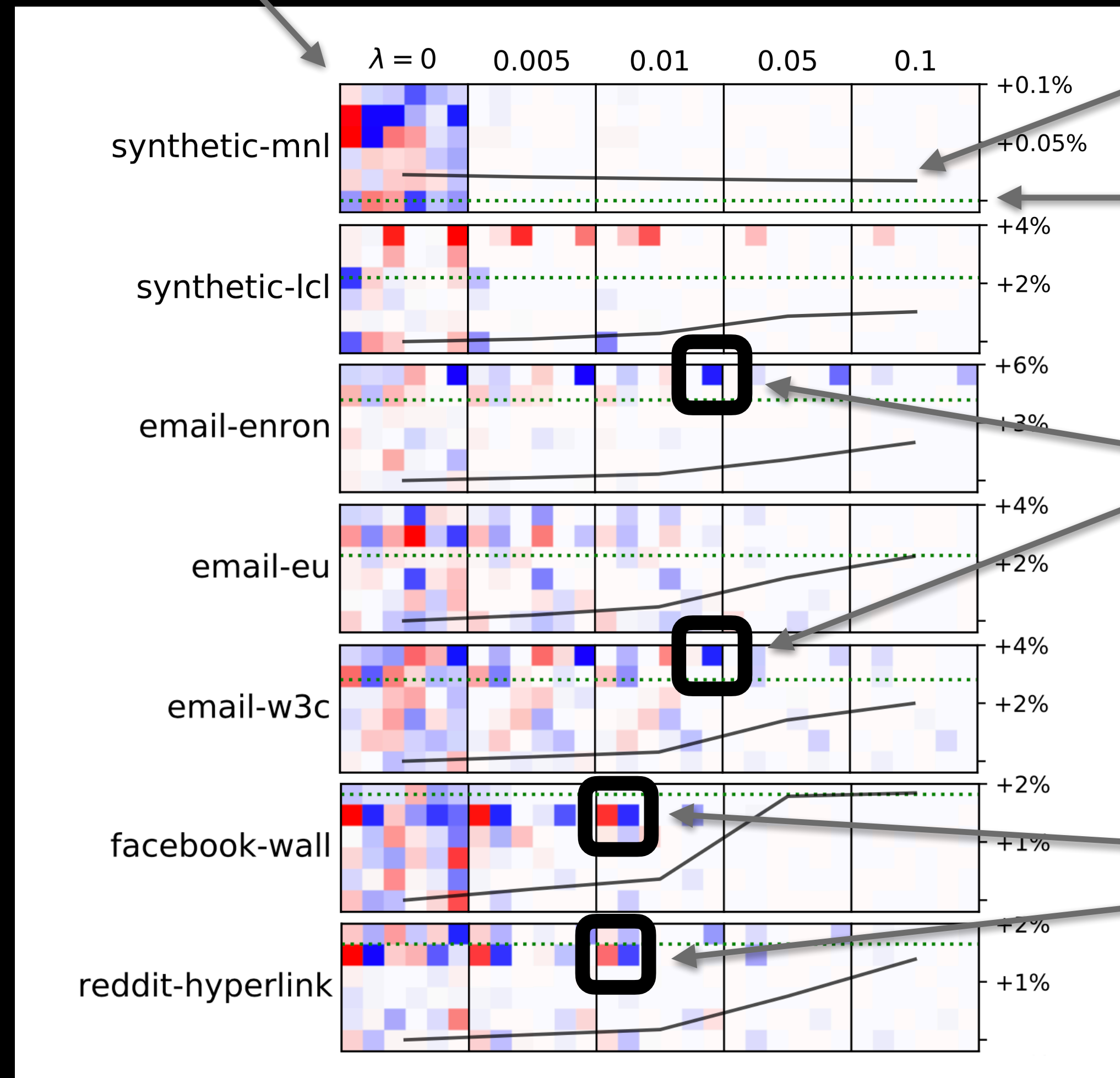
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increasing  $L_1$  regularization on  $A$

## Node features (left-right, top-bottom)

1. in-degree
2. shared neighbors
3. reciprocal weight
4. send recency
5. receive recency
6. reciprocal recency



NLL (lower = better fit)

$p = 0.001$  LRT threshold

*“cluttered inbox”*

high choice set reciprocal recency  
 → in-degree less important

red: *“cocktail party introduction”*

high choice set in-degree  
 → shared neighbors more important

blue: *“familiarity saturation”*

high choice set shared neighbors  
 → shared neighbors less important



# Concluding thoughts

Code: [bit.ly/lcl-code](https://bit.ly/lcl-code)  
Data: [bit.ly/lcl-data](https://bit.ly/lcl-data)  
Slides: [bit.ly/lcl-kdd-slides](https://bit.ly/lcl-kdd-slides)

## Key takeaways

*Feature context effects* extend item-level effects  
LCL offers an interpretable and tractable way to reveal them

## Future work

Non-linear context effects  
Negative sampling  
Discovering relational effects

## Causal context effects?

See our other KDD '21 paper:  
“Choice Set Confounding in Discrete Choice”

**Submit to our NeurIPS '21 workshop!**  
[bit.ly/WHMD2021](https://bit.ly/WHMD2021)

## Thank you!

More questions or ideas?  
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 @kiran\_tomlinson

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